

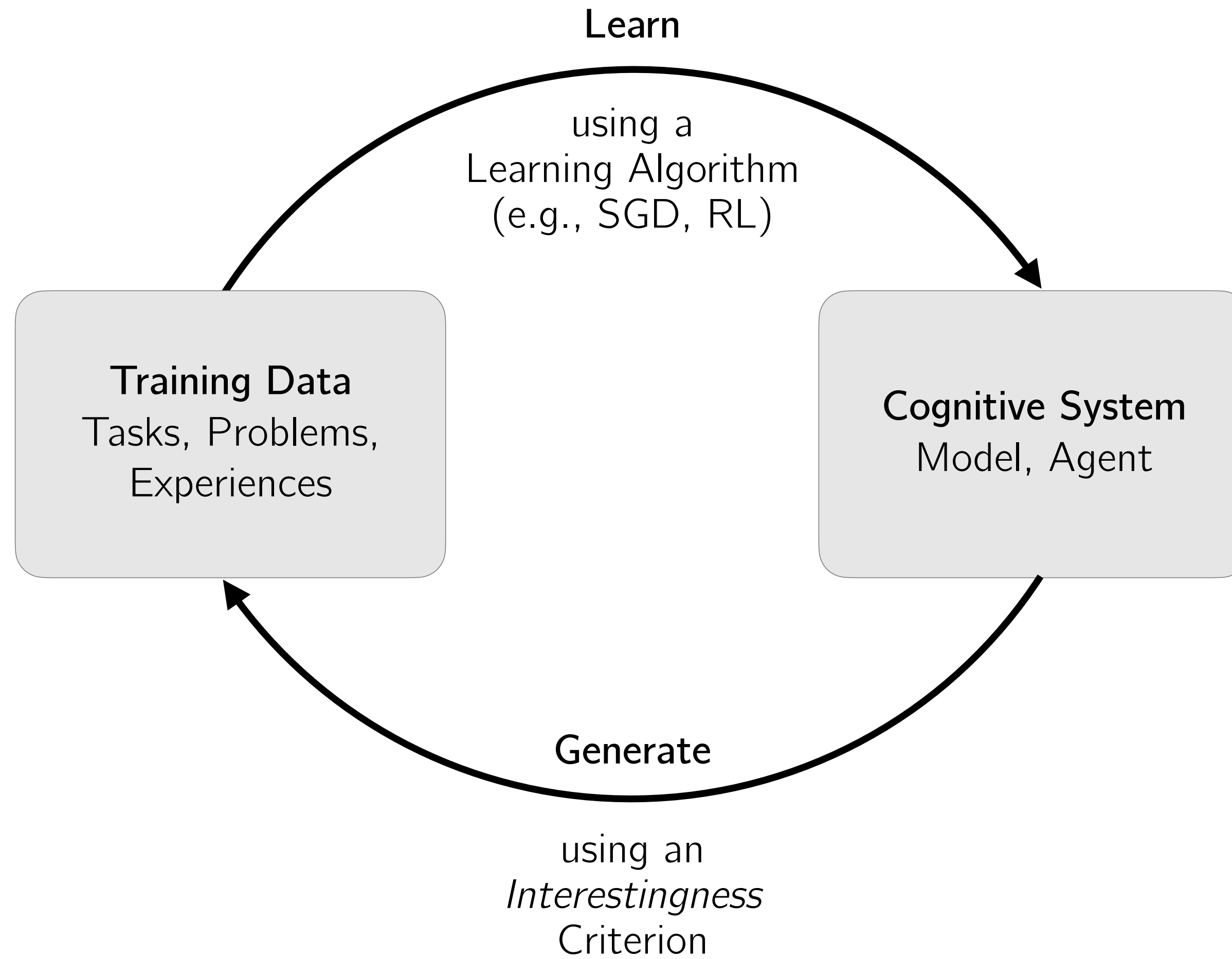
# Position: Interestingness is an Inductive Heuristic for Future Compression Progress



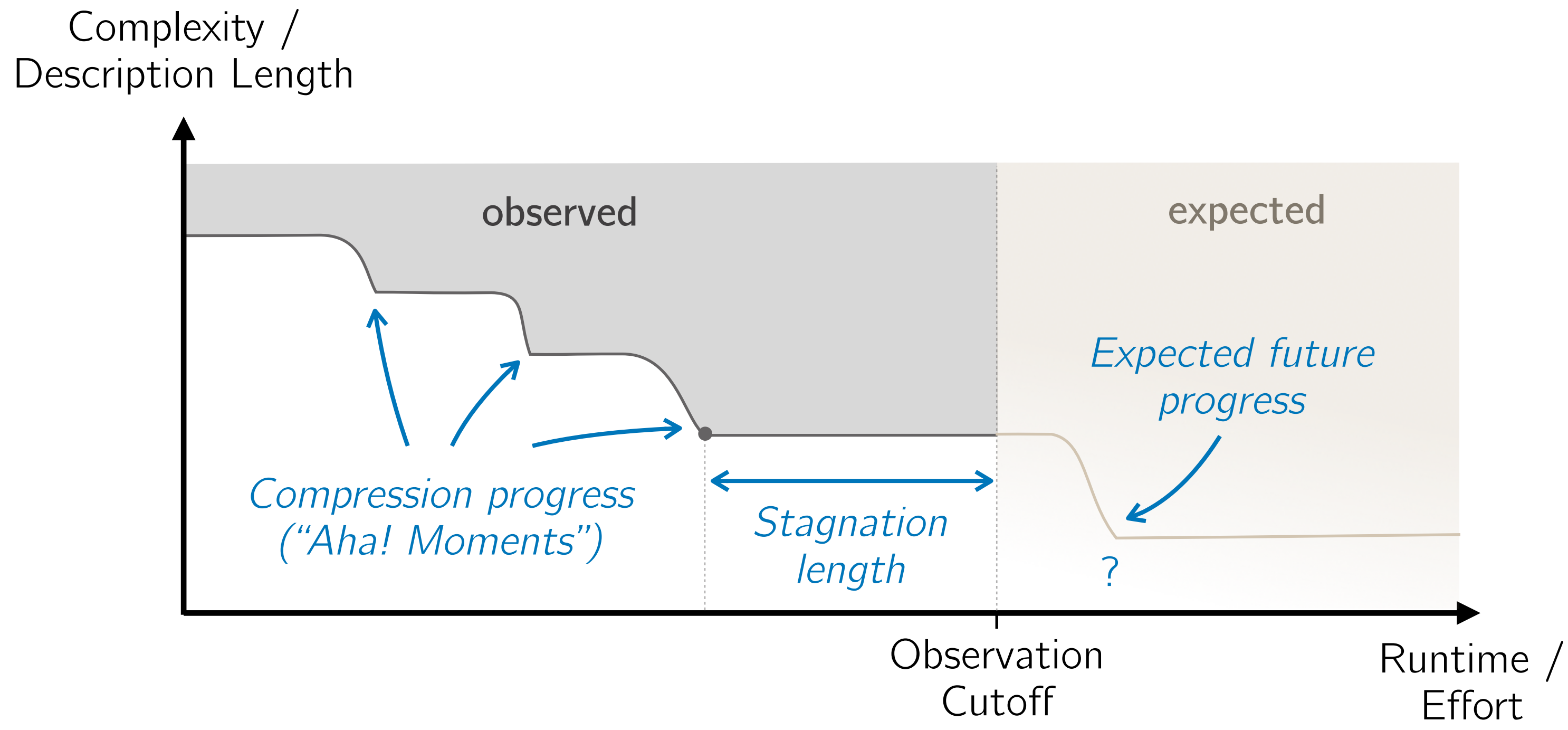
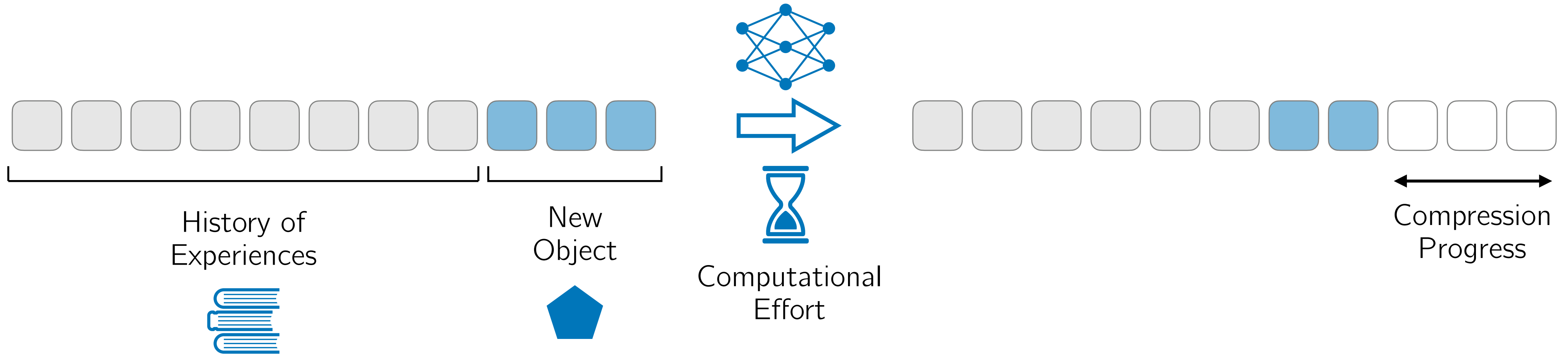
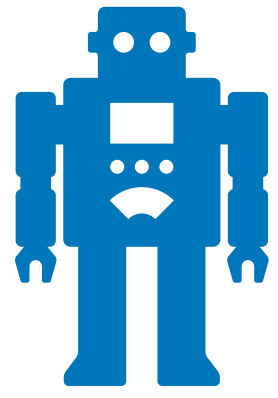
Vincent  
Herrmann



Jürgen  
Schmidhuber

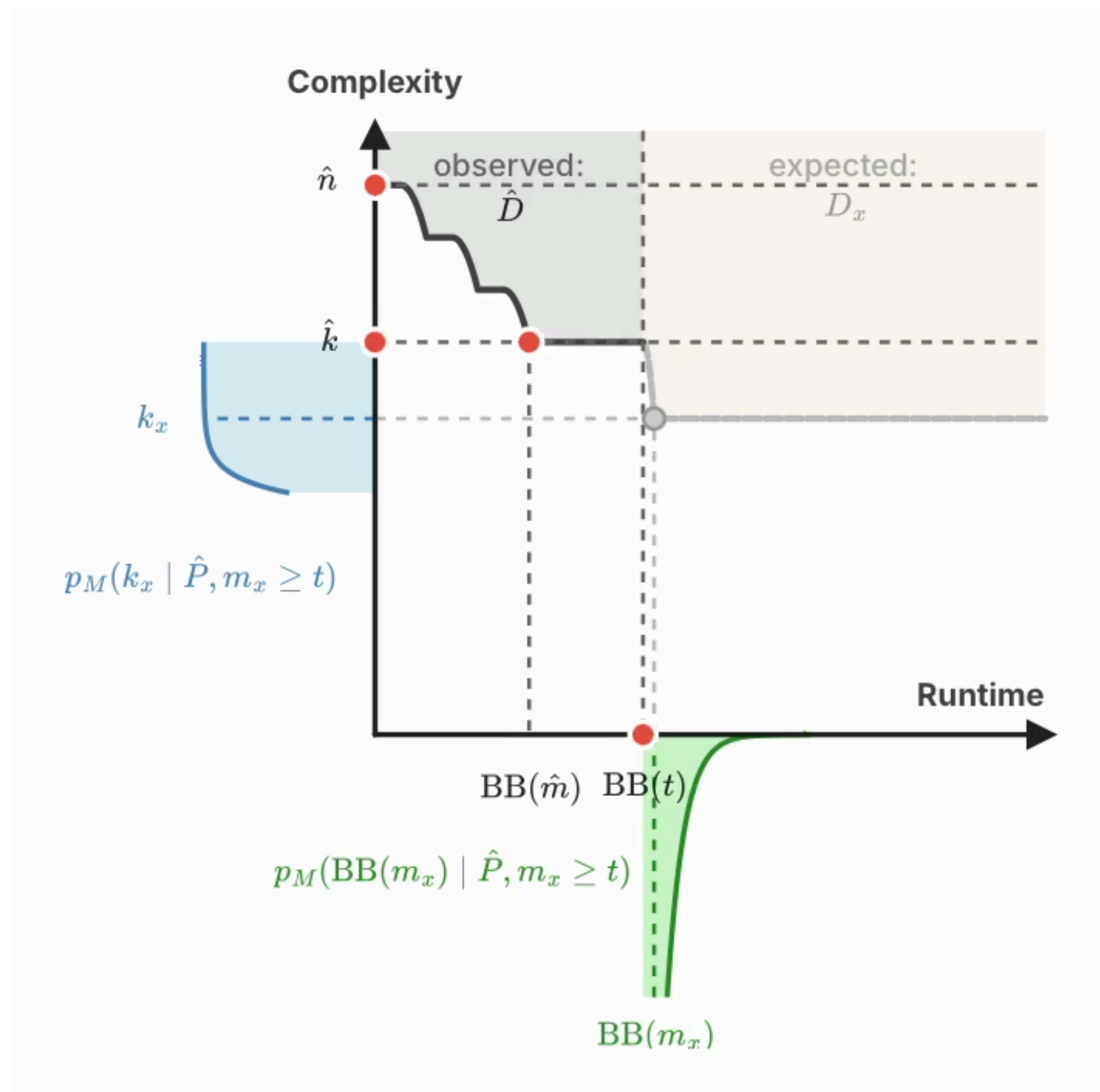


*Interestingness* is the expected future compression progress under further engagement



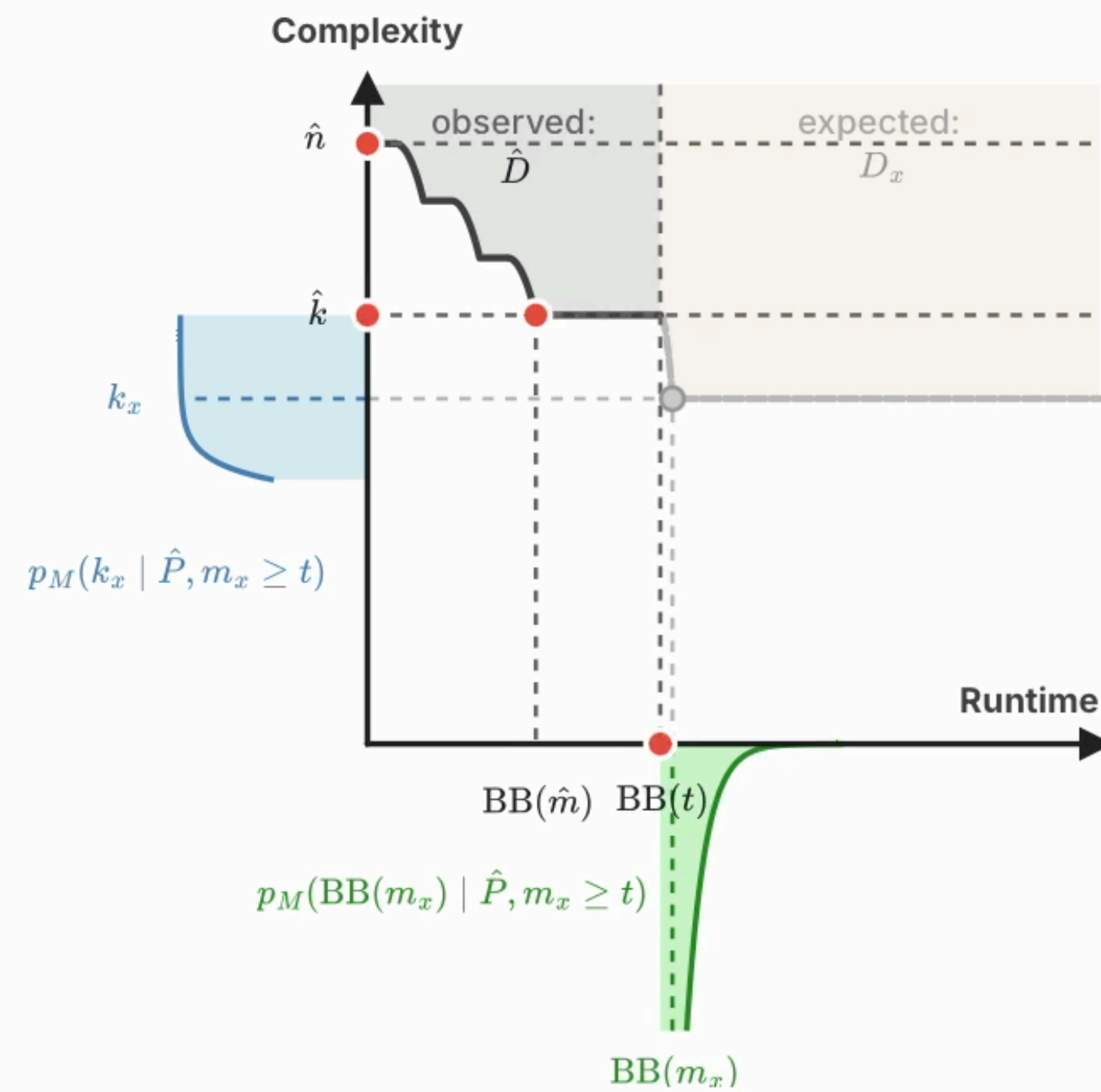
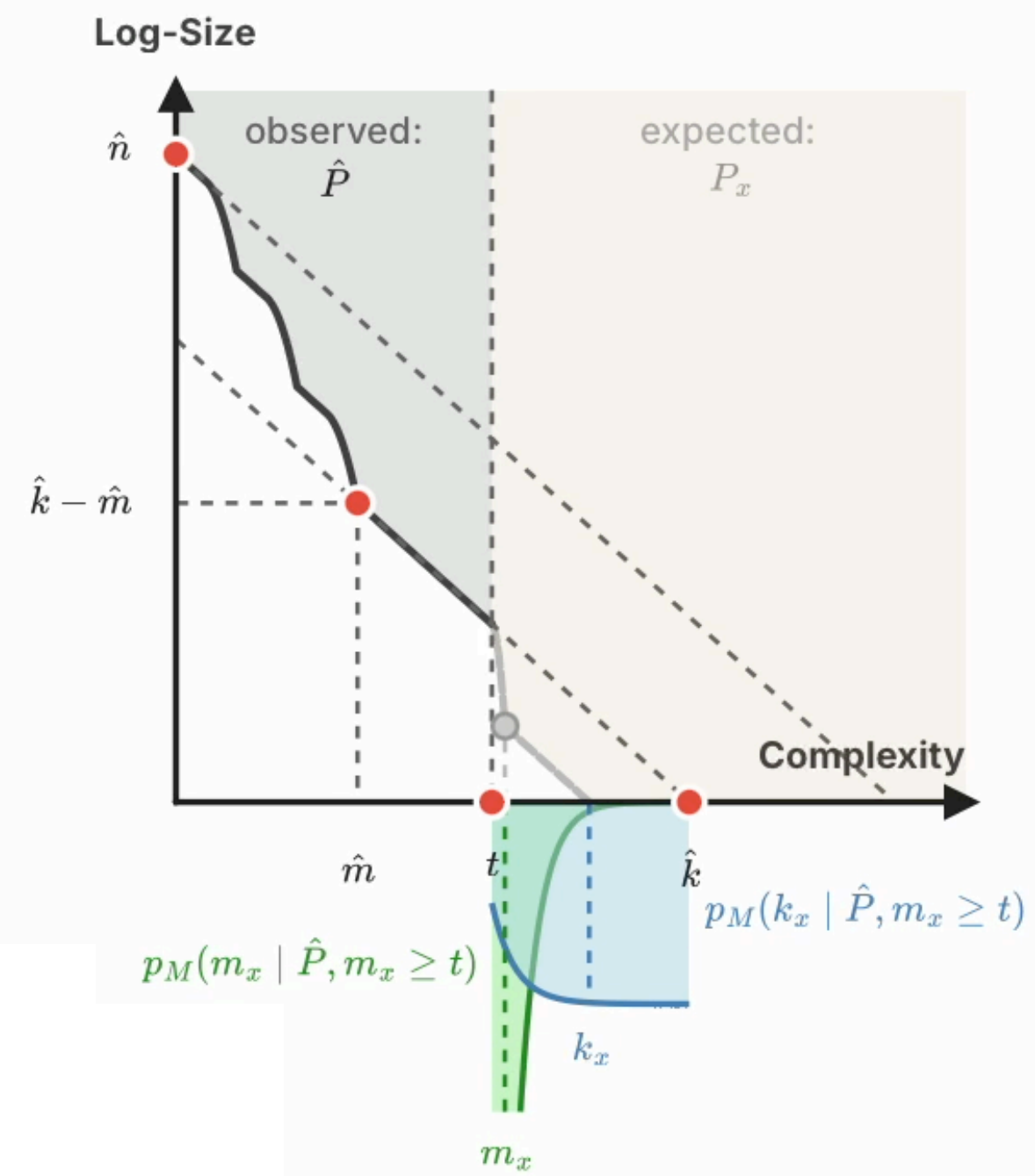
Algorithmic Prior over all computable objects:

$$M(x) = \sum_{p:U(p)=x} 2^{-|p|}$$



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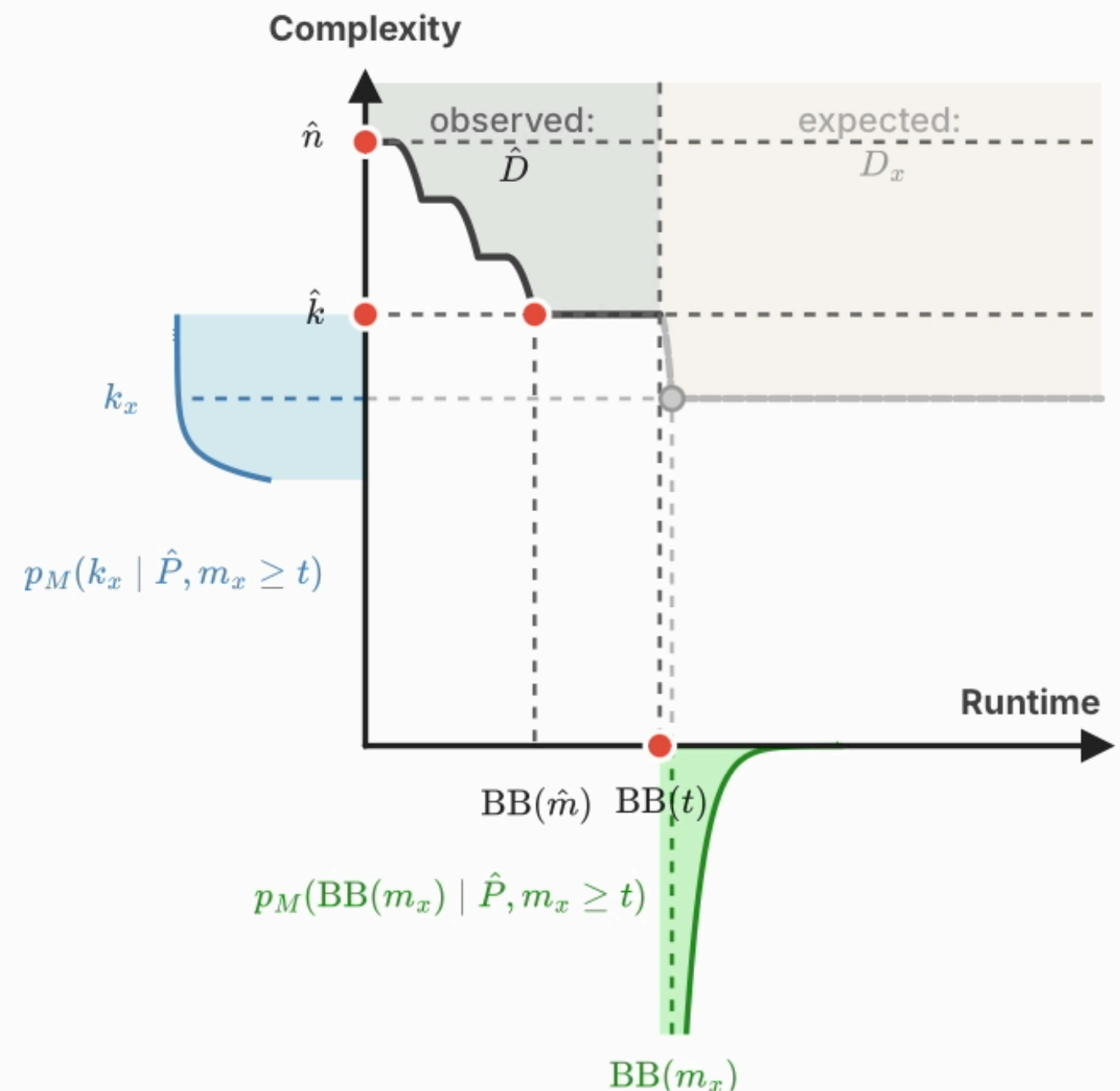


Algorithmic Prior over all computable objects:

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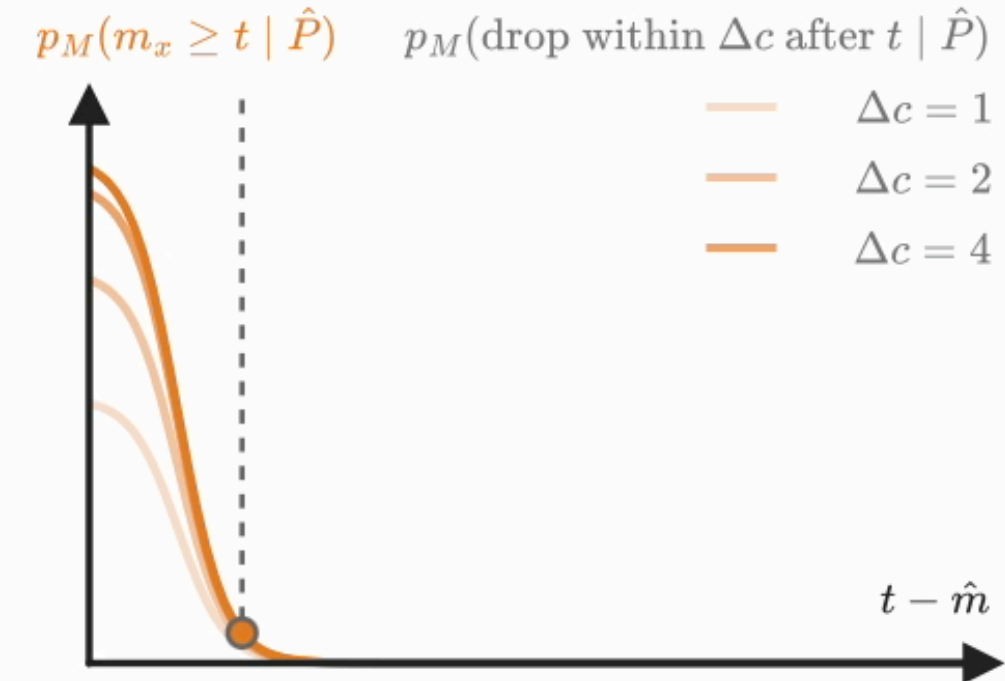
Affine Transform

$$(i, j) \mapsto (\text{BB}(i), i + j)$$

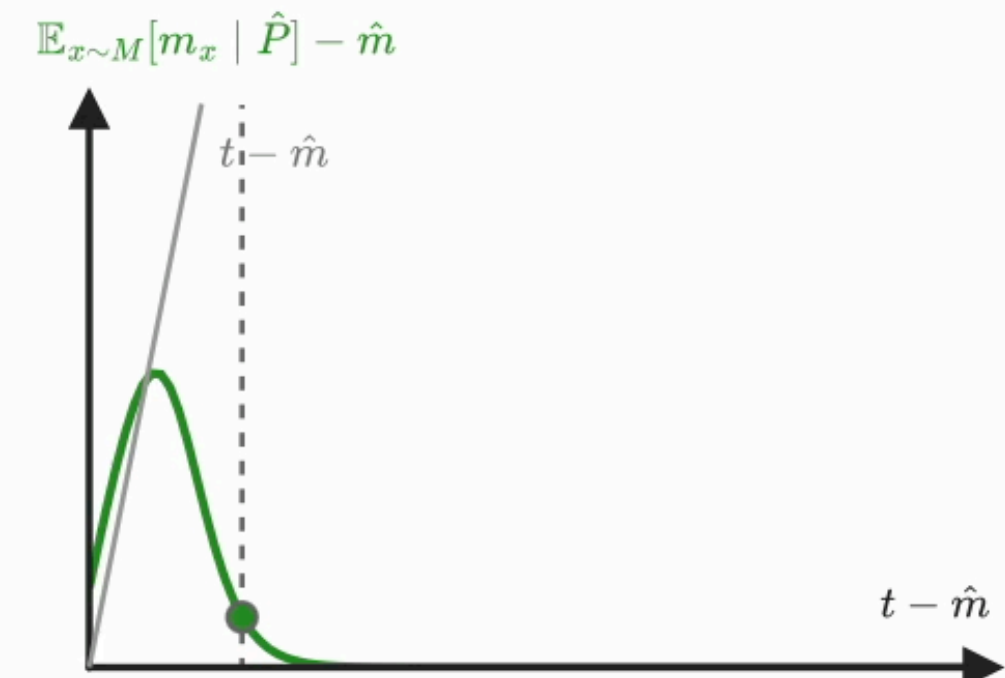


Prior: Algorithmic  $x \sim M$

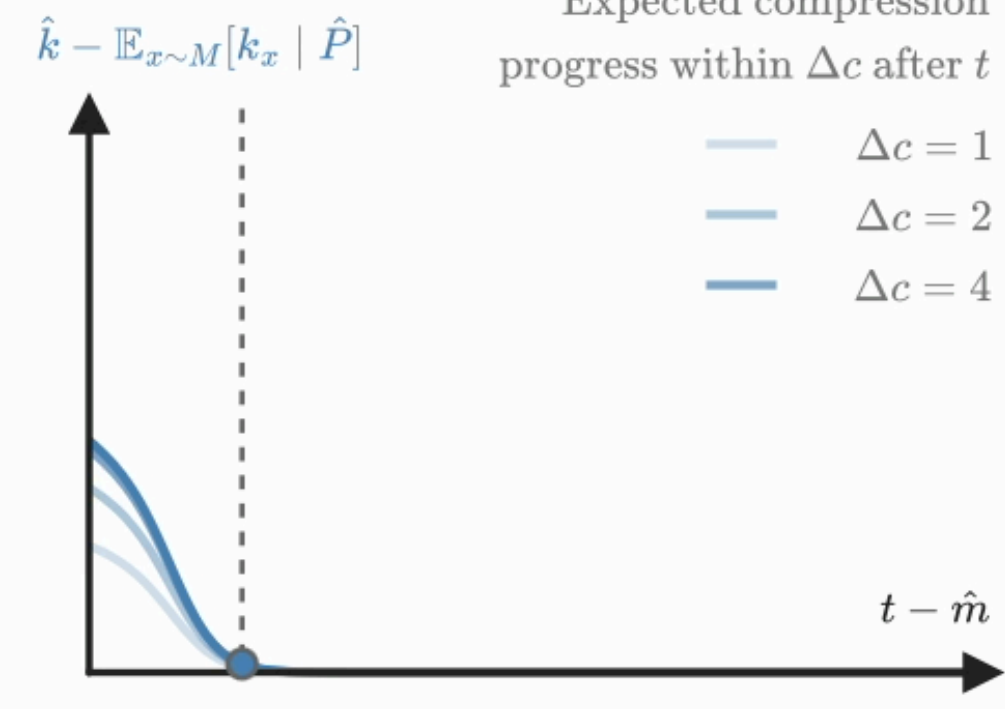
Probability of future drop



Expected relative position of last drop



Expected future compression progress



**Proposition 4.2.** Assuming the object  $x$  producing  $\hat{P}$  is sampled from the Algorithmic Prior  $M$  rather than the Length Prior, the following properties hold:

(i) The probability of a further drop at or after complexity  $t$  is:

$$p_M(m_x \geq t | \hat{P}) = \frac{1}{1 + \frac{2^{-\hat{m} + O(\log \hat{n})}}{2^{-t + O(\log \hat{n})} + 2^{-\hat{k} + O(\log \hat{n})}}}$$

which, for  $\hat{k} - t \gg 0$ , simplifies to:

$$p_M(m_x \geq t | \hat{P}) \approx 2^{-(t - \hat{m}) + O(\log \hat{n})}$$

(ii) Given the existence of such a drop, the expected complexity of  $x$  is:

$$\mathbb{E}_{x \sim M}[k_x | \hat{P}, m_x \geq t] \approx \frac{\hat{k} + t}{2}$$

(iii) Without conditioning on the existence of a further drop, the expected further compression progress is:

$$\hat{k} - \mathbb{E}_{x \sim M}[k_x | \hat{P}] \approx 2^{-(t - \hat{m}) + O(\log \hat{n})}$$

**Proposition 4.3.** Assuming  $\hat{k} - t \gg 0$  and  $t - \hat{m} \gg \log \hat{n}$ , the Algorithmic Prior  $M$  yields significantly higher expectations for future insight than the Length Prior  $L$ . Specifically, the following ratios hold:

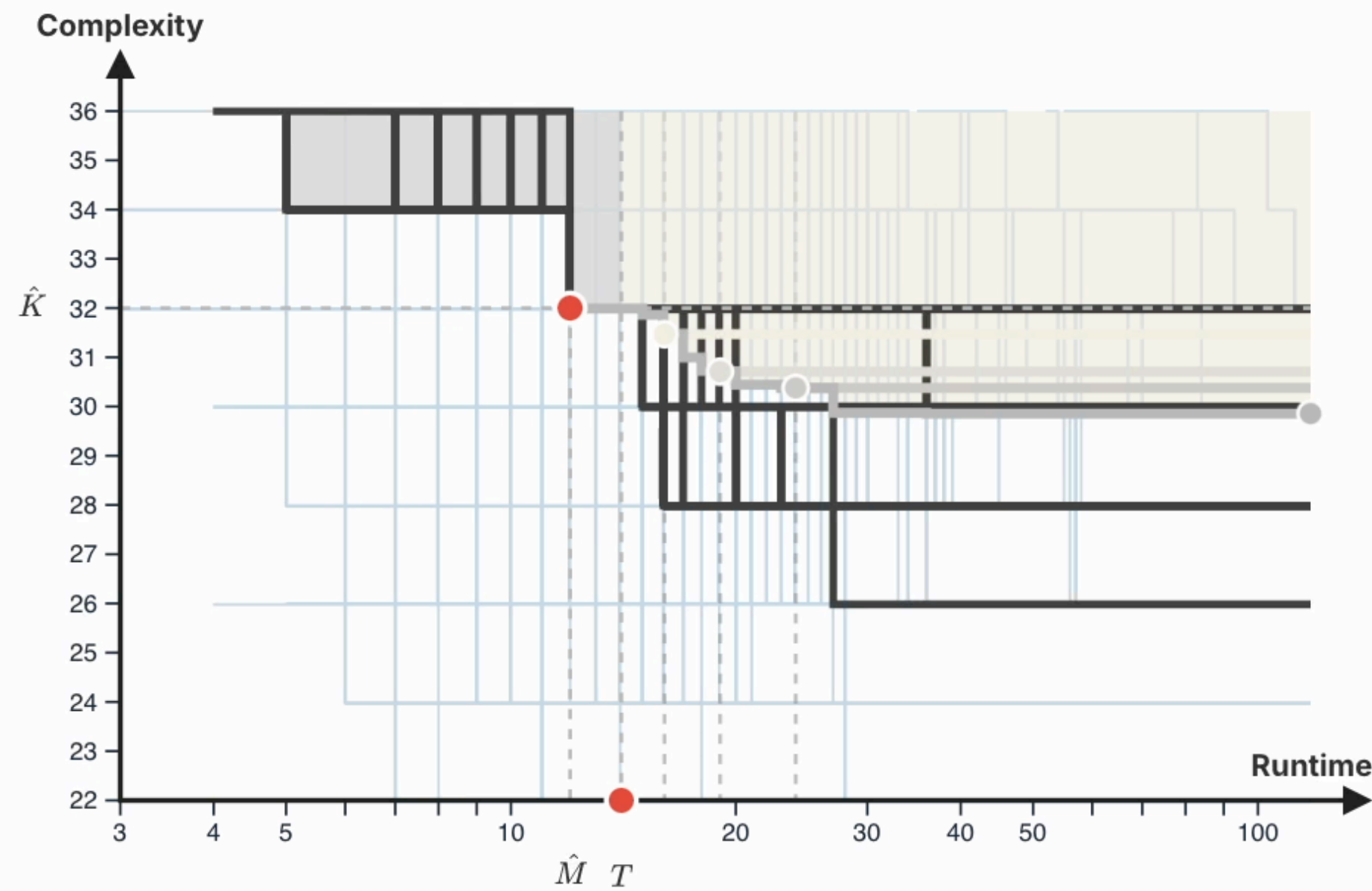
(i) The ratio between the further drop probabilities is approximately linear in the remaining complexity gap  $\hat{k} - t$ :

$$\frac{p_M(m_x \geq t | \hat{P})}{p_L(m_x \geq t | \hat{P})} \approx \hat{k} - t - 1$$

(ii) The ratio between the expected further compression progress is approximately quadratic in the remaining complexity gap:

$$\frac{\hat{k} - \mathbb{E}_{x \sim M}[k_x | \hat{P}]}{\hat{k} - \mathbb{E}_{x \sim L}[k_x | \hat{P}]} \approx \frac{1}{4} (\hat{k} - t - 1) (\hat{k} - t)$$

# 2-Tag System



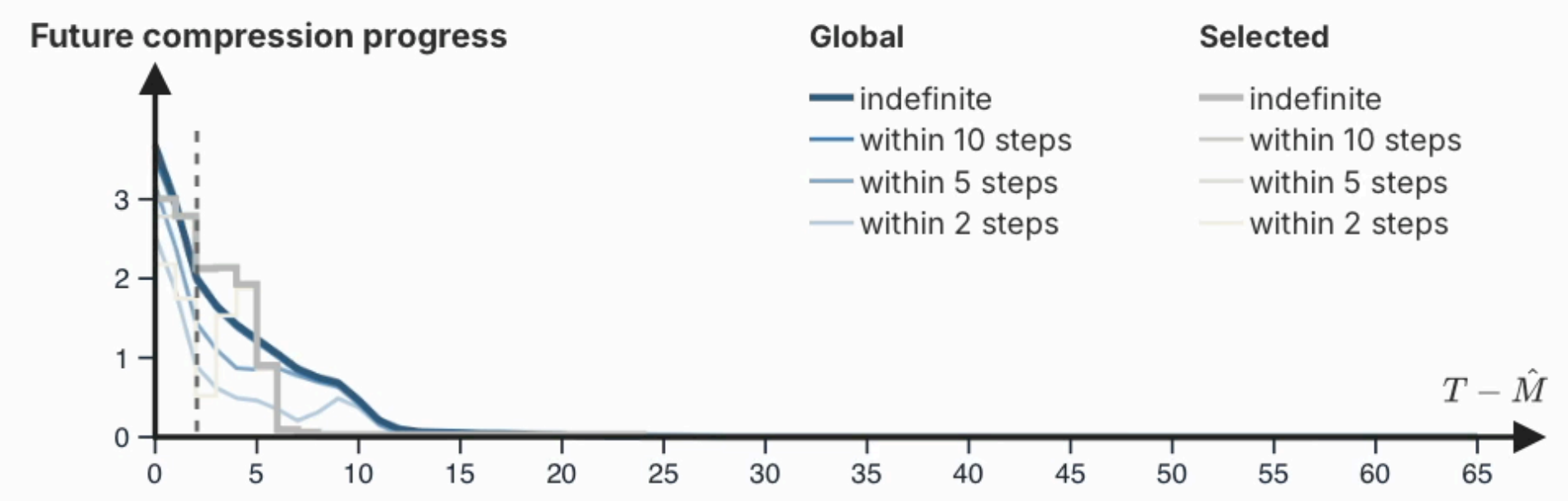
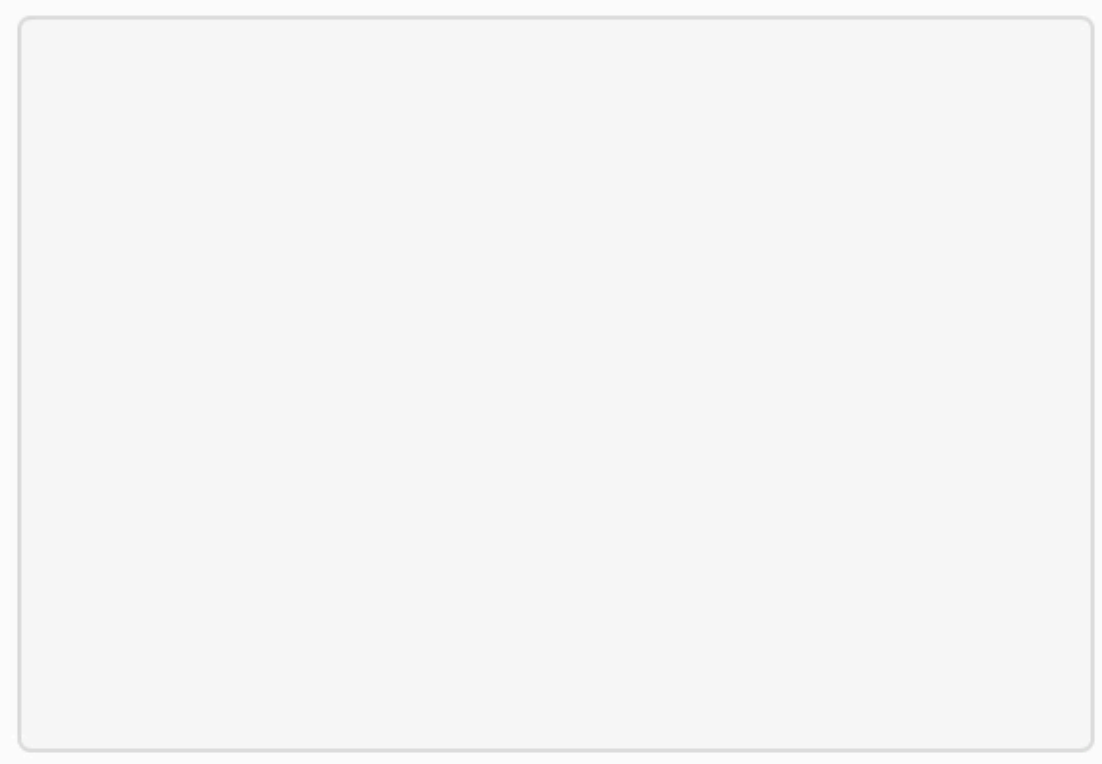
Prior: Algorithmic  $x \sim M$

### Outputs $x$

- Hbacbabacbabbbb
- Hcabcacabcacccc
- HcHcHcHccccccH
- HcHcHcHcccccc
- HbbbbHccbhhhHcc
- HbbcbHccbcbHcbcbH
- HcbbbHcccbbbHcbbbHc
- HcbcbHcccbbHcbcbHcbcbH

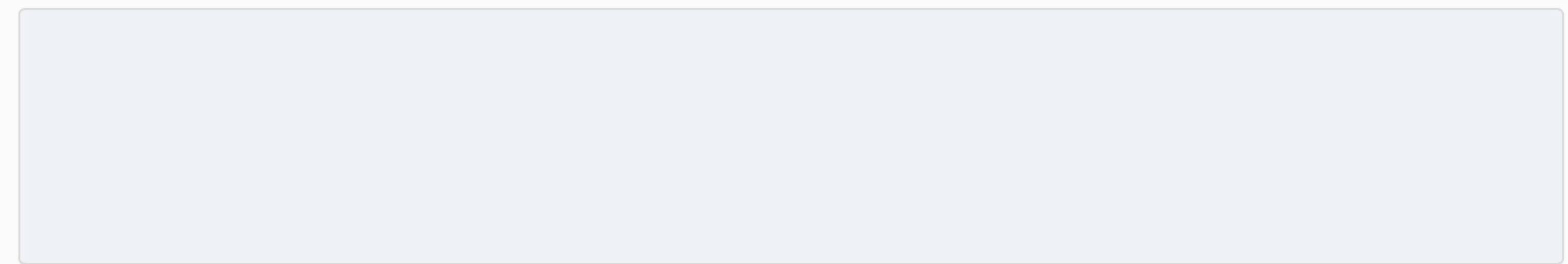
### Programs computing $x$

Runtime

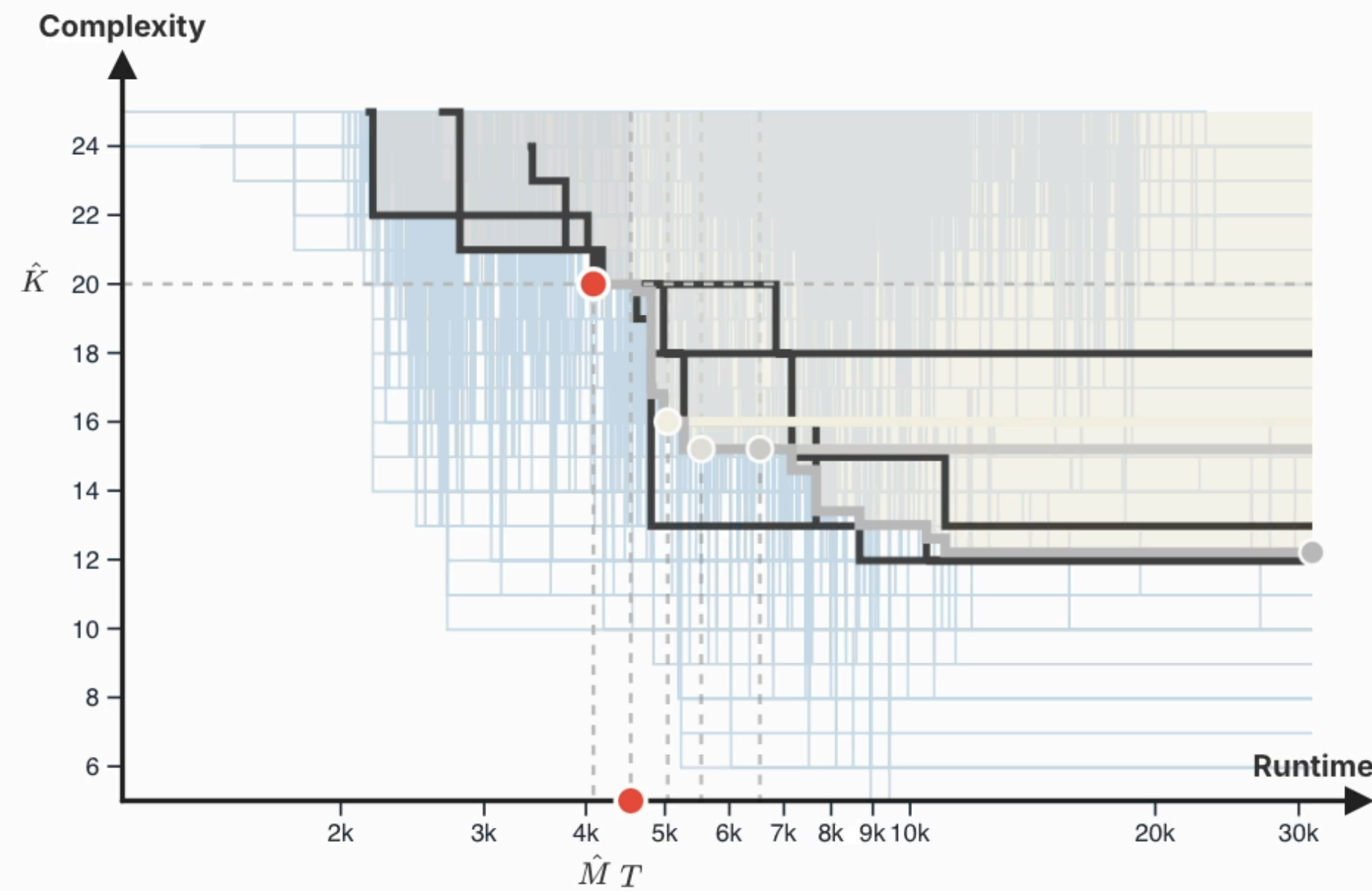


### 2-Tag Machine Execution (Initial: aaa)

a:  b:  c:



# Rule 110 ECA



Prior: Algorithmic  $x \sim M$

Outputs  $x$

```

11011111000100110111111000100110111110001001
1000100110111111000100110111110001001101111
1111000100110111111000100110111110001001101
1111100010011011111100010011011111000100110

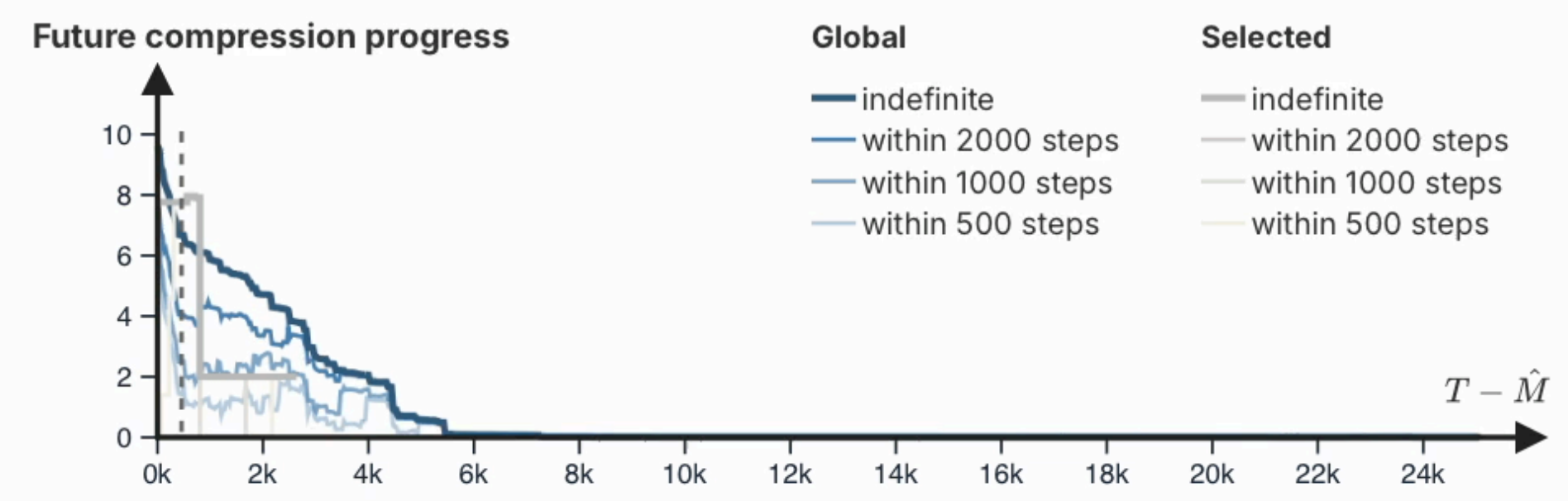
```

Programs computing  $x$

Runtime

```


```



## Rule 110 Cellular Automaton (Tape: 512 bits)

Tape:

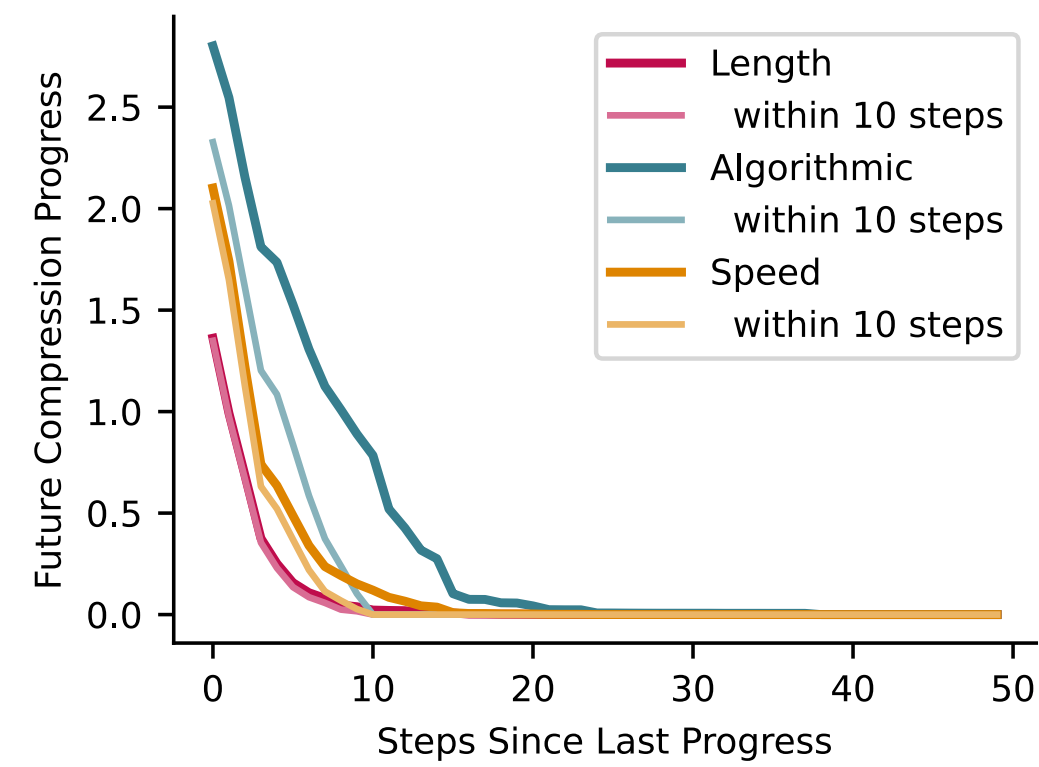
**Run (Max 20k steps)**

```

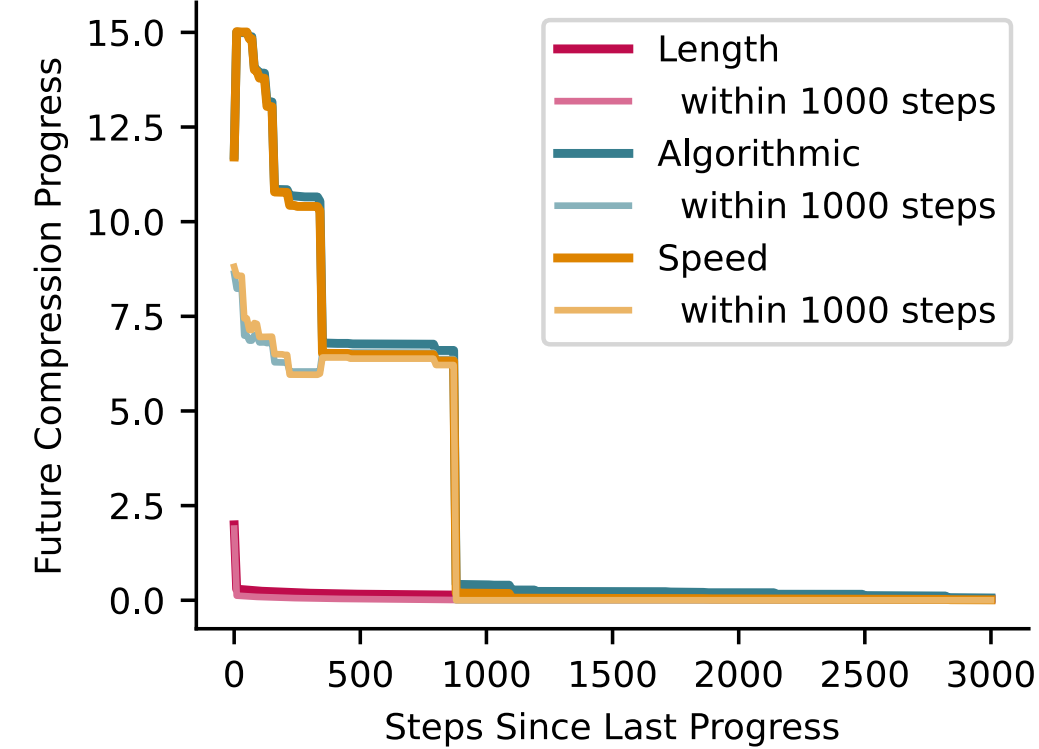

```

2-Tag

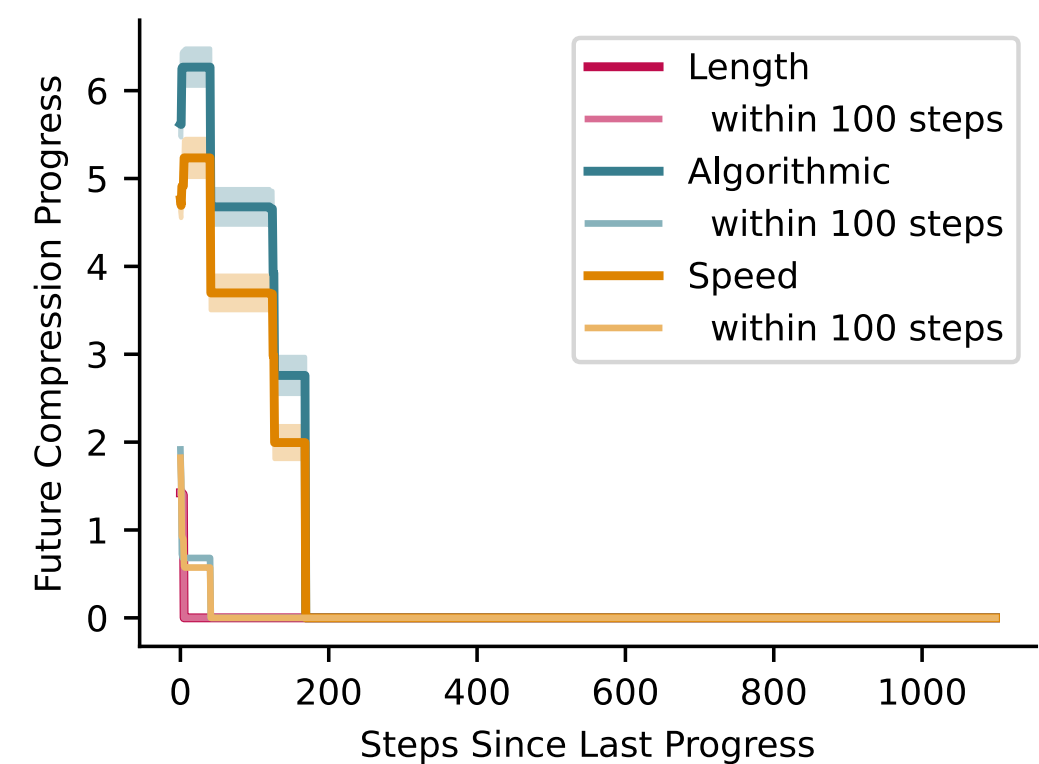
## Experiments



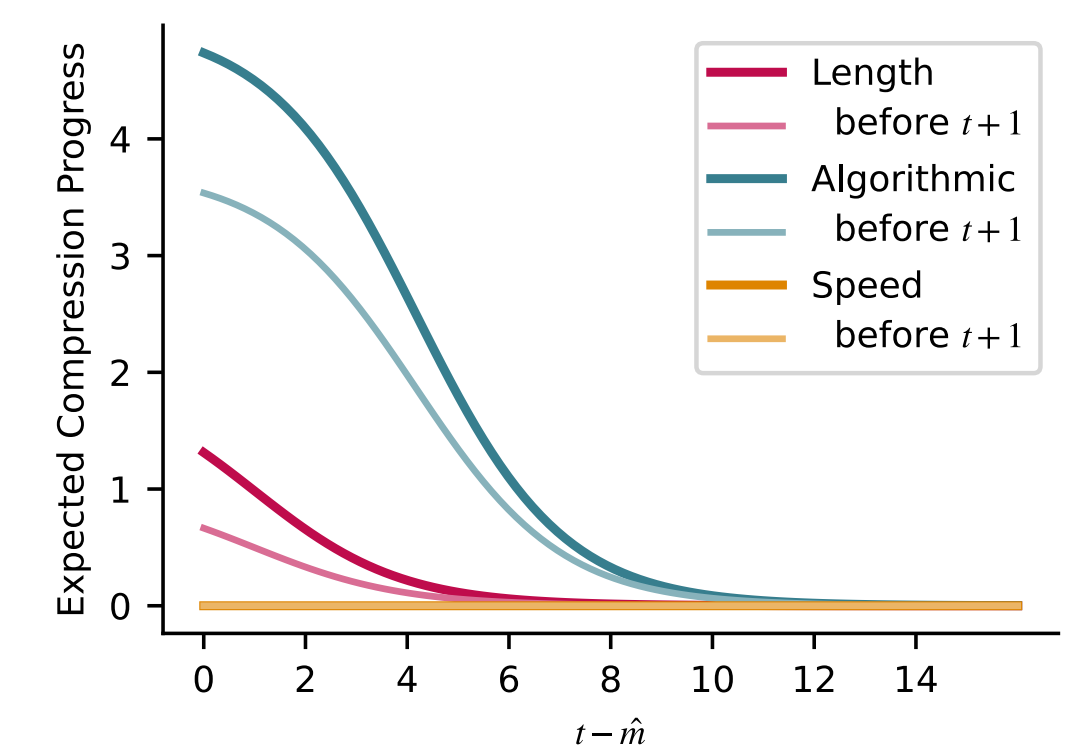
Rule 110



Brainfuck



## Theory



- Neural Networks & Stochastic Gradient Descent
- Reasoning & Chain-of-Thought
- Model Scaling
- ...



*Paper &  
Interactive Diagrams*

<https://inductive-interestingness-2026.github.io/future-compression-progress/>