类脑认知计算与脑机融合智能
Brain-inspired Cognitive Computation and
Brain-computer Integrative Intelligence

中国科学院脑科学与智能技术卓越创新中心
(神经科学研究所)
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Functional Building Blocks of Neural Networks: From Network Motifs to Collective Dynamics

Jian Zhang†, Yue Sun†, Wangzi Yao†, Tielin Zhang*

Connectivity and motifs in neural network

- ***Connectivity*** is fundamental to neural computation, shaping ***rich collective dynamics*** for deep learning progress
- ***High-dimensional global connectivity*** lacks interpretability, motivating the study of ***low-dimensional network motifs***.

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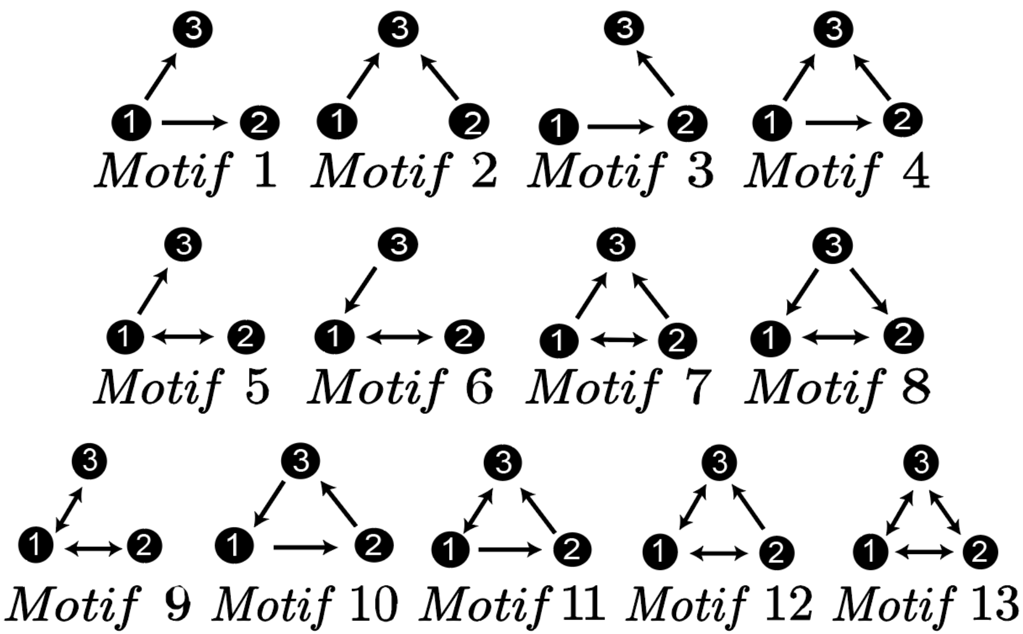
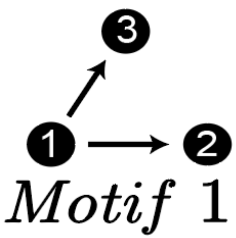
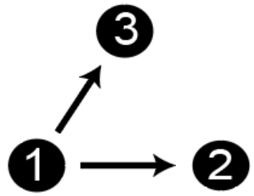




$$\begin{bmatrix} W_{11} & 0 & 0 \\ W_{21} & W_{22} & 0 \\ W_{31} & 0 & W_{33} \end{bmatrix}$$

- This study ***bridges local motifs*** and ***whole-network computation*** to selectively enhance robustness or flexibility.

Network motif training



Motif 1

$$\begin{bmatrix} W_{11} & 0 & 0 \\ W_{21} & W_{22} & 0 \\ W_{31} & 0 & W_{33} \end{bmatrix}$$

$$c_1(W) = \frac{1}{2} \sum_{i,j,k} \overline{W}_{ji} \ddddot{W}_{ij} \ddddot{W}_{ik} \overline{W}_{ki} \ddddot{W}_{kj} \ddddot{W}_{jk}$$

$$= \frac{1}{2} \sum_{j,k} \left\{ \sum_{i} [\overline{W} \otimes \ddddot{W}^T]_{ji} [\ddddot{W} \otimes \overline{W}^T]_{ik} \right\} [\ddddot{W}^T \otimes \ddddot{W}]_{jk}$$

$$= \frac{1}{2} \sum_{j,k} [(\overline{W} \otimes \ddddot{W}^T)(\ddddot{W} \otimes \overline{W}^T)]_{jk} [\ddddot{W}^T \otimes \ddddot{W}]_{jk}$$

$$= \frac{1}{2} L^T [(\overline{W} \otimes \ddddot{W}^T)(\ddddot{W} \otimes \overline{W}^T) \otimes \ddddot{W}^T \otimes \ddddot{W}] L,$$

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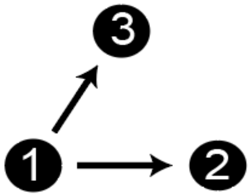

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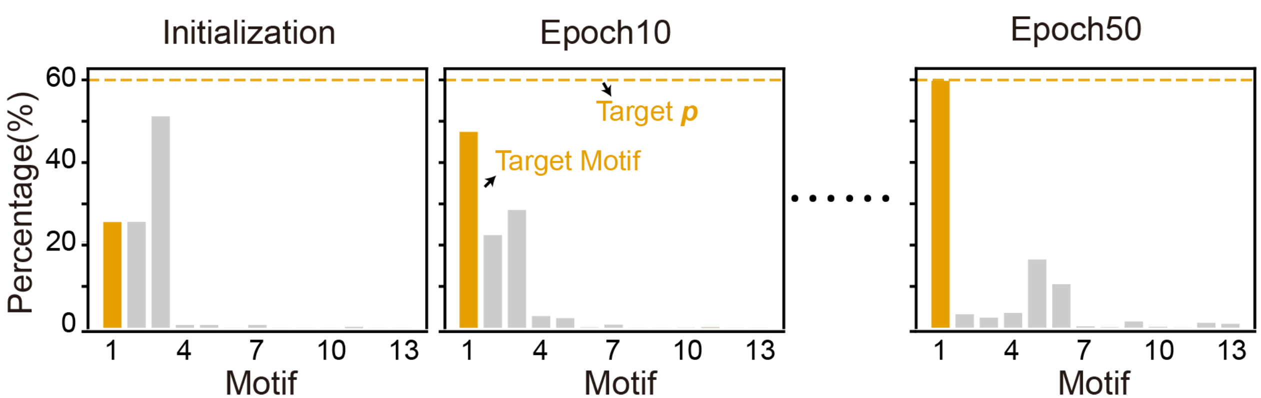
- The target motifs can be embedded through ***loss function***:

$$p_m = \frac{c_m}{\sum_{i=1}^{13} c_i}, \quad \mathcal{L}_{\text{motif}}^m = (p_m - p_m^*)^2,$$



Local dynamics of three-node motifs

➢ By the Hartman–Grobman theorem, ***the local behavior near attractors*** of nonlinear dynamics is governed by the linearized dynamics.

Linear dynamical system

$$\frac{dX}{dt} = \begin{bmatrix} W_{11} & W_{12} & W_{13} \\ W_{21} & W_{22} & W_{23} \\ W_{31} & W_{32} & W_{33} \end{bmatrix} X.$$

Nonlinear dynamical system

$$\frac{dh_i}{dt} = -\frac{h_i}{\tau} + \sum_{j=1}^{3} W_{ij}^{(m)} \phi(h_j) + I_i(t) + \eta_i(t),$$

The local behavior of the nonlinear system by linearizing it around an equilibrium h^*: using Jacobian matrix $J_{ij}(\mathbf{h}^*) = \partial \dot{h}_i / \partial h_j |_{\mathbf{h}=\mathbf{h}^*}$

$$\begin{bmatrix} -\frac{1}{\tau} + W_{11}[1 - \tanh^2(h_1^*)] & W_{12}[1 - \tanh^2(h_2^*)] & W_{13}[1 - \tanh^2(h_3^*)] \\ W_{21}[1 - \tanh^2(h_1^*)] & -\frac{1}{\tau} + W_{22}[1 - \tanh^2(h_2^*)] & W_{23}[1 - \tanh^2(h_3^*)] \\ W_{31}[1 - \tanh^2(h_1^*)] & W_{32}[1 - \tanh^2(h_2^*)] & -\frac{1}{\tau} + W_{33}[1 - \tanh^2(h_3^*)] \end{bmatrix},$$

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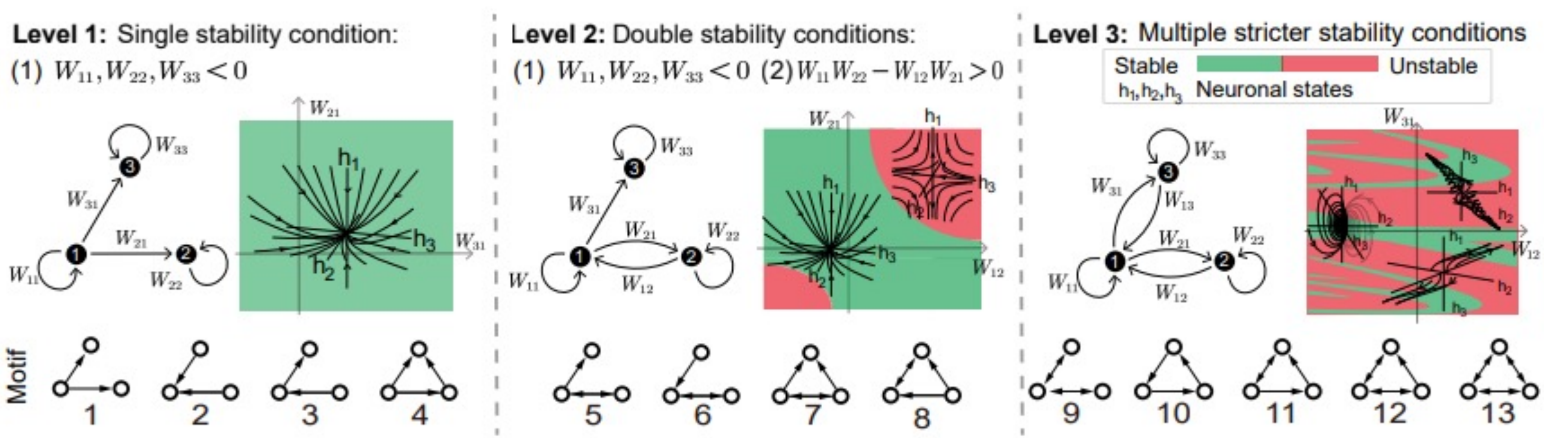
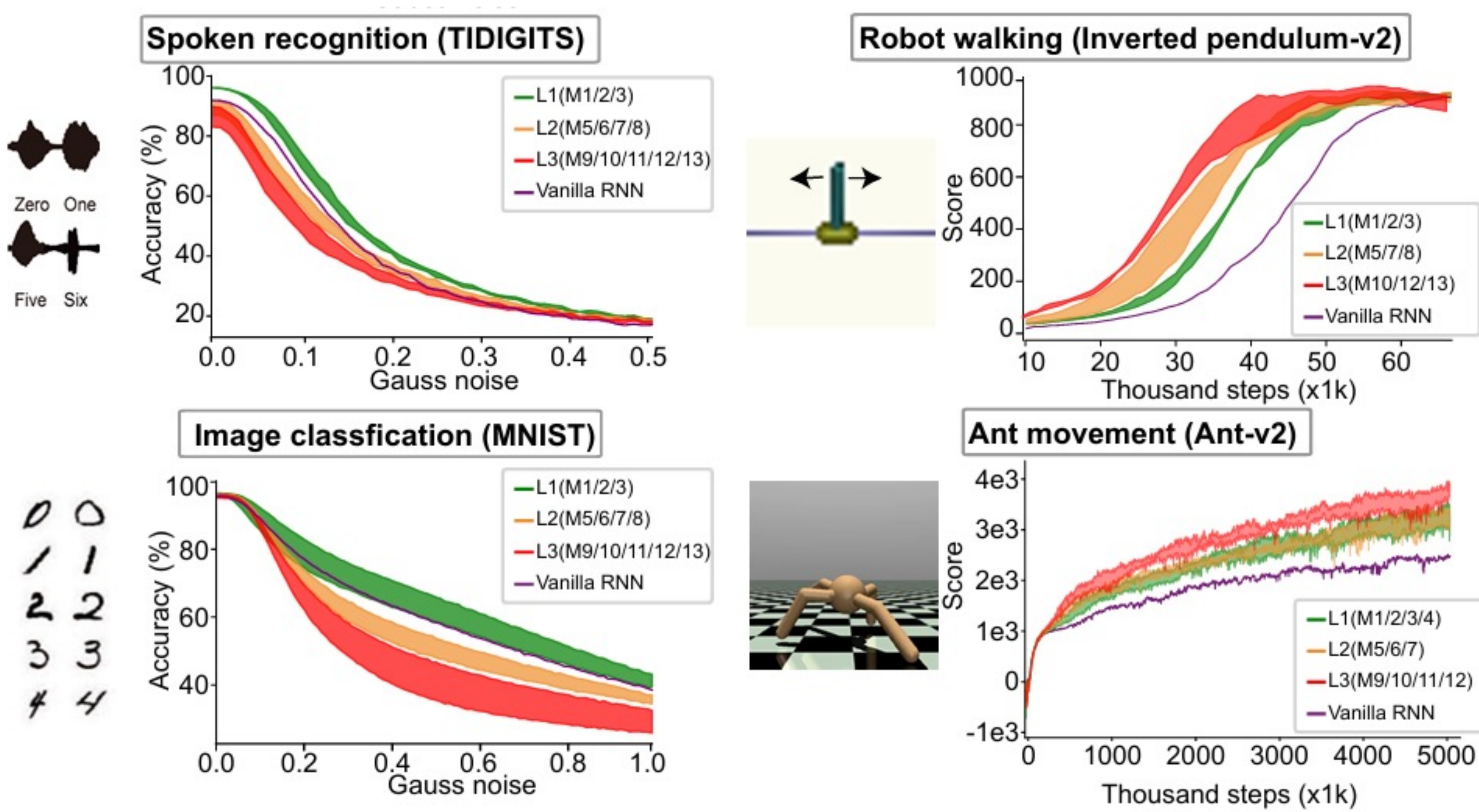
- This allows the 13 directed three-node motifs to be grouped into ***three levels*** according to their ***stability conditions***.



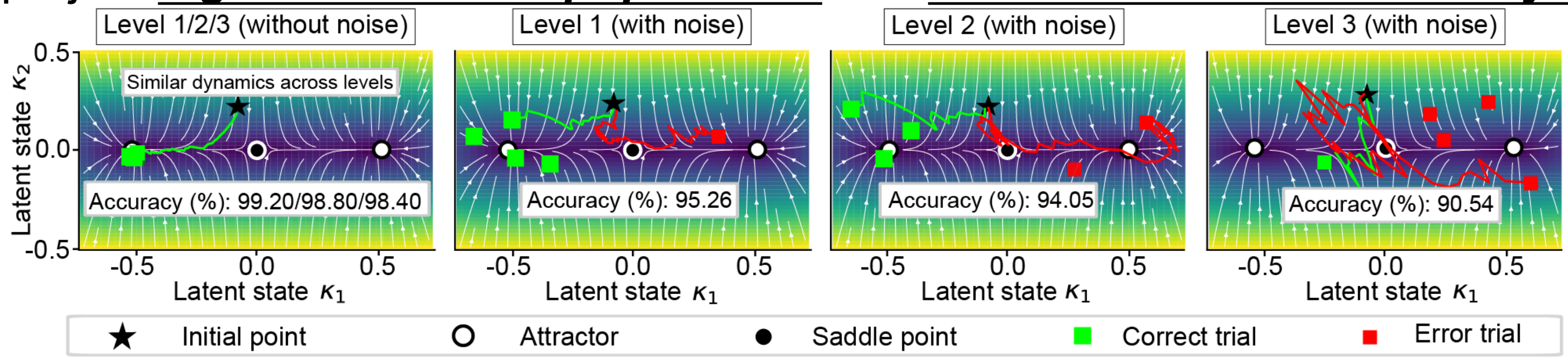
Computational Capabilities of Motif-Endowed Networks



Networks enriched with **Level-1 motifs** enhance **accuracy under noise**, while networks with **Level-3 motifs accelerate reinforcement learning**.

Collective dynamics of Motif-Endowed Networks

We project ***high-dimensional population*** onto ***a low-dimensional latent dynamics***:



Networks enriched ***with Level-1 motifs*** form ***latent trajectories*** that ***are more resistant to noise perturbations***

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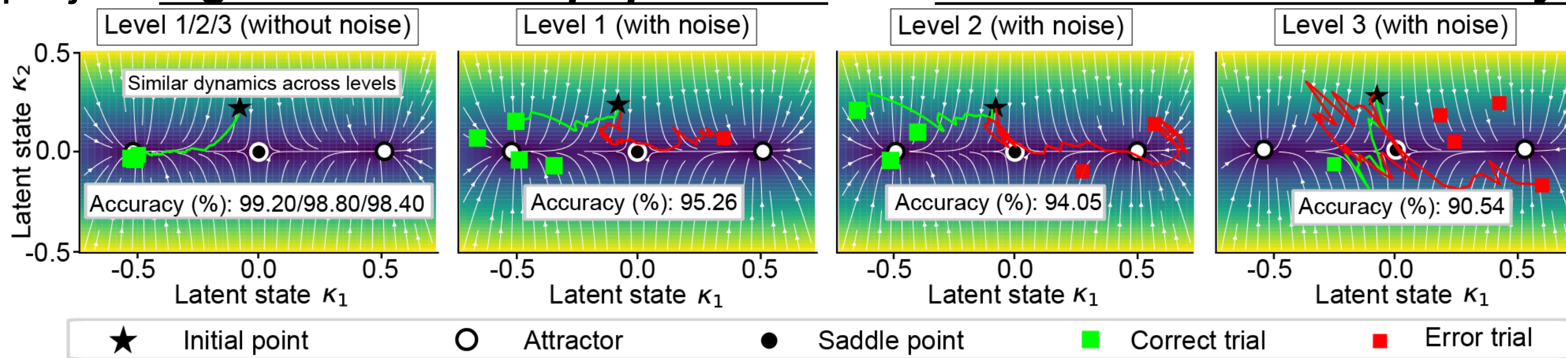
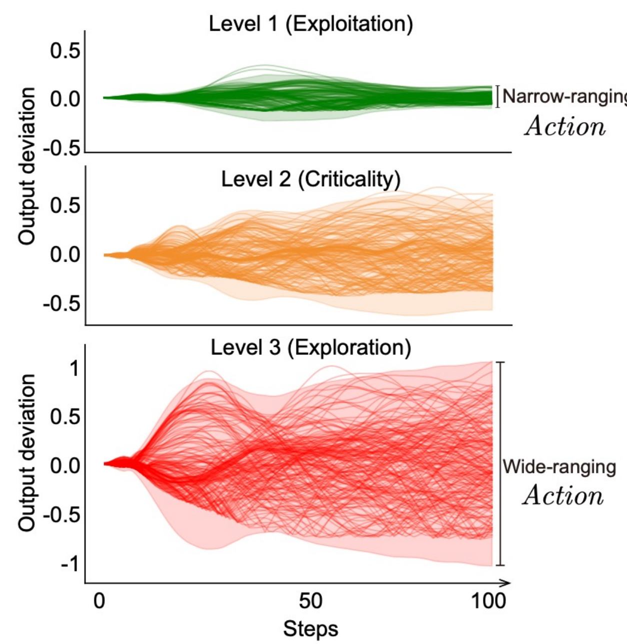


Networks enriched ***with Level-1 motifs*** form ***latent trajectories*** that ***are more resistant to noise perturbations***

- We track the step-by-step output deviation observations over 100 steps
- Networks enriched with ***Level-3 motifs*** sustain ***wide-ranging output deviations*** that ***facilitate exploration***.



Conclusion and future direction

- We developed scalable, differentiable framework for proactive network motifs of ANNs
- The 13 motifs are mathematically stratified into a three-tiered stability hierarchy.
- Altering motif composition shapes global dynamics to balance robustness and flexibility.
- Bridges local circuits to network capabilities, scaling to larger subgraphs, MLPs, and Transformers.

Paper

Lab

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