

Learning Permutation from Structure Without Supervision

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- Sorting: monotone sequence
- Jigsaw: smooth image boundaries
- TSP: short valid tours

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Key Idea

A neural network predicts an ordering; loss is then calculated by measuring the deviation from the expected structure.

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Temperature Controls Sharpness

A single scalar governs how soft or hard each assignment is.

- Low temperature: commit to one assignment
- High temperature: explore more options

The Limitation

Global Temperature

Standard methods sharpen every assignment at the same rate.

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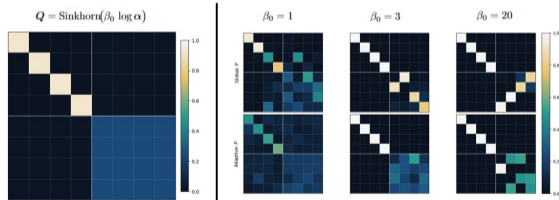
- Some assignments are already decided
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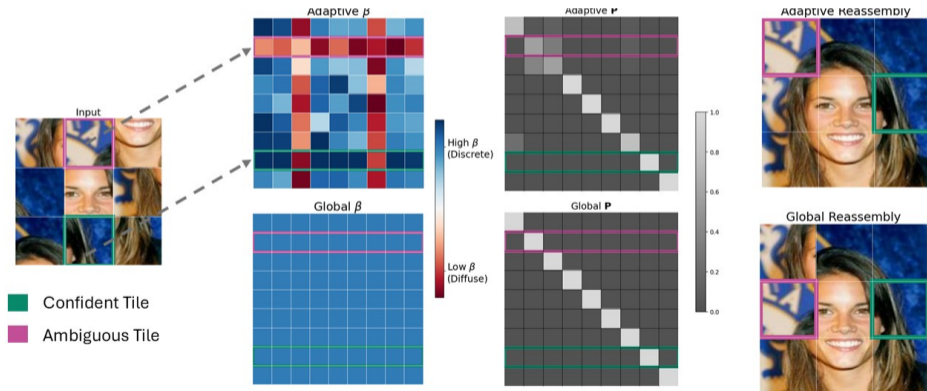
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Our Method

Adaptive Temperature

Each assignment gets its own temperature: confident assignments converge early; uncertain ones keep exploring.



Sorting

Higher Kendall τ is better.

Method	$n = 100$	$n = 200$	$n = 300$
Global GS	0.74	0.60	0.06
Adaptive GS	0.80	0.67	0.15

Results

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TSP

Method	100	200	500
UTSP	45.08	50.68	66.61
GS UTSP	31.24	38.51	52.95
+ DV-TA	30.14	37.67	49.06

Relative optimality gap (%); lower is better.

- 1 Unsupervised permutation learning: expected structure can be used as a training signal.
- 2 **Adaptive temperature** lets confident assignments commit early while uncertain ones keep exploring.

Thank you!

For more details, check out our paper -

