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On the Provable Suboptimality of Momentum SGD in Nonstationary Stochastic Optimization

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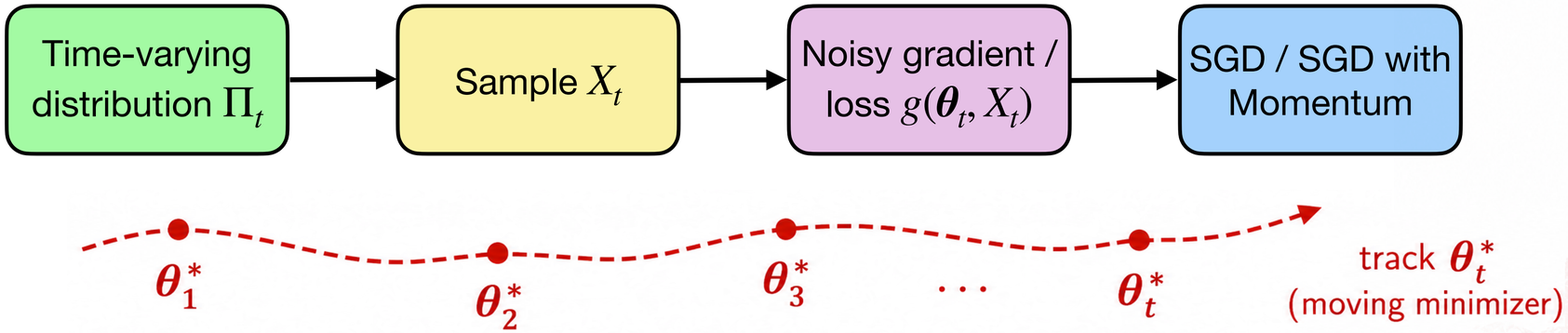
Motivation

- **Problem Setting:** We study stochastic optimization when the data distribution changes over time. At time t, a μ-strongly convex, L-smooth objective is

$$\boldsymbol{\theta}_t^\star = \arg\min_{\boldsymbol{\theta}\in\mathbb{R}^d} G_t(\boldsymbol{\theta}), \quad G_t(\boldsymbol{\theta}) = \mathbb{E}_{X_t\sim\Pi_t}\left[g(\boldsymbol{\theta}, X_t)\right]$$

Here $g(\boldsymbol{\theta}, X_t)$ is a noisy sample loss and $X_t \sim \Pi_t$ where Π_t is a time-varying data distribution.



- **Goal:** Track the moving minimizer sequences $\left\{\boldsymbol{\theta}_t^\star\right\}_{t=1}^T$ for finite $T < \infty$.

- **Practical relevance:** Time-varying distribution Π_t appears in stochastic tracking and concept drift and captures many relevant settings such as policy optimization, online recommendation, continual learning, and federated learning [Kushner & Yin, 1997; Sayed, 2003; Gama et al., 2014; Hsu et al., 2021; Kakade, 2002; Schulman et al., 2015; Li et al., 2010; Parisi et al., 2019; Kairouz et al., 2021].

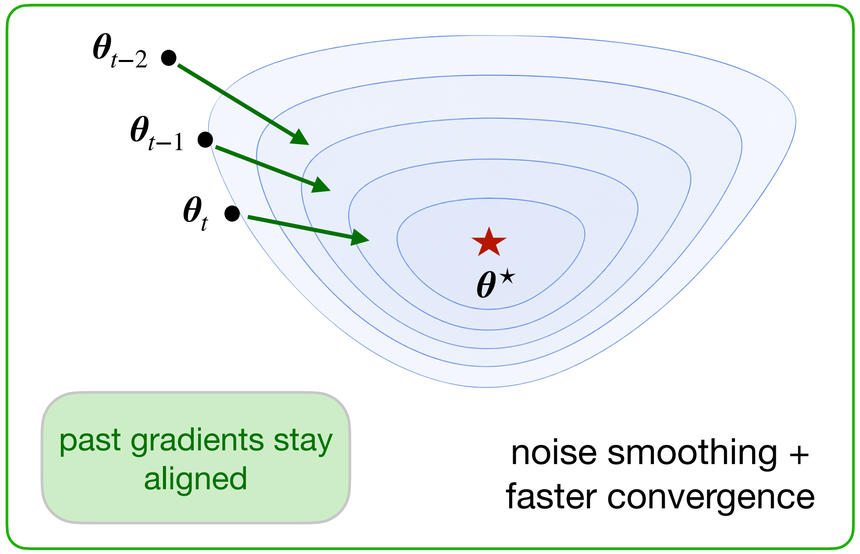
Motivation

- **Momentum in SGD:** Momentum methods average past gradients to reduce noise and often speed up optimization

$$\boldsymbol{v}_t = \beta \boldsymbol{v}_{t-1} + \nabla_{\boldsymbol{\theta}_t} g(\boldsymbol{\theta}_t, X_t), \quad \boldsymbol{\theta}_{t+1} = \boldsymbol{\theta}_t - \gamma \boldsymbol{v}_t$$

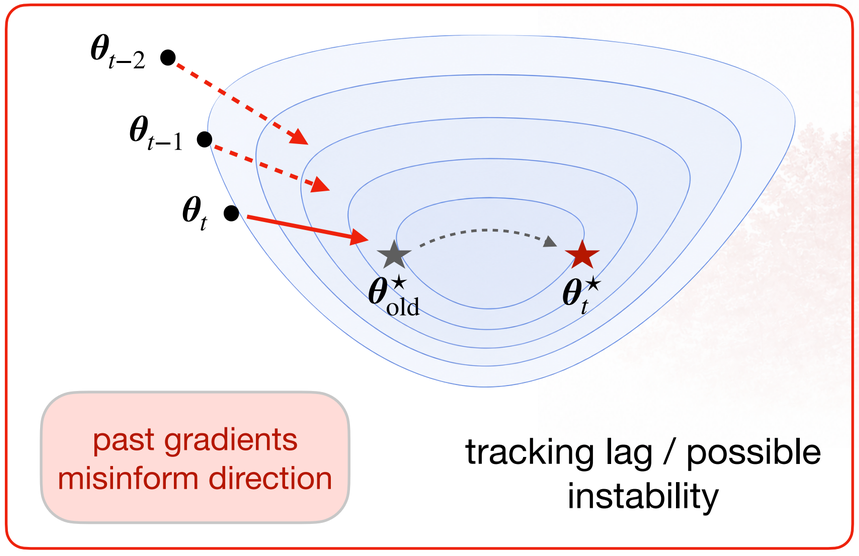
Stationary objective

fixed distribution / fixed minimizer



Nonstationary objective

drifting distribution / moving minimizer



past gradients from Π_{t-k} for $k = 0, \ldots, t$ may misalign with the current objective G_t.

- **Main question:** *When does momentum accelerate tracking, and when does uninformative gradient information create unavoidable lag in nonstationary stochastic optimization?*

Main Results (I) - Expectation Tracking Bound

- **A1:** Second-moment bounds
  - **Drift:** $\boldsymbol{\Delta}_t = \boldsymbol{\theta}_t^\star - \boldsymbol{\theta}_{t+1}^\star, \quad \mathbb{E}\left[\|\boldsymbol{\Delta}_t\|_2^2\right] \le \Delta^2$
  - **Noise:** $\mathbb{E}\left[\|\boldsymbol{\xi}_{t+1}(\boldsymbol{\theta}_t)\|^2\right] \le \sigma^2$ and $\mathbb{E}\left[\|\boldsymbol{\xi}_{t+1}(\boldsymbol{\psi}_t)\|^2\right] \le \sigma^2$

Tracking error bound in expectation for SGD

Under **A1**, for any $t \ge 0$ and constant stepsize $\gamma \le \min\left\{\mu/L^2, 1/L\right\}$,

$$\mathbb{E}\left[\left\|\boldsymbol{\theta}_t - \boldsymbol{\theta}_t^\star\right\|^2\right] \lesssim \underbrace{\left(1 - \frac{\gamma\mu}{2}\right)^t \left\|\boldsymbol{\theta}_0 - \boldsymbol{\theta}_0^\star\right\|^2}_{\text{optimization error / transient decay}} + \underbrace{\frac{\Delta^2}{\gamma^2\mu^2}}_{\text{tracking floor from drift}} + \underbrace{\frac{\sigma^2\gamma}{\mu}}_{\text{noise floor}}.$$

- **Takeaway:** The error decomposes into initialization error, a floor due to nonstationary drift, and a floor due to stochastic gradient noise.

Main Results (I) - Expectation Tracking Bound

Tracking error bound in expectation for SGDM

Under **A1**, for any $t \geq 0$ and constant stepsize $\gamma \leq \mu(1-\beta)^2/4L^2$,

$$\mathbb{E}\left[\left\| \boldsymbol{\theta}_t - \boldsymbol{\theta}_t^\star \right\|^2\right] \lesssim \underbrace{\frac{1}{(1-\beta)^2}\exp\left(-\frac{\gamma\mu t}{1-\beta}\right)\left\| \boldsymbol{\theta}_0 - \boldsymbol{\theta}_0^\star \right\|^2}_{\text{sensitivity to initialization}} + \frac{(2+\beta)^2}{\gamma^2\mu^2}\Delta^2 + \underbrace{\frac{\sigma^2\gamma}{\mu(1-\beta)}}_{\text{inflates noise floor}}.$$

- Tracking error admits the same three-term decomposition as SGD but with explicit momentum penalties.
- Letting $\kappa := L/\mu$ be the condition number, $\gamma\mu = \mathcal{O}(1/\kappa)$ so ill-conditioned problems can have a long burn-in time before reaching steady state.
- Classical Nesterov acceleration improves deterministic GD from $\mathcal{O}(\kappa)$ to $\mathcal{O}(\sqrt{\kappa})$ [Nesterov, 1983; Nesterov, 2014].

Main Results (II) - High Probability Tracking Bounds

- **A2:** Conditional sub-Gaussian noise. There exists a constant $\sigma > 0$ such that for all $t \geq 0$,
$\left\| \boldsymbol{\xi}_{t+1}(\boldsymbol{\theta}_t) \mid \mathcal{F}_t \right\|_{\Psi_2} \leq \sigma$ and $\left\| \boldsymbol{\xi}_{t+1}(\boldsymbol{\psi}_t) \mid \mathcal{F}_t \right\|_{\Psi_2} \leq \sigma$

High probability tracking error bound for SGD

Under **A2,** for all $t \in [T]$, $\gamma \leq \min\left\{\mu/L^2, 1/L\right\}$, and $\delta \in (0,1)$, with probability at least $1-\delta$,

$$\left\| \boldsymbol{\theta}_t - \boldsymbol{\theta}_t^\star \right\|^2 \lesssim \left(1 - \frac{\gamma\mu}{2}\right)^t \left\| \boldsymbol{\theta}_0 - \boldsymbol{\theta}_0^\star \right\|^2 + \frac{\mathfrak{D}_t}{\gamma\mu} + \frac{d\sigma^2\gamma}{\mu} + \left(d\sigma^2\gamma^2 + \frac{\sigma^2\gamma}{\mu} + \gamma^2\sigma^2\mathfrak{D}_t^{(2)}\right)\log\frac{2T}{\delta},$$

where $\mathfrak{D}_t := \sum_{\ell=0}^{t-1} (1-\gamma\mu/2)^{t-\ell-1} \left\| \boldsymbol{\Delta}_\ell \right\|^2$ and $\mathfrak{D}_t^{(2)} := \sum_{\ell=0}^{t-1} (1-\gamma\mu/2)^{2(t-\ell-1)} \left\| \boldsymbol{\Delta}_\ell \right\|^2$.

- Our bound is *drift-adaptive and time-resolved* and captures that **(i)** only recent nonstationarity drift affects the guarantee and **(ii)** drift amplifies stochastic fluctuations by increasing the tracking mismatch.

Main Results (II) - High Probability Tracking Bounds

High probability tracking error bound for SGDM

Under **A2,** for all $t \in [T]$, $\gamma \leq \min\left\{1/L, \mu(1-\beta)^2/4L^2\right\}$, and $\delta \in (0,1)$, with probability at least $1-\delta$,

$$\left\| \boldsymbol{\theta}_t - \boldsymbol{\theta}_t^\star \right\|^2 \lesssim \frac{1}{(1-\beta)^2} \exp\left(-\frac{\gamma^2\mu^2}{4(1-\beta)^2} t\right) \left\| \boldsymbol{\theta}_0 - \boldsymbol{\theta}_0^\star \right\|^2 + \frac{1}{\gamma\mu} \cdot \frac{1}{1-\beta} \mathfrak{D}_t^{\mathrm{lag}} + \frac{d\sigma^2}{\mu^2} + \left(\frac{d\sigma^2\gamma^2}{(1-\beta)^2} + \frac{\sigma^2}{\mu^2} + \frac{\sigma^2\gamma^2}{(1-\beta)^2} \mathfrak{D}_t^{\mathrm{lag},(2)}\right) \log \frac{2T}{\delta},$$

where $\mathfrak{D}_t^{\mathrm{lag}} := \sum_{\ell=0}^{t-1} \tilde{\rho}^{\,t-\ell-1} \|\boldsymbol{b}_\ell\|^2$ and $\mathfrak{D}_t^{\mathrm{lag},(2)} := \sum_{\ell=0}^{t-1} \tilde{\rho}^{\,2(t-\ell-1)} \|\boldsymbol{b}_\ell\|^2$ with $\tilde{\rho} := 1 - \gamma^2\mu^2/4(1-\beta)^2$ and $\boldsymbol{b}_\ell$ depends on lagged drift terms $\boldsymbol{\Delta}_\ell, \boldsymbol{\Delta}_{\ell-1}$.

- Momentum adds an inertia horizon of order $\mathcal{O}(1/(1-\beta))$, amplifies the drift-noise coupling by $\mathcal{O}(1/(1-\beta)^2)$ and slows transient decay and prevents SGDM from adapting to drift in ill-conditioned regimes ($\kappa \gg 1$).
- Explains why SGD can be strictly more robust in nonstationary, ill-conditioned settings: it avoids the compounded inertia and variance penalties induced by momentum.

Main Results (III) - Minimax Lower Bounds

- **Setting:** Dynamic regret for non stationary stochastic optimization under distribution shift.
  - **Dynamic regret:** $\mathscr{R}_T^{\pi}(G) := \mathbb{E}^{\pi}\left[\sum_{t=0}^{T-1}\left(G_{t+1}(\boldsymbol{\theta}_t) - G_{t+1}(\boldsymbol{\theta}_{t+1}^{\star})\right)\right]$, **Minimax risk:** $\mathfrak{M}_T(\Pi_\beta, \mathbb{V}_T) := \inf_{\pi\in\Pi_\beta}\sup_{G:\,\mathrm{GVar}_{p,q}(G)\le\mathbb{V}_T}\mathscr{R}_T^{\pi}(G)\,.$
- **Function class (gradient variation budget):** for $1 \le p, q \le \infty$, the $L_{p,q}$ gradient-variational functional of $g = (g_1, \ldots, g_T)$ is:

$$\mathrm{GVar}_{p,q}(g) := \begin{cases} \left(\frac{1}{T}\sum_{t=1}^{T-1}\left\|\nabla g_{t+1} - \nabla g_t\right\|_p^q\right)^{1/q} & q < \infty \\ \max_{1\le t\le T-1}\left\|\nabla g_{t+1} - \nabla g_t\right\|_p & q = \infty, \end{cases} \qquad \|g\|_p := \begin{cases} \left(\int_\Theta \|g(\theta)\|_2^p\, d\theta\right)^{1/p} & p < \infty \\ \sup_{\theta\in\Theta}\|g(\theta)\|_2 & p = \infty\,. \end{cases}$$

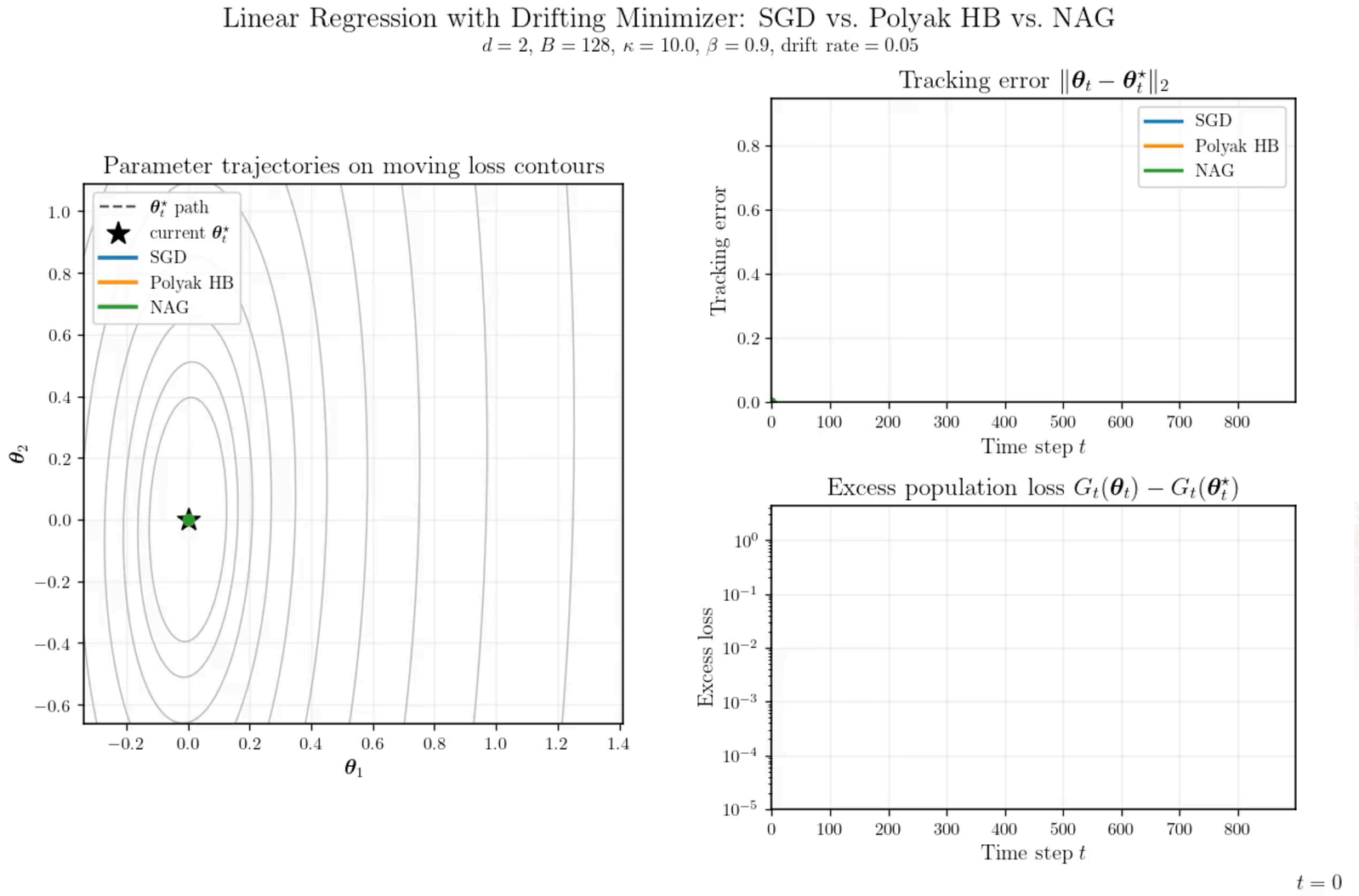
Minimax lower bound for strongly-convex function sequences using SGDM

Fix arbitrary $1 \le p, q \le \infty$. Consider the class Π_β of $\mathrm{SGDM}(\beta)$ policies with constant step size $\gamma \le c_0(1-\beta)^2/L$. Then there exists a class $\mathscr{G}_{p,q}(\mathbb{V}_T)$ of μ-strongly convex, L-smooth function sequences whose gradient-variational functional budget satisfies $\mathrm{GVar}_{p,q}(G) \le \mathbb{V}_T$ such that

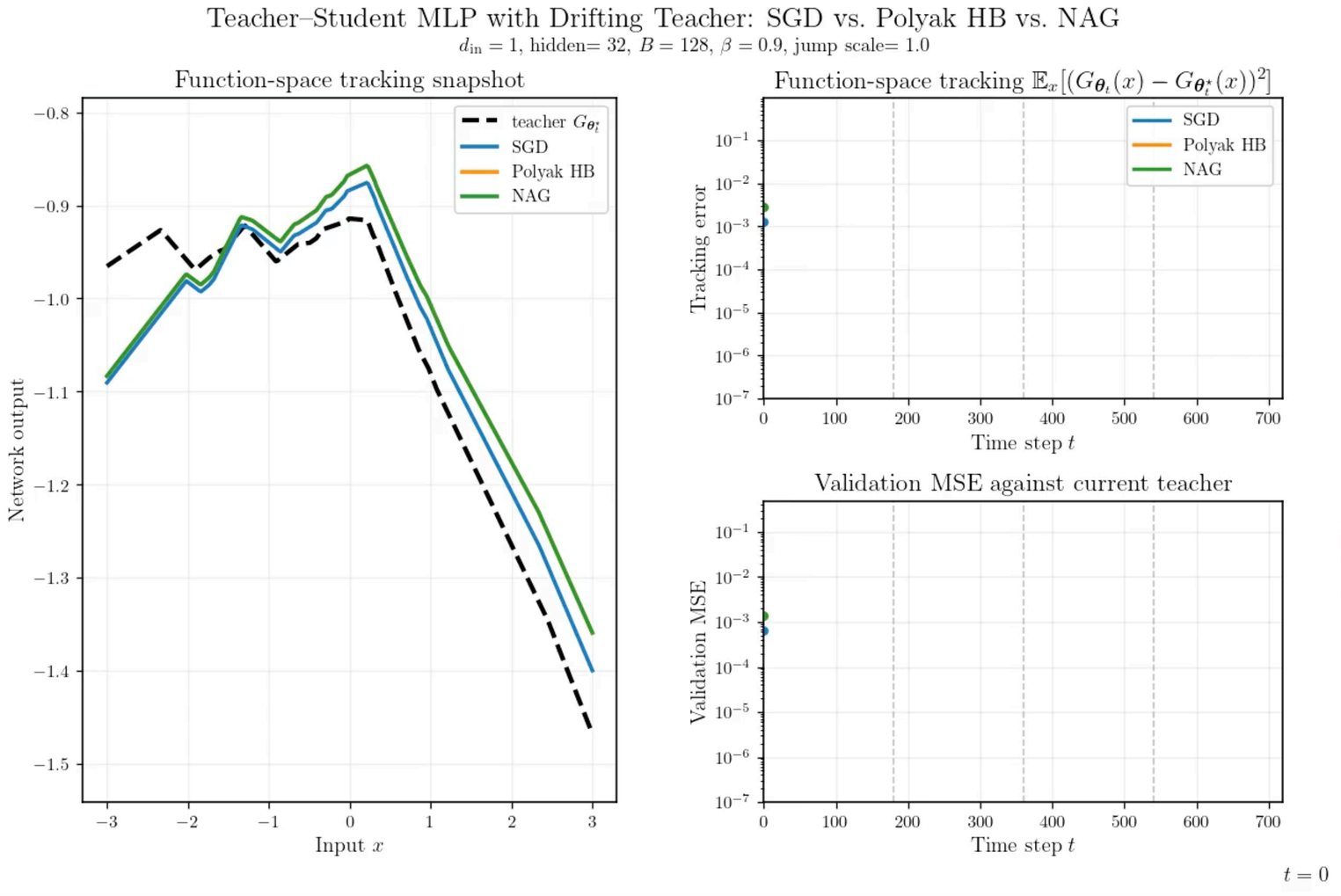
$$\mathfrak{M}_T(\Pi_\beta, \mathscr{V}_T) \gtrsim \max\Big\{\underbrace{(1-\beta)^{-2/(\alpha q+2)}\sigma^{4/(\alpha q+2)}\mu^{(\alpha q-2q-2)/(\alpha q+2)}\mathbb{V}_T^{2q/(\alpha q+2)}T^{\alpha q/(\alpha q+2)}}_{\text{Noise-limited regime}}, \underbrace{(1-\beta)^{-2/(\alpha q)}\mu^{(\alpha q-2q-2)/(\alpha q)}L^{2/(\alpha q)}\mathbb{V}_T^{2/\alpha}T^{1-2/(\alpha q)}}_{\text{Drift-limited regime}}\Big\}\,.$$

- **Takeaway:** No sequence of functions $\{g_t\}_{t=1}^T \subset \mathscr{G}_{p,q}(\mathbb{V}_T)$ using SGDM with any constant stepwise can achieve smaller regret and must incur penalties depending on $(1-\beta)^{-1}$.

Empirical Results - Linear Regression



Empirical Results - Teacher Student MLP



Thank you for listening!

For further details, please visit:

- Our paper: https://openreview.net/forum?id=U1bxeLQLaK
- Our poster: https://icml.cc/virtual/2026/poster/63752