Trainable Nonexpansive Denoisers for Contractive Image Reconstruction

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Motivation: learned PnP needs stability

Inverse problem

$$y = Ax + e, \qquad \text{recover } x.$$

PnP methods use a denoiser inside an iterative solver instead of an explicit regularizer.

- Powerful CNN denoisers improve image quality.
- But repeated application can make reconstruction dynamics unstable.
- Empirical Lipschitz penalties are usually training-sample based.
- Directly controlling the NN reduces performance.

Question

Can we design a trainable denoiser that is nonexpansive *by construction*, and still competitive in reconstruction?

Target guarantee

$$\|D(x) - D(x')\|_2 \leq \|x - x'\|_2$$

for all images, not only training samples.

Key idea: Unconstrained NN, constrain the operator

- Use a set of permutations $\mathcal{G}$ acting on the image lattice.
- Two images: input image x and a reference image ξ
- The Neural Network compares

$$\xi \quad \text{and} \quad \pi\xi.$$

- The denoiser aggregates permuted copies of the input image x.

Expressive

Learnable Neural Network

Controllable

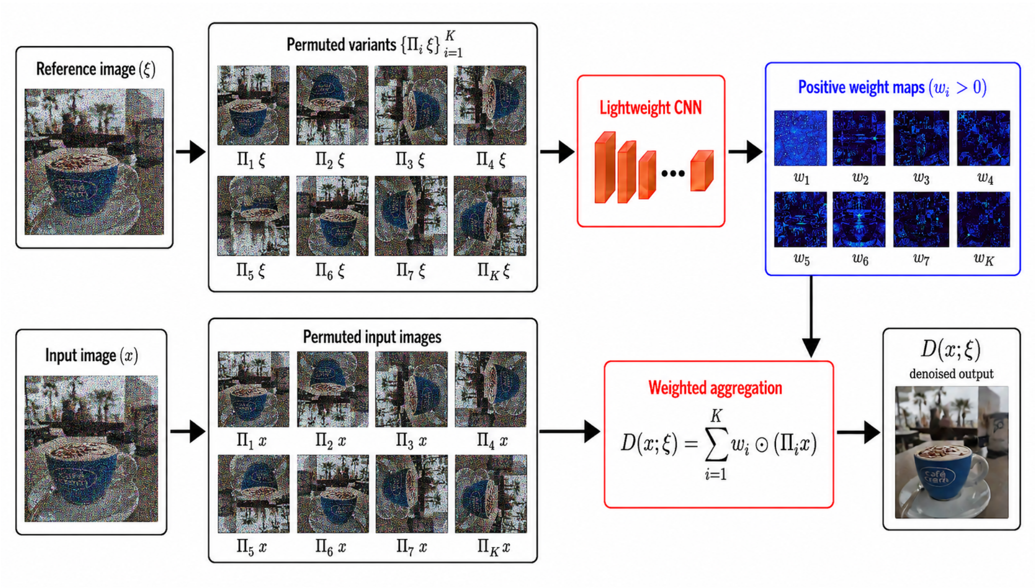
structured aggregation operator

Unnormalized aggregation

$$K(x;\xi) = \sum_{\pi \in \mathcal{G}} \mathcal{N}_\theta(\xi, \pi\xi) \odot (\pi x).$$

Normalized denoiser

$$D(x;\xi) = C(\xi)^{-1} K(x;\xi),$$
$$C(\xi) = K(e;\xi).$$


Reference image (ξ)
Permuted variants $\{\Pi_i \xi\}_{i=1}^{K}$
$\Pi_1 \xi$
$\Pi_2 \xi$
$\Pi_3 \xi$
$\Pi_4 \xi$
$\Pi_5 \xi$
$\Pi_6 \xi$
$\Pi_7 \xi$
$\Pi_K \xi$
Lightweight CNN
Positive weight maps ($w_i > 0$)
w_1
w_2
w_3
w_4
w_5
w_6
w_7
w_K
Input image (x)
Permuted input images
$\Pi_1 x$
$\Pi_2 x$
$\Pi_3 x$
$\Pi_4 x$
$\Pi_5 x$
$\Pi_6 x$
$\Pi_7 x$
$\Pi_K x$
Weighted aggregation
$D(x;\xi) = \sum_{i=1}^{K} w_i \odot (\Pi_i x)$
$D(x;\xi)$
denoised output

Assumptions to impose control

(A1) Set structure:
$\pi_{id} \in G$, and G is closed under inversion, i.e., if $\pi \in G$, then $\pi^{-1} \in G$.

(A2) Positivity:
$N_\theta(\xi, \xi') > 0$ for all $\xi, \xi' \in X$.

(A3) Inverse-Consistency Condition (ICC):
For all $\pi \in G$: $N_\theta(\xi, \pi \cdot \xi) = \pi \cdot N_\theta(\xi, \pi^{-1} \cdot \xi)$.

(A4) Transitivity:
For all $i, j \in \Omega$, there exist $\pi_1, \ldots, \pi_k \in G$ such that $\pi_k \cdots \pi_1(i) = j$.

(A5) Nonannihilating forward model:
$Ae \neq 0$.

Symmetrization of the Denoiser

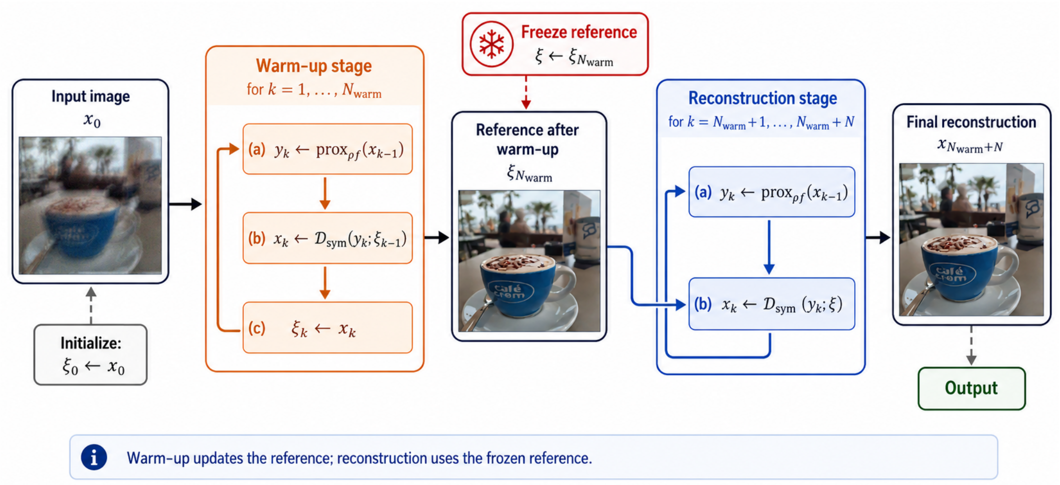
$$D_{\mathrm{sym}}(x;\xi) = \frac{1}{\|\hat{e}\|_\infty} C(\xi)^{\frac{1}{2}} \odot D(x \oslash C(\xi)^{\frac{1}{2}};\xi) + \left(e - \frac{\hat{e}}{\|\hat{e}\|_\infty}\right) \odot x,$$

Lemma (Nonexpansivity of the Denoiser)

Suppose ***(A1)***,***(A2)*** *and* ***(A3)*** *hold. Then,*

$$\|D_{\mathrm{sym}}(\cdot;\xi)\|_2 = 1.$$

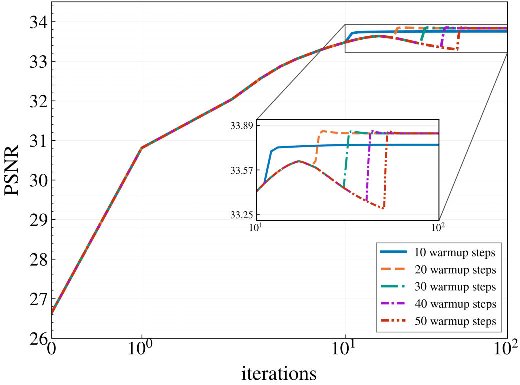
Reconstruction with HQS



Contractivity of the IR operator

Theorem (Contractivity of the reconstruction operator)

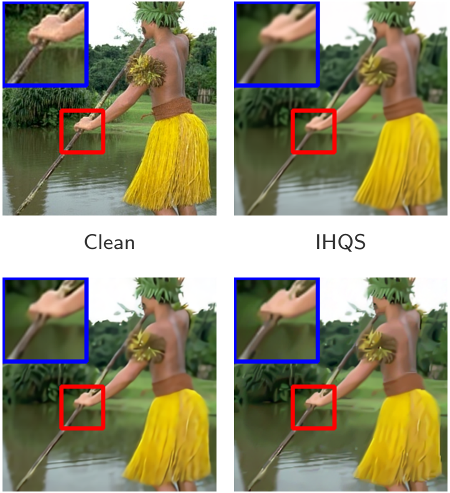
*Under assumptions **(A1)–(A5)**, for any ξ, the operator $D_{\mathrm{sym}} \circ \mathrm{prox}_{\rho f}$ is contractive.*



Experimental validation

Task	Input	Ours	Best baseline
Deblur, $\nu = 0.03$	20.32	**28.29**	29.09
Deblur, $\nu = 0.05$	19.44	**26.75**	27.43
SR avg.	21.88	**24.95**	25.35
Denoise $\sigma = 50/255$	14.14	**26.42**	27.13

The method is competitive with strong convergent/stability-oriented baselines while providing stronger global guarantees.



Clean IHQS

CoCo-ADMM Ours