

RSA-CP: Efficient Conformal Prediction in Small-Sample Regimes via Random Score Alignment

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Conformal Prediction

Split Conformal prediction (SCP) sets a threshold from calibration scores.

1. Fit any predictor

Train a model on training data.

2. Calibration

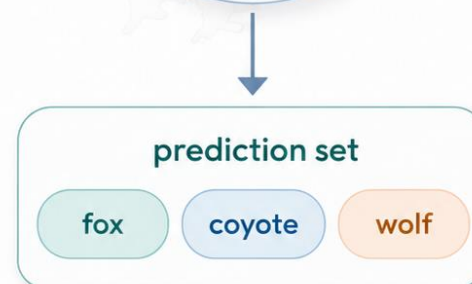
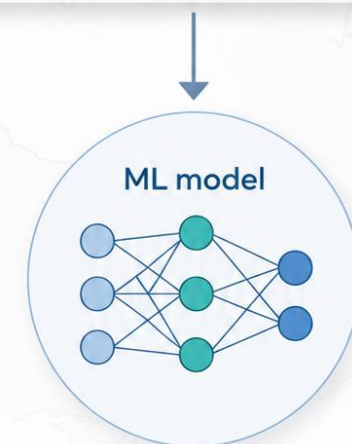
Compute nonconformity scores
 $S_i = \text{score}(x_i, y_i)$.

3. Predict a set

Choose a quantile threshold q
and include labels with score $\leq q$.

Finite-sample guarantee

Under exchangeability, the prediction set covers the true output with probability at least $1 - \alpha$. (**High Coverage**)



Why small calibration sets are a problem?

Split Conformal prediction (SCP) sets a threshold from calibration scores.

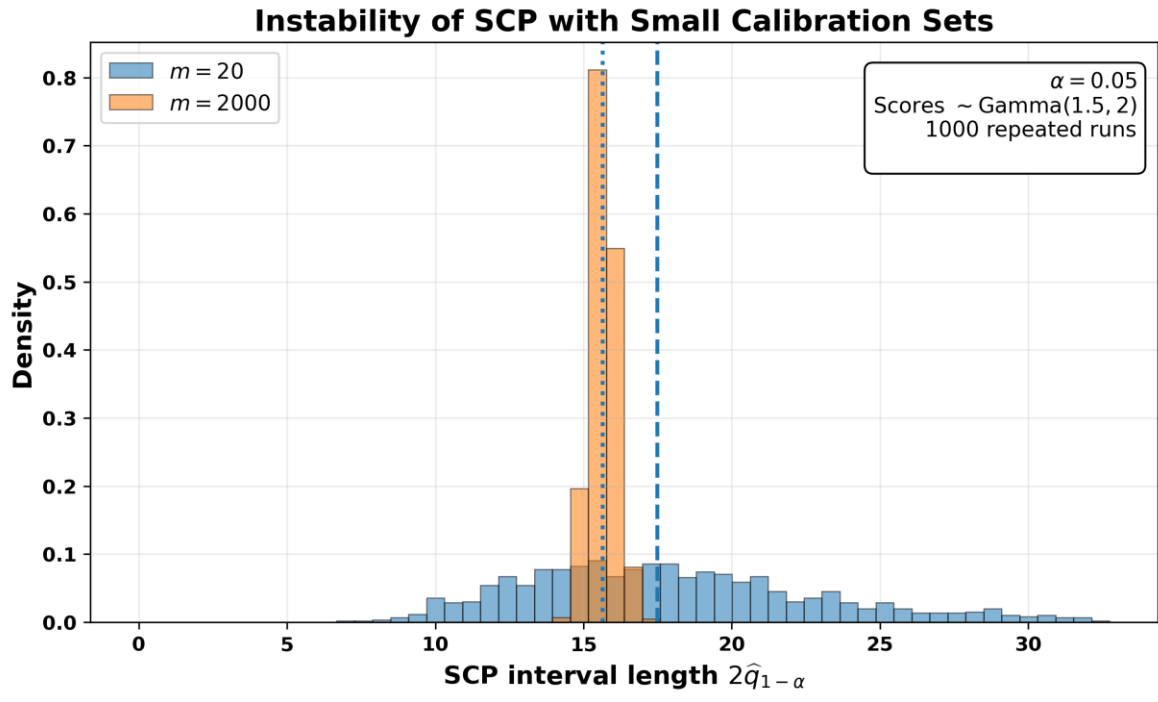
Only a few calibration scores



Threshold choices are coarse.
The method may jump to a very large set.

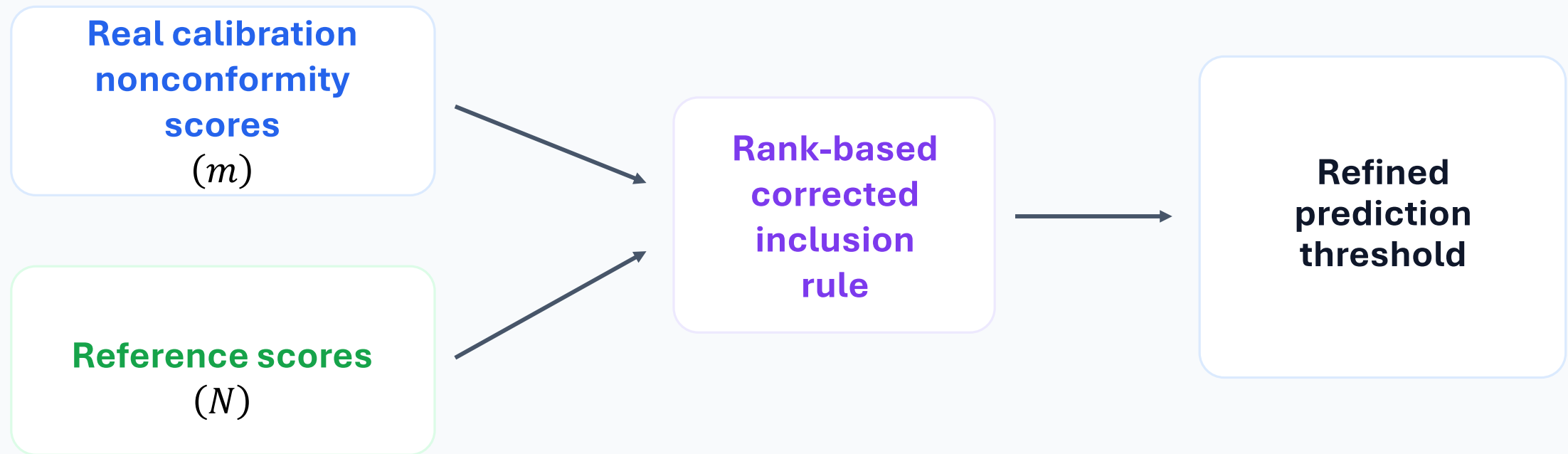
Consequence

Prediction sets become larger than necessary, or even uninformative.



Main idea: augment additional scores, **but carefully!**

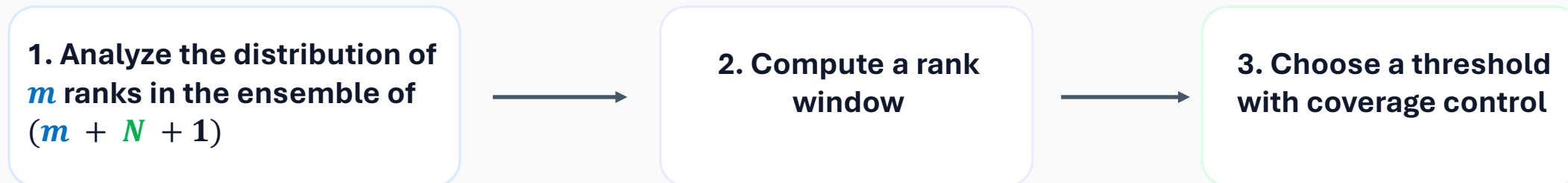
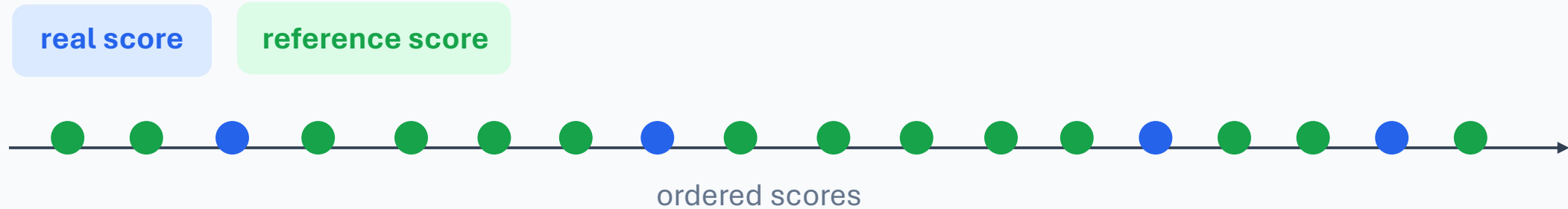
RSA-CP supplements real calibration scores with reference scores.



Reference sources can include simulators, fitted models, previous studies, or random generators.

Main idea: How does RSA-CP work?

The method works with order, not absolute score values.



The correction is conservative enough to protect coverage, but flexible enough to exploit informative reference scores.

What the theory says?

Coverage bounds

Coverage bounds remain computable for any reference-score choice.

$$LB(m, \beta) \leq \mathbb{P} \left(y \in \hat{C}_{RSA}(x) \right) \leq UB(m, \beta)$$

Robustness

Mismatch is not ignored; its effect appears in distribution-robust bounds.

$$|\mathbb{P} \left(y \in \hat{C}_{RSA}(x) \right) - (1 - \alpha)| \leq \frac{1}{m+N+1} + \varepsilon(TV)$$

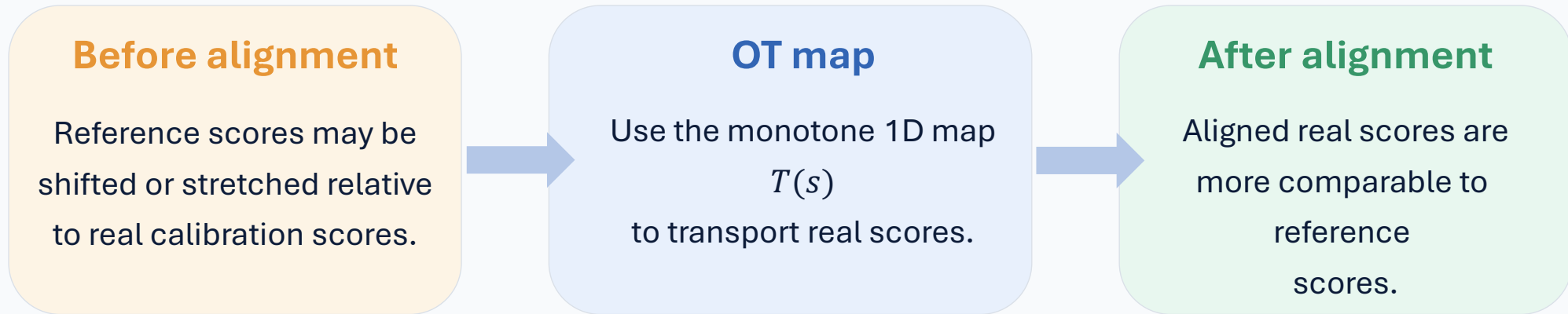
Efficiency Gains

If reference scores resemble real scores, prediction sets can shrink.

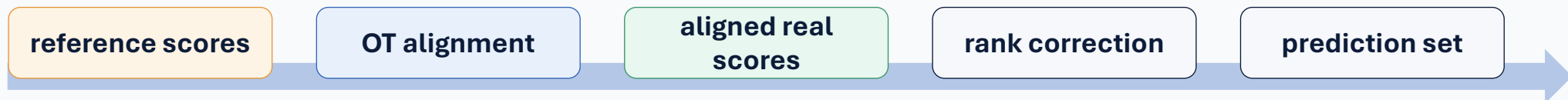
Takeaway: RSA-CP provides coverage bounds for any reference-score augmentation, while informative reference scores can yield more efficient prediction sets.

Score Alignment via Optimal Transport (OT)

Monotone Transform: real score \rightarrow aligned real score = $T(\text{real score})$

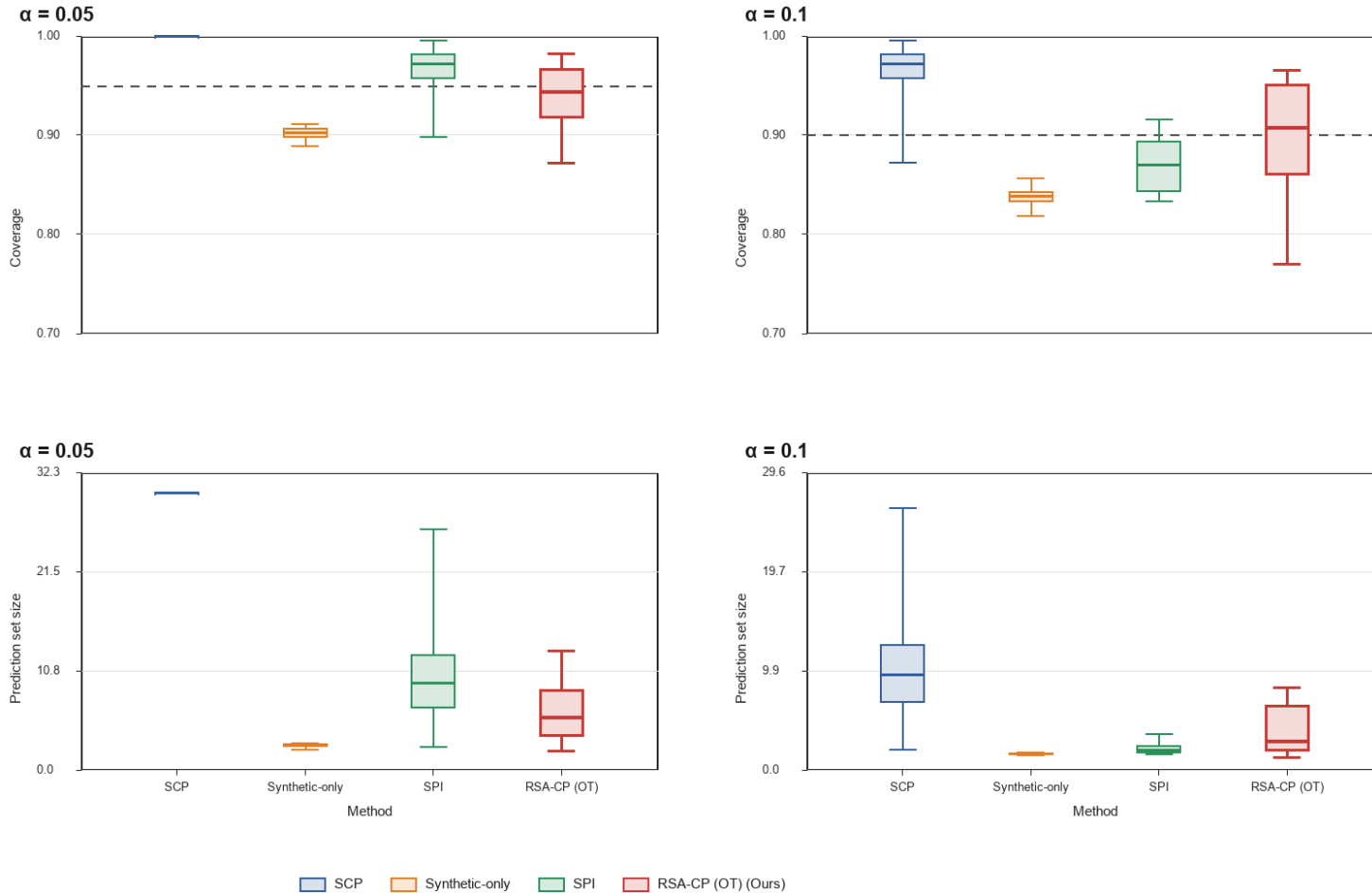


Then use inside RSA-CP:



Takeaway: RSA-CP (OT) provides better score alignment leading to improved efficiency, while satisfying coverage bounds.

Empirical results: RSA-CP (OT) leads to informative prediction sets

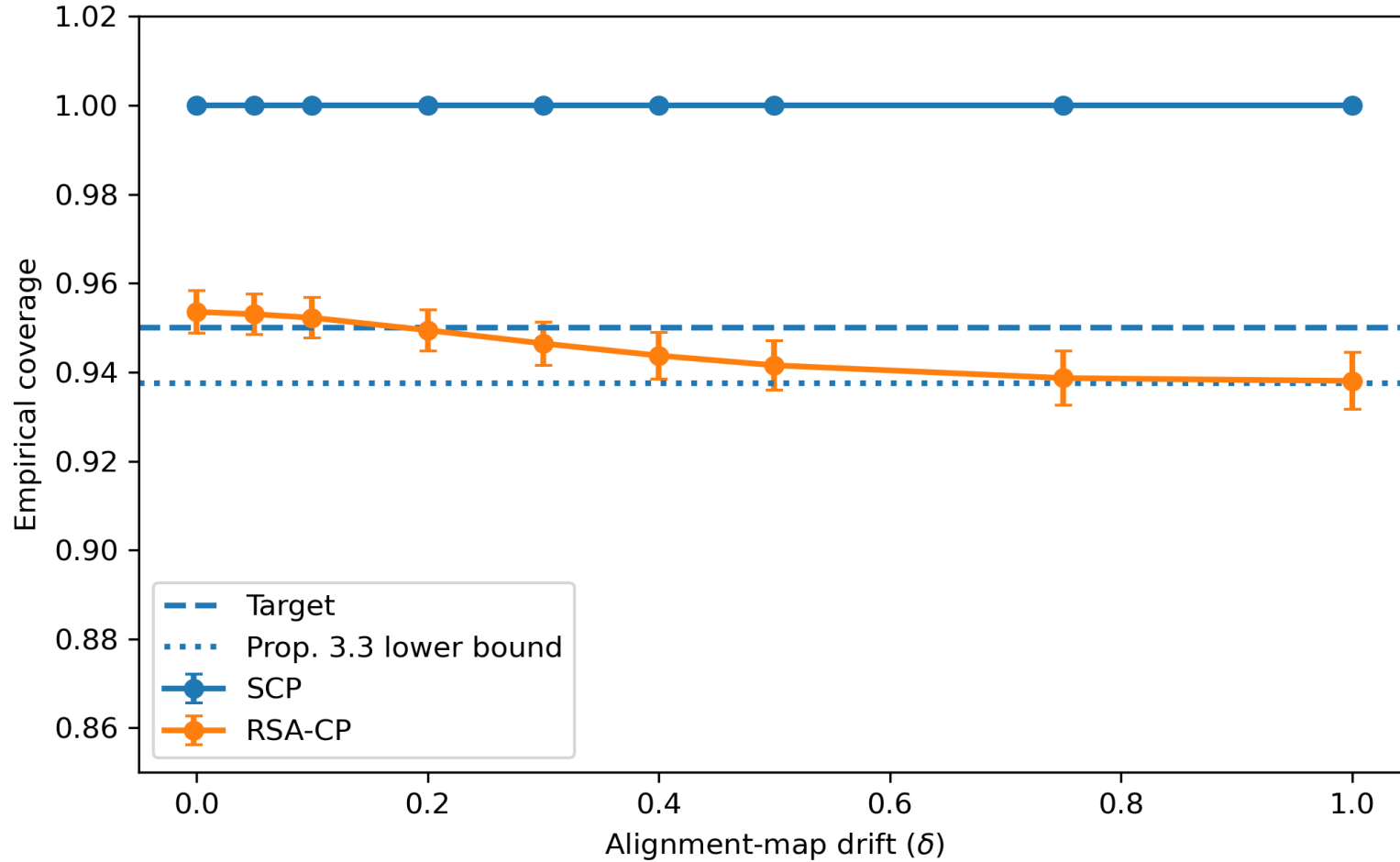


RSA-CP (OT) coverage remains close to the targeted level.

**ImageNet
Image Classification
Task**

Prediction set size: smaller is better when coverage is controlled.

Empirical results: RSA-CP (OT) satisfies theoretical coverage bounds even under mismatch

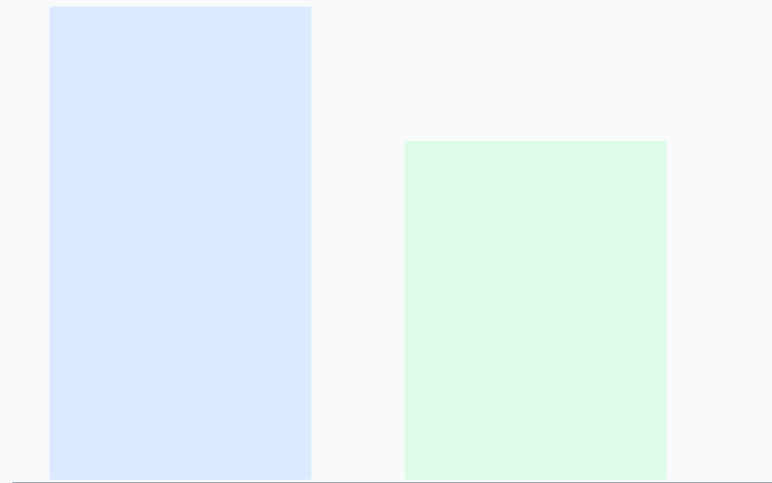


RSA-CP (OT) satisfies coverage bounds even when OT alignment is not perfect!

Simulation study

In Summary,

Prediction-set size



SCP

RSA-CP

smaller is better
when coverage is controlled

- When calibration data are scarce, RSA-CP can reduce conservativeness.
- RSA-CP (OT) satisfies distribution-free deterministic coverage bounds.
- Efficiency gains are significant when reference and real score distributions are close.



Read more here

Get in touch ...



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Thank you!