Parsimonious Learning-Augmented Online Metric Matching

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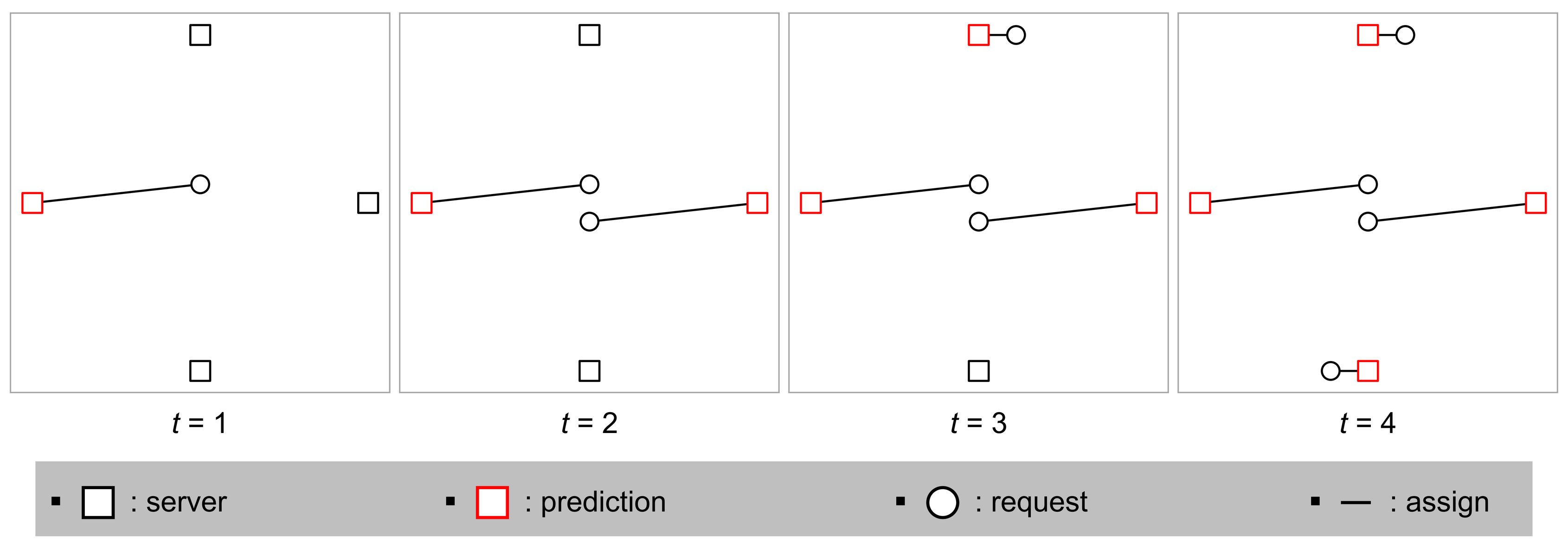
Learning-Augmented Online Metric Matching

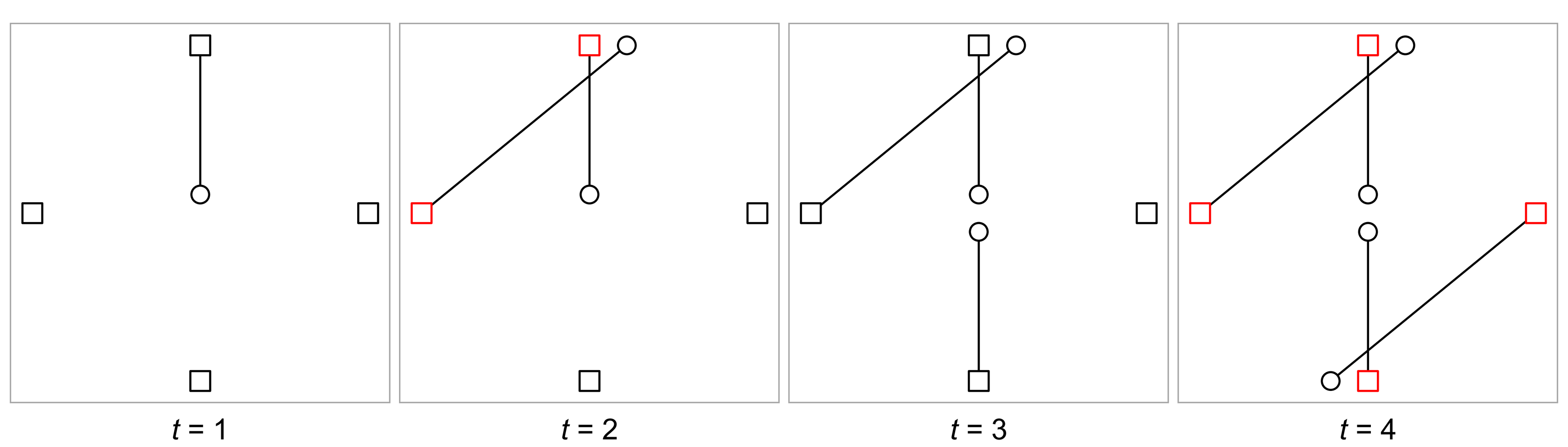
- Input: n servers and n requests lying on a metric space
- Servers given upfront; requests given one by one
- Whenever a new request arrives:
  - Given a (possibly wrong) prediction on servers matched so far [ACE+23]
  - Irrevocably match the request to a currently available server
- Goal: minimize the total distance of the final matching



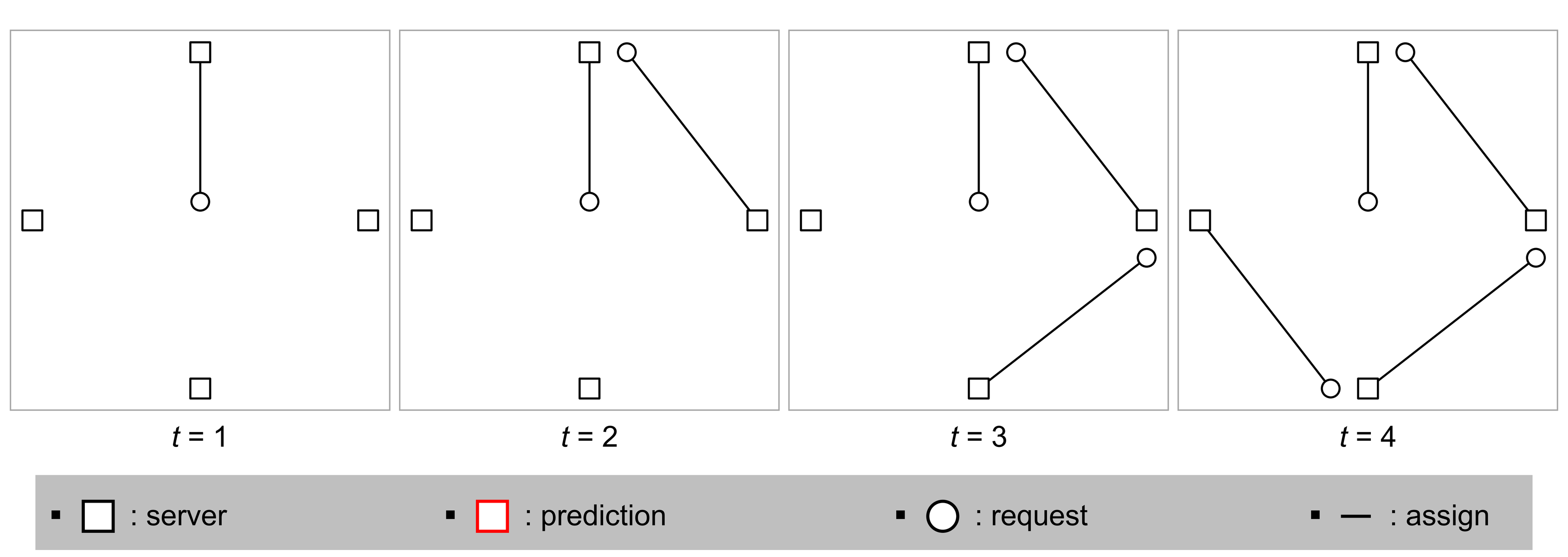
However, generating predictions is expensive!
[IKPP22, SA24]

Well-Separated Queries to Prediction Oracle

- Actual predictions are provided only in every k-th time step [SA24]
- $k = 2$



- $k = \infty$



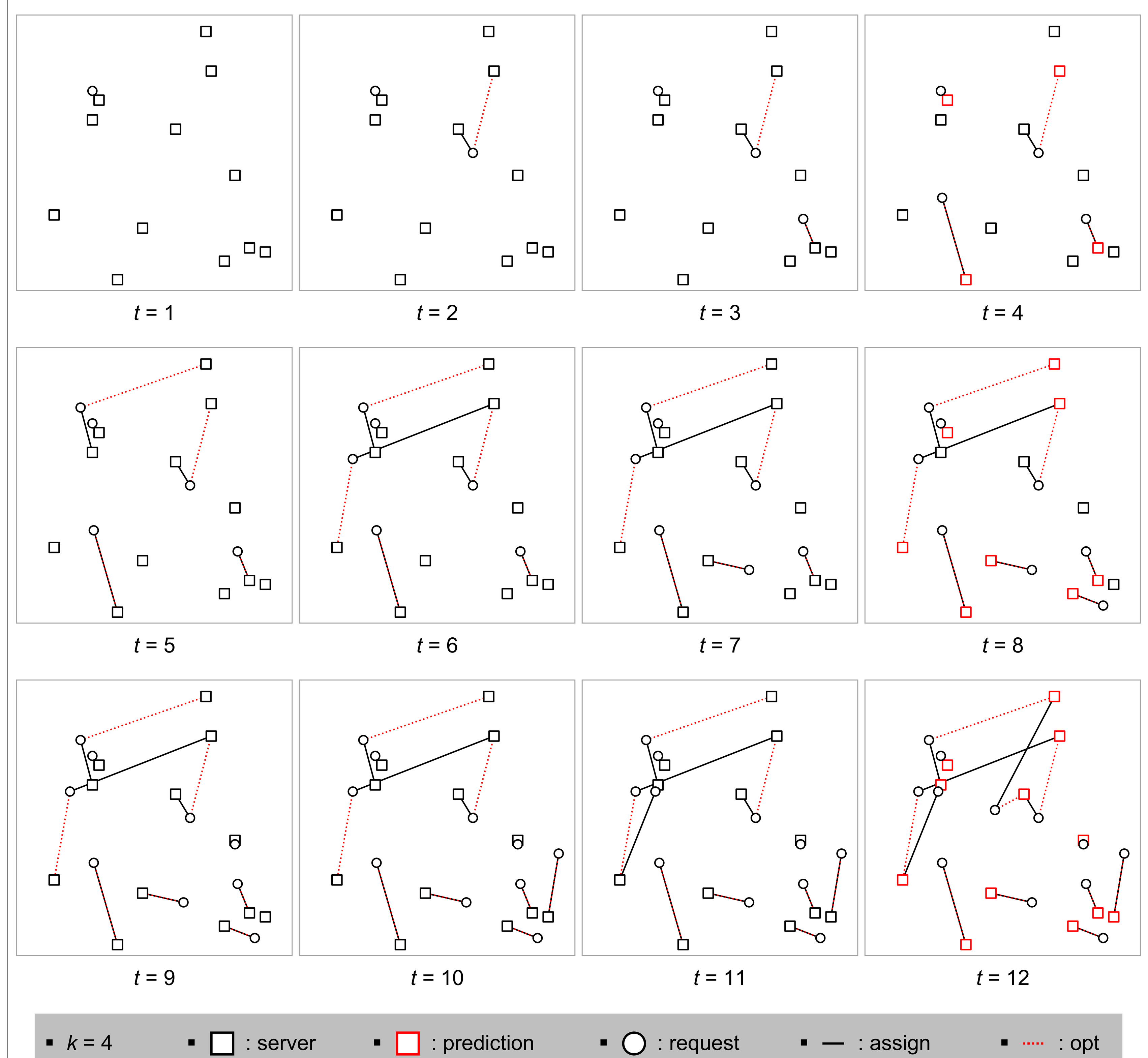
Poster

Full paper (arXiv)

How to solve online matching in metric spaces using predictions parsimoniously?

Our Proposed Algorithm

Follow a competitive algorithm maintaining good *intermediate* matchings between actual predictions + Follow actual predictions [ACE+23]



Adherence & Strong Competitiveness

- γ-adherent if
  (opt match dist btw matched servers & reqs at time t) ≤ γ (opt cost at time t)
- Strongly ρ-competitive if
  (cost of alg at time t) ≤ $\rho(t)$·(opt cost at time t)
- Many algorithms already exhibit these properties

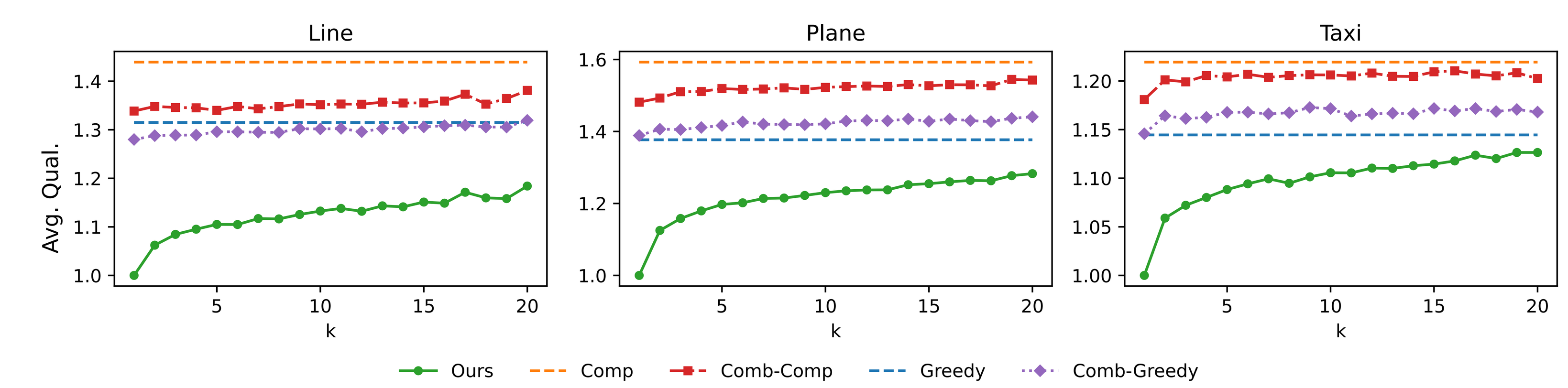
Algorithm	Metric	Det?	γ	$\rho(x)$
[KP93], [KMV94]	General	Yes	1	$2x - 1$
[NR17]	General	Yes	3	$O(\mu_M(S) \log^2 x)$
[Rag18]	Line	Yes	3	$O(\log x)$
[BBGN14]	2-HST	No	1	$O(\log x)$

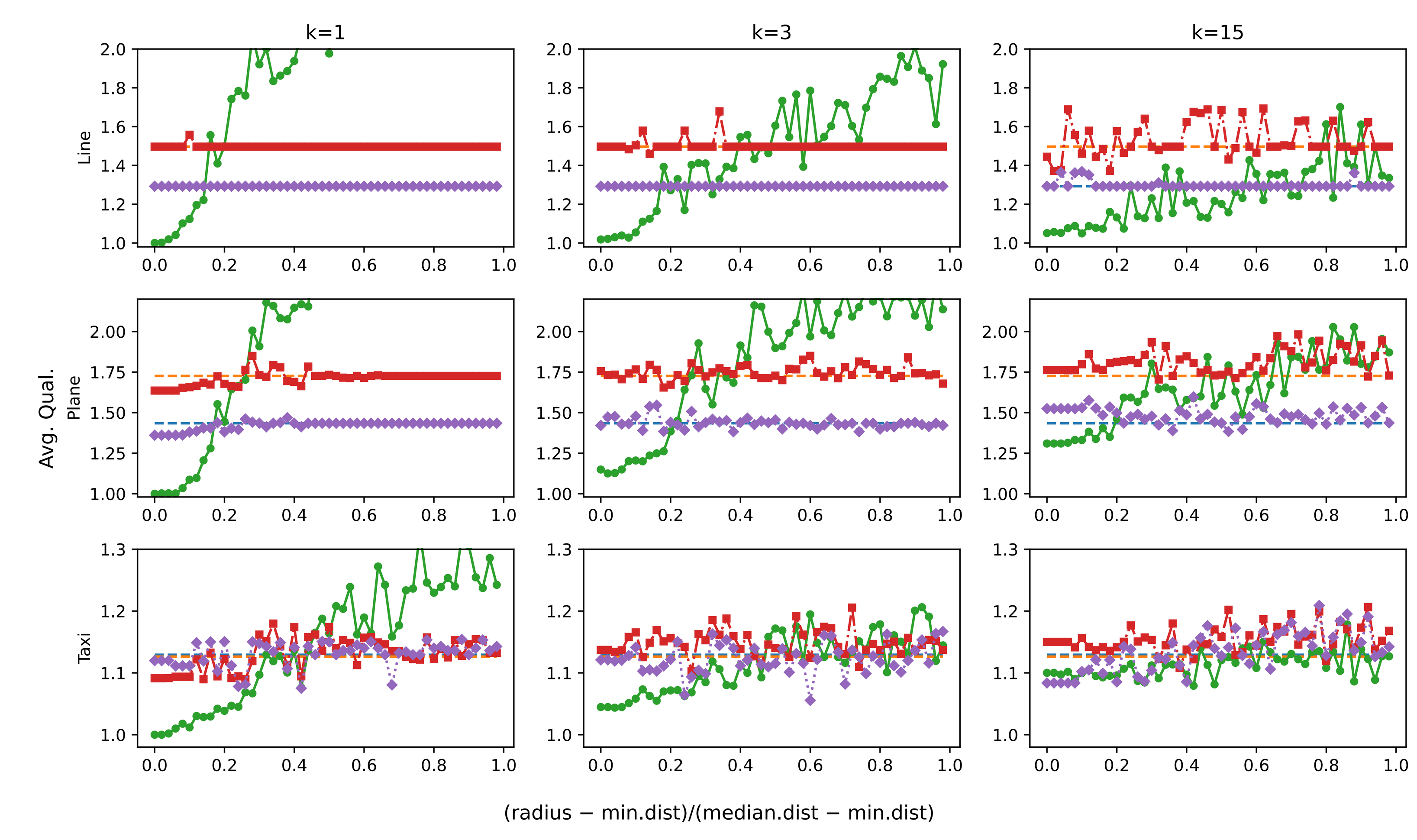
Our Contributions (General Metric)

	k	1 [ACE+23]	k	∞ (Prediction-free setting)
Det	UB	$1+2\eta/\mathrm{OPT}$ [ACE+23]	$(2k-1)+2k\eta/\mathrm{OPT}$	$2n-1$ [KP93, KMV94]
	LB	$2k-1$		
Rand	UB	$O(\log n \log k)\,(1+\eta/\mathrm{OPT})$		$O(\log^2 n)$ [BBGN14]
	LB	$\Omega(\log k)$		$\Omega(\log n)$ [MNP06]

- η : total error of predictions
- Can make the above algorithms robust via a standard combination technique with competitive algorithms at factor 9 in total cost [FRR94, ACE+23]
- Lower bounds are obtained from star graphs with perfect predictions

Experiments





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