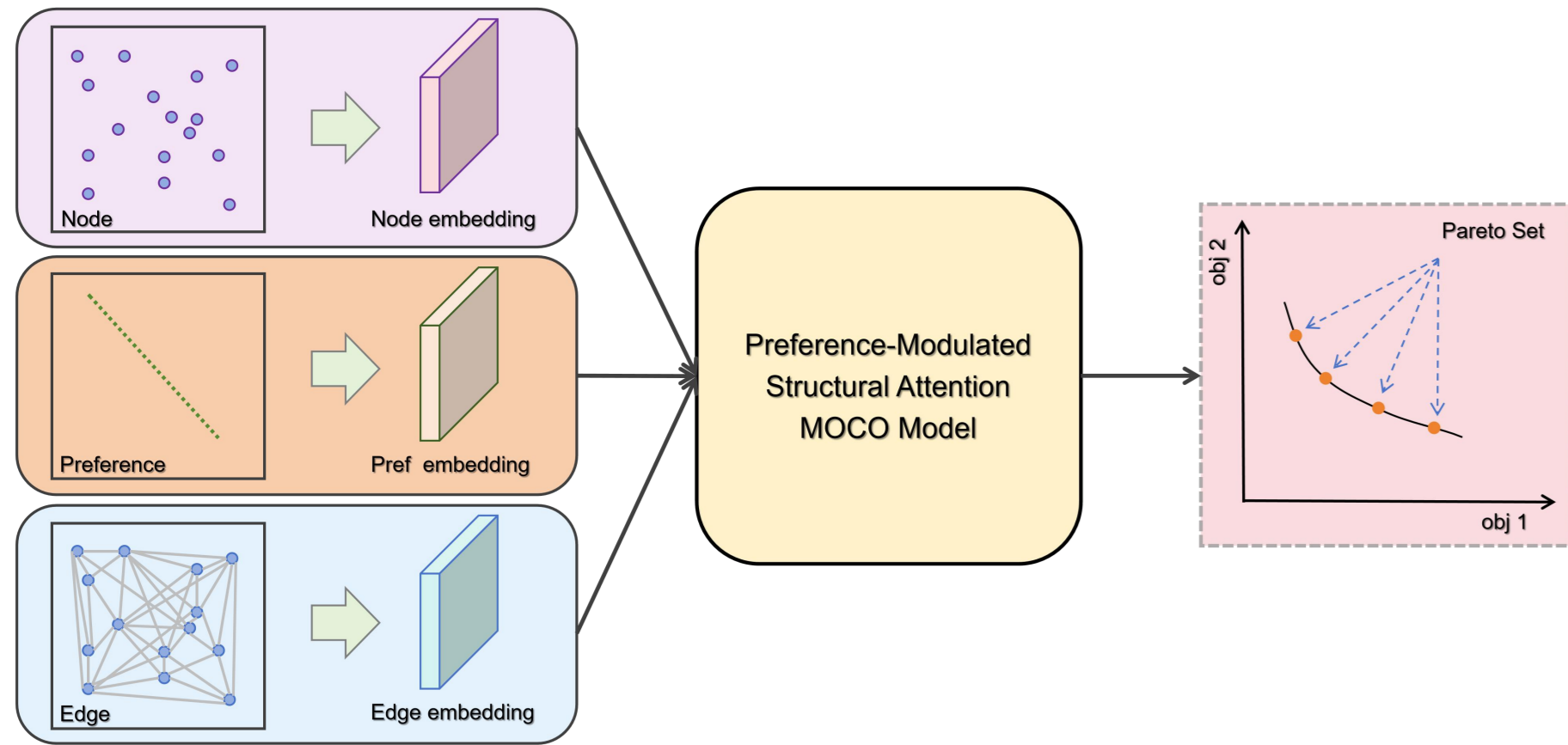


## 1. Introduction

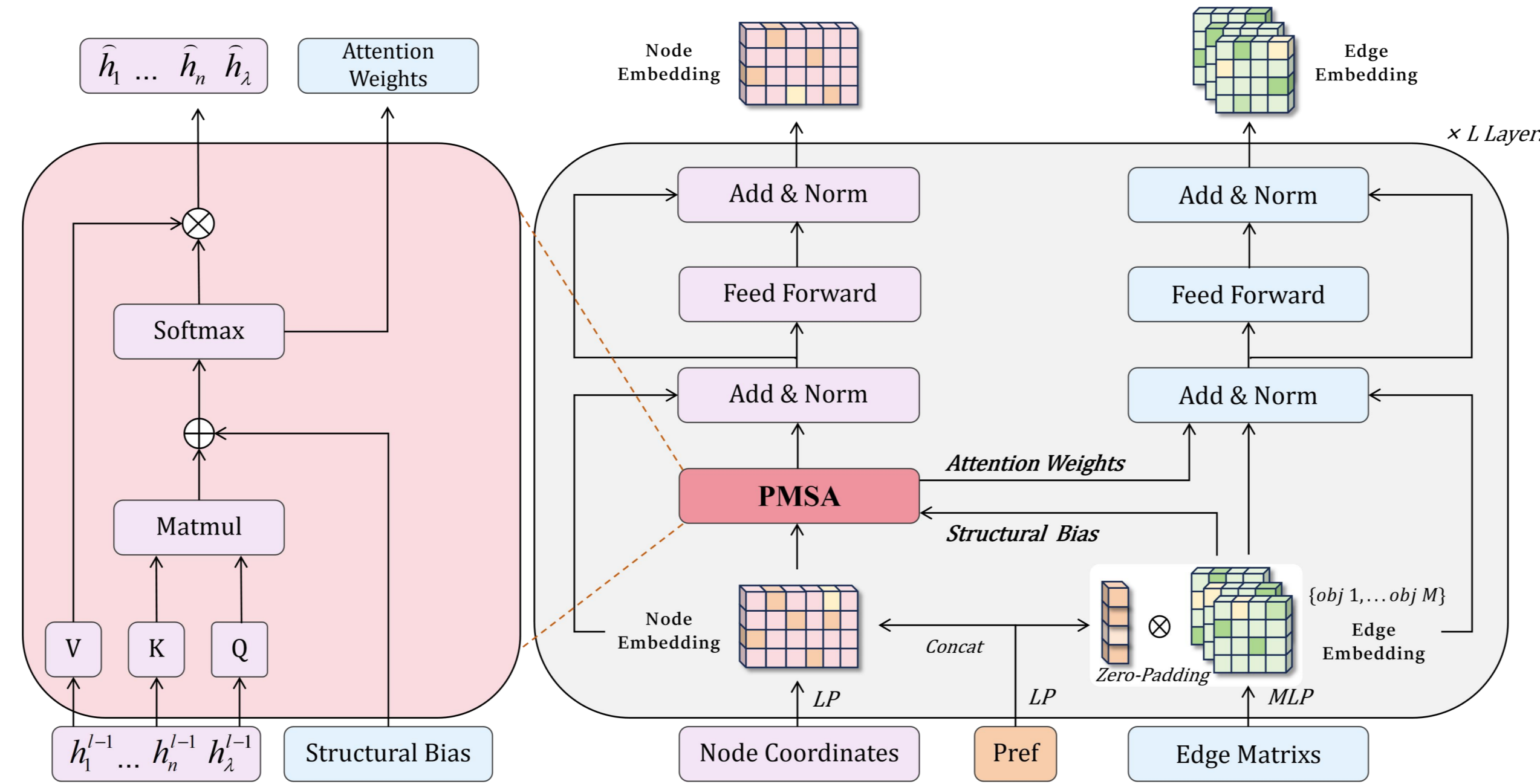


➤ We devise **PMSA** that explicitly injects edge attributes into attention scores as a prior to capture preference-specific topologies.

➤ We design a node-guided dynamic **edge feature aggregation strategy** where node features actively refine edge representations during encoding, enhancing the perception of sub-problem structures.

➤ Extensive evaluations on Bi-TSP and Bi-CVRP benchmarks confirm that our approach outperforms **state-of-the-art** baselines efficiency.

## 2. Method: PMSA



### Preference-Modulated Edge Feature Construction:

$$e_{ij}^{(m)} = \left\| x_i^{(m)} - x_j^{(m)} \right\|_2^2$$

$$h_e = W_2 \sigma \left( W_1 \left[ e_{ij}^{(1)}, \dots, e_{ij}^{(M)} \right]^T + b_1 \right) + b_2$$

$$B_{ij} = \begin{cases} h_{ij}^e \odot \text{Linear}(\lambda), & \text{if } i, j \leq N \\ 0, & \text{otherwise} \end{cases}$$

### Edge Feature Aggregation:

$$A_{ij} = \frac{(W_Q h_i)^T (W_K h_j)}{\sqrt{d_k}} + B_{ij} \quad w_{ij} = W_e \text{Softmax} \left( A_{ij}^{(l)} \right) + b_e$$

$$\tilde{B}_{ij}^{(l)} = \text{Norm} \left( B_{ij}^{(l)} + w_{ij} \right) \quad B_{ij}^{(l+1)} = \text{Norm} \left( \tilde{B}_{ij}^{(l)} + \text{FFN} \left( \tilde{B}_{ij}^{(l)} \right) \right)$$

## 3. Main Result

| METHOD          | Bi-TSP20      |               |          | Bi-TSP50      |              |          | Bi-TSP100     |               |          |
|-----------------|---------------|---------------|----------|---------------|--------------|----------|---------------|---------------|----------|
|                 | HV (↑)        | GAP (↓)       | TIME (↓) | HV (↑)        | GAP (↓)      | TIME (↓) | HV (↑)        | GAP (↓)       | TIME (↓) |
| WS-LKH          | 0.6270        | 0.00%         | 10M      | <b>0.6415</b> | <b>0.00%</b> | 1.8H     | <b>0.7090</b> | <b>-0.10%</b> | 6H       |
| MOEA/D          | 0.6241        | 0.46%         | 1.7H     | 0.6316        | 1.54%        | 1.8H     | 0.6899        | 2.60%         | 2.2H     |
| NSGA-II         | 0.6258        | 0.19%         | 6.0H     | 0.6120        | 4.60%        | 6.1H     | 0.6692        | 5.52%         | 6.9H     |
| MOGLS           | <b>0.6279</b> | <b>-0.14%</b> | 1.6H     | 0.6330        | 1.33%        | 3.7H     | 0.6854        | 3.23%         | 11H      |
| PPLS/D-C        | 0.6256        | 0.22%         | 26M      | 0.6282        | 2.07%        | 2.8H     | 0.6844        | 3.37%         | 11H      |
| DRL-MOA         | 0.6257        | 0.21%         | 6S       | 0.6360        | 0.86%        | 9S       | 0.6970        | 1.60%         | 16S      |
| MDRL            | 0.6271        | -0.02%        | 5S       | 0.6364        | 0.80%        | 8S       | 0.6969        | 1.61%         | 14S      |
| EMNH            | 0.6271        | -0.02%        | 5S       | 0.6364        | 0.80%        | 8S       | 0.6969        | 1.61%         | 15S      |
| PMOCO           | 0.6259        | 0.18%         | 6S       | 0.6351        | 1.00%        | 12S      | 0.6957        | 1.78%         | 26S      |
| CNH             | 0.6270        | 0.00%         | 14S      | 0.6387        | 0.44%        | 17S      | 0.7019        | 0.90%         | 29S      |
| WE-CA           | 0.6270        | 0.00%         | 7S       | 0.6392        | 0.36%        | 10S      | 0.7034        | 0.69%         | 21S      |
| <b>PMSA</b>     | <b>0.6272</b> | <b>-0.03%</b> | 8S       | 0.6412        | 0.05%        | 19S      | 0.7070        | 0.18%         | 38S      |
| MDRL-AUG        | 0.6271        | -0.02%        | 34S      | 0.6408        | 0.11%        | 1.7M     | 0.7022        | 0.86%         | 14M      |
| EMNH-AUG        | 0.6271        | -0.02%        | 34S      | 0.6408        | 0.11%        | 1.7M     | 0.7023        | 0.85%         | 14M      |
| PMOCO-AUG       | 0.6270        | 0.00%         | 1.0M     | 0.6395        | 0.31%        | 3.2M     | 0.7016        | 0.95%         | 15M      |
| CNH-AUG         | 0.6271        | -0.02%        | 1.5M     | 0.6410        | 0.08%        | 4.1M     | 0.7054        | 0.41%         | 16M      |
| WE-CA-AUG       | 0.6271        | -0.02%        | 1.0M     | 0.6413        | 0.03%        | 3.3M     | 0.7066        | 0.24%         | 16M      |
| <b>PMSA-AUG</b> | 0.6270        | 0.00%         | 1.2M     | <b>0.6415</b> | <b>0.00%</b> | 5.8M     | <b>0.7083</b> | <b>0.00%</b>  | 22M      |

