Incremental BPE Tokenization

by Shenghu Jiang (姜圣虎) and Ruihao Gong (龚睿昊)

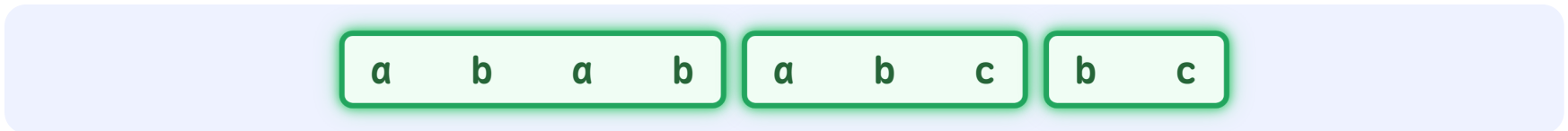
BUAA & SenseTime

Byte-Pair Encoding (BPE)

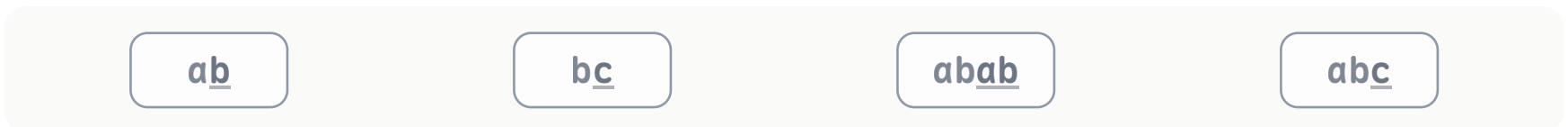
1. By Priority

2. Left-to-Right

3. Non-Overlap



Current Token Sequence

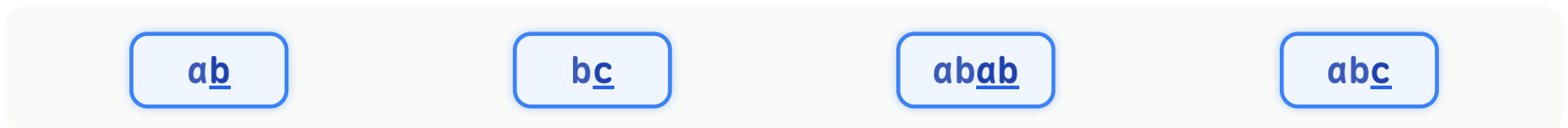


Merge Rules: `["a b", "b c", "ab ab", "ab c"]`

Token-wise Truncation

Token-wise truncation $\Longleftrightarrow$ Tokenization of the corresponding string

Current Token Sequence



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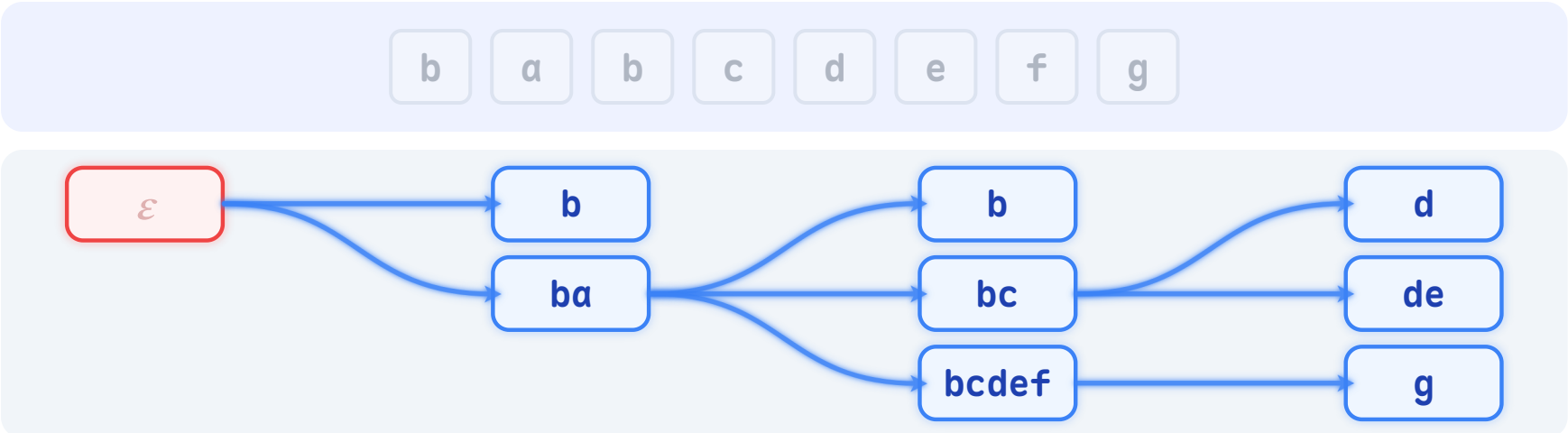
Prefix Tree of Tokens



Prefix Tree of Tokens for `babcdefg`

Merge Rules

Incremental Objective

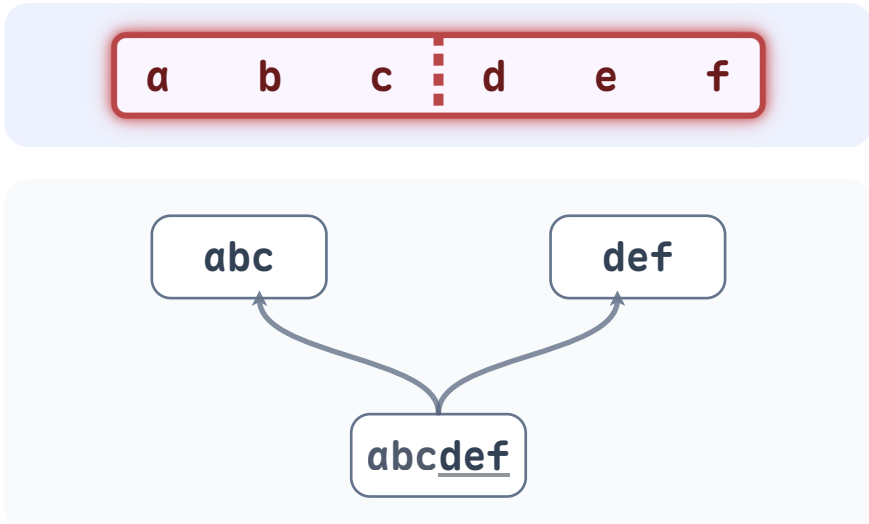
The "Last Token"

Incrementally maintain the "**last token**" $\theta(p)$ for *each prefix* p of the input string s.

> **Note**
> Here we can naturally derive the "eager output" mechanism. Check out the "Eager Output" section in the paper for details!

Predecessors & Successors

- Predecessor: $\text{pre}(t)$
- Successor: $\text{suc}(t)$
- $t = \text{concatenate}(\text{pre}(t), \text{suc}(t))$
- Deterministic & token-wise truncation ->
  - The relationship is **unique**!



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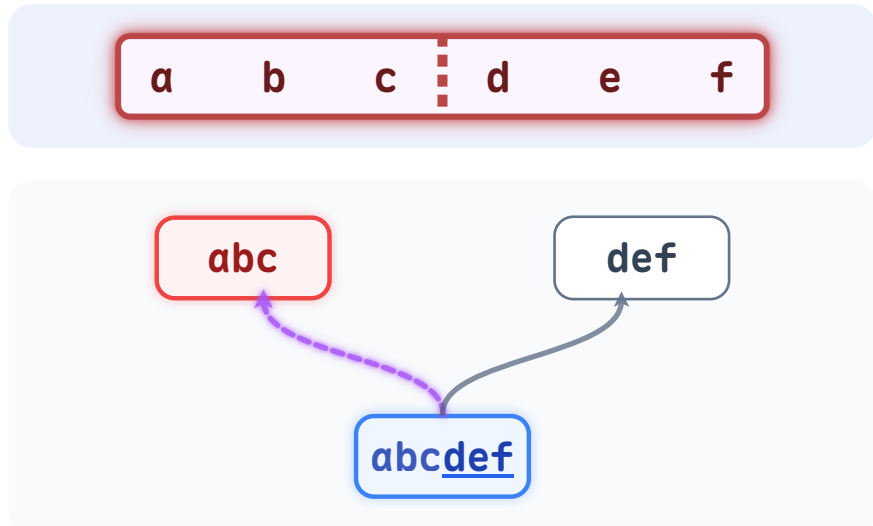

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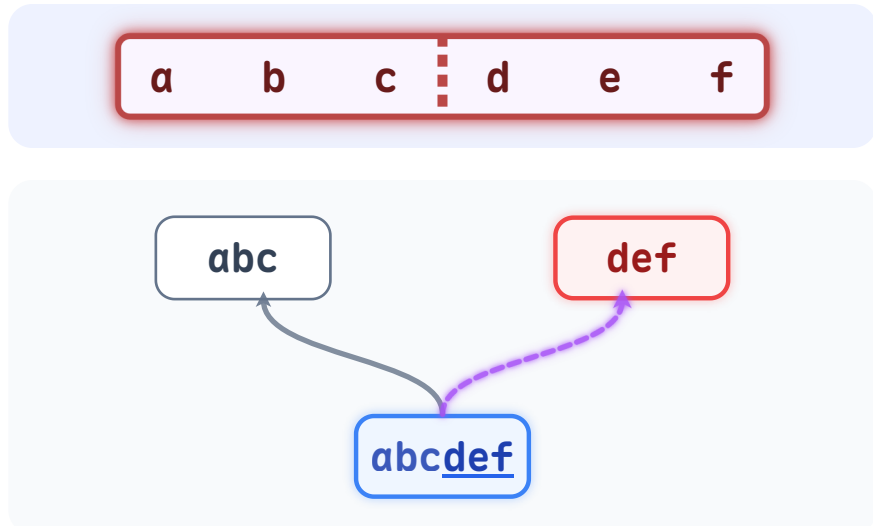
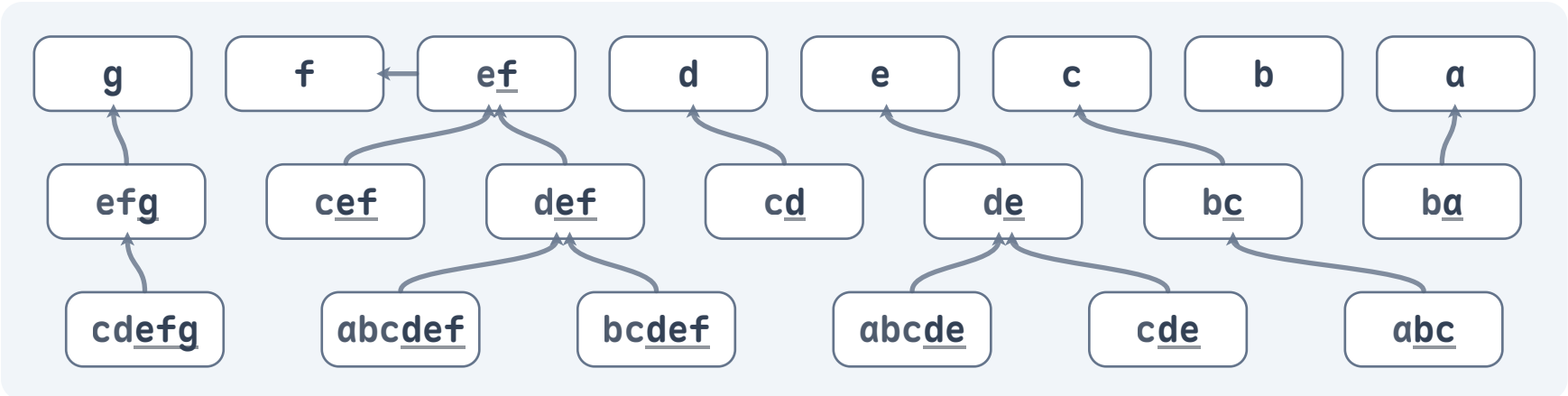


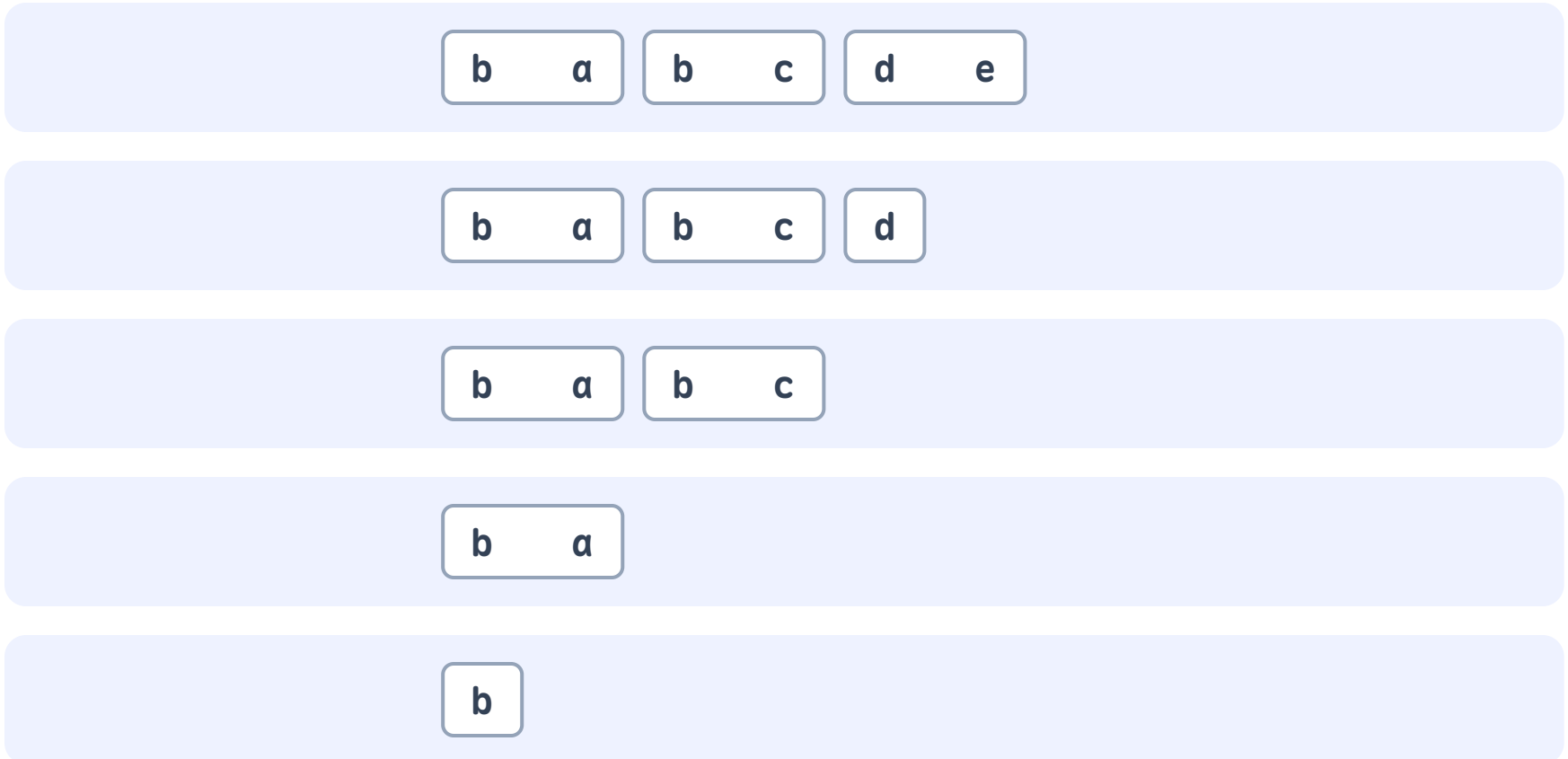
Successor Forest



Successor Forest

Merge Rules

Which One is the Answer?



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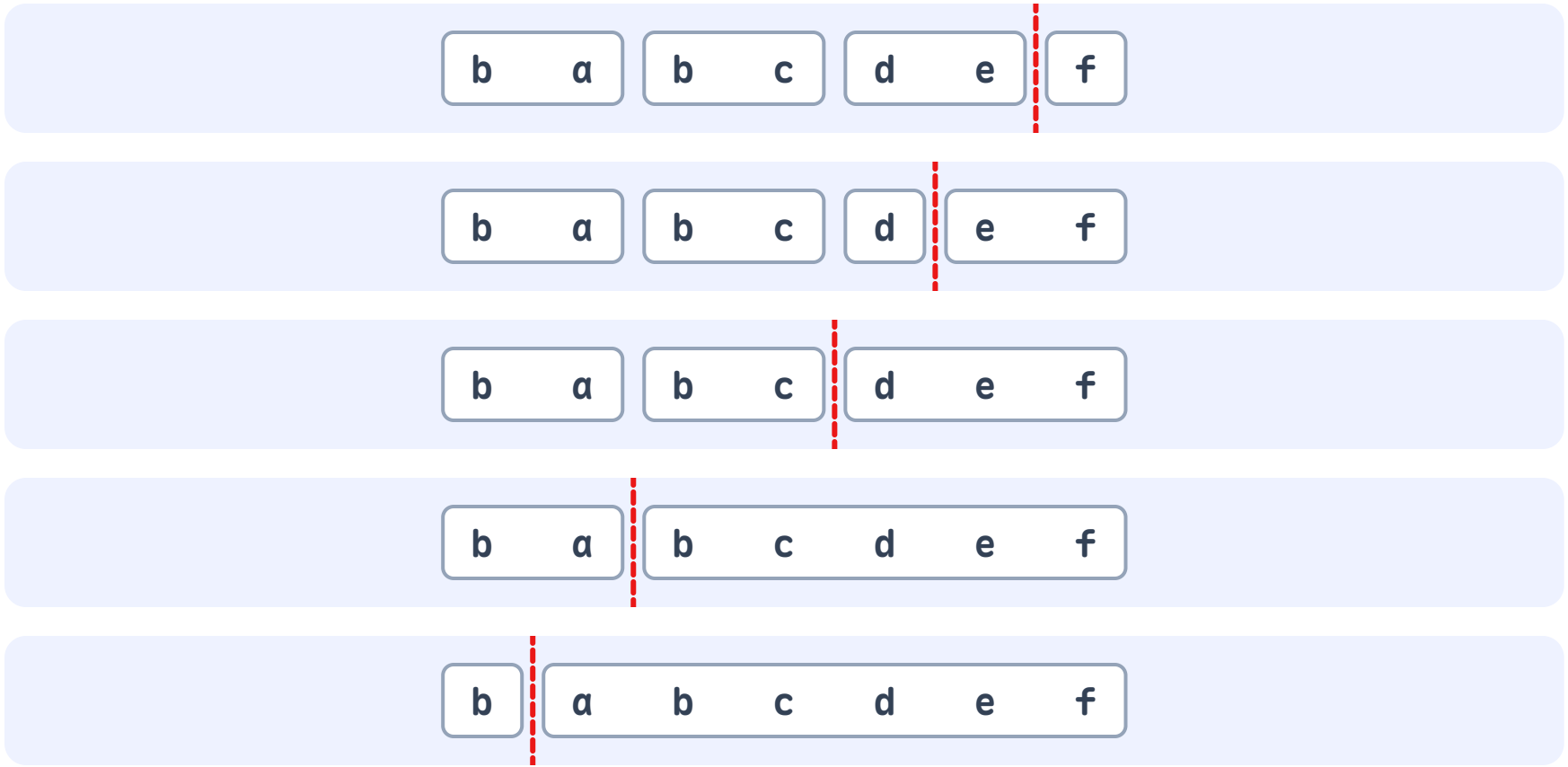

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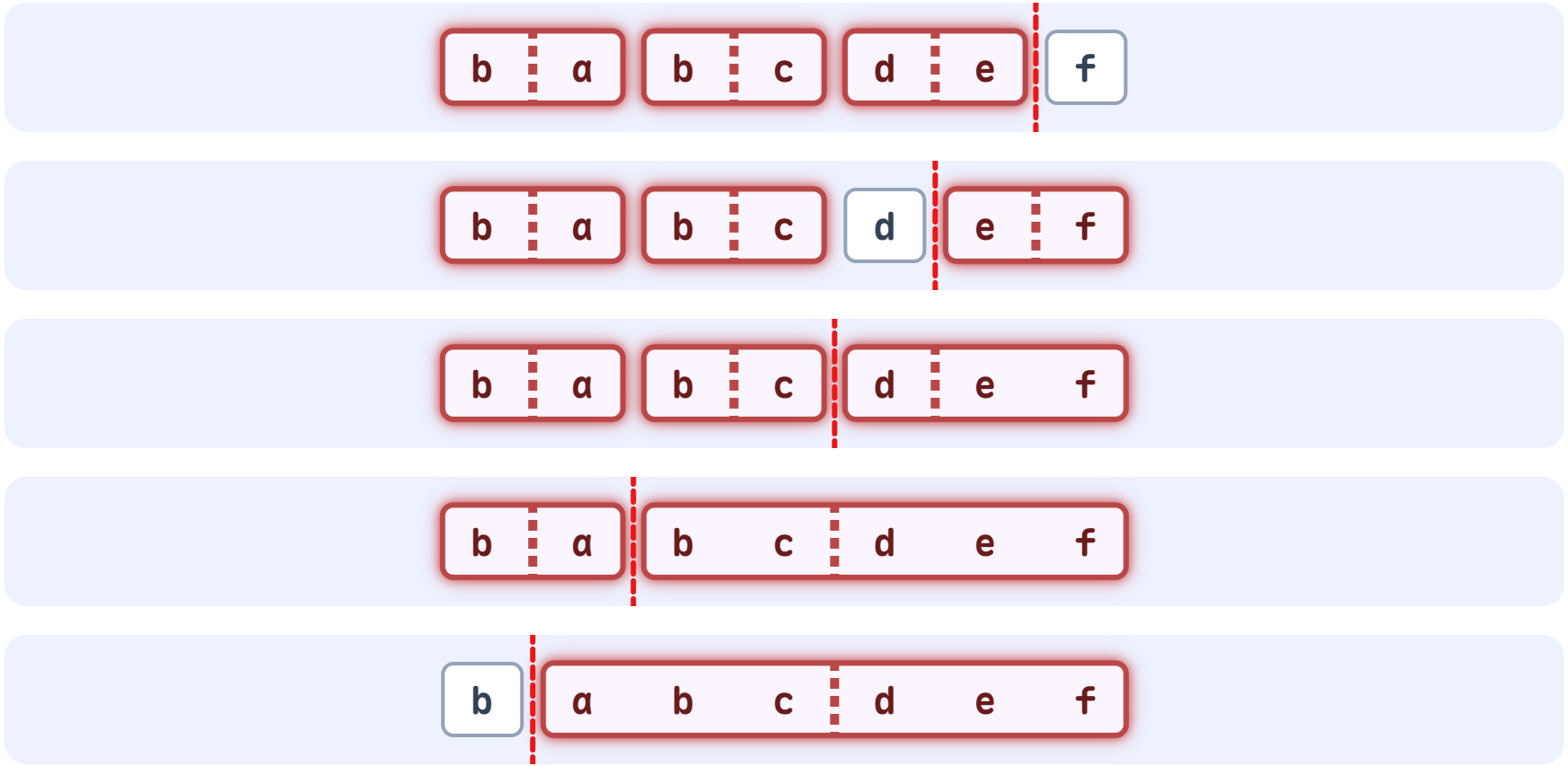

Which One is the Answer?

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Monotonic Path: Chain to the "Last Token"

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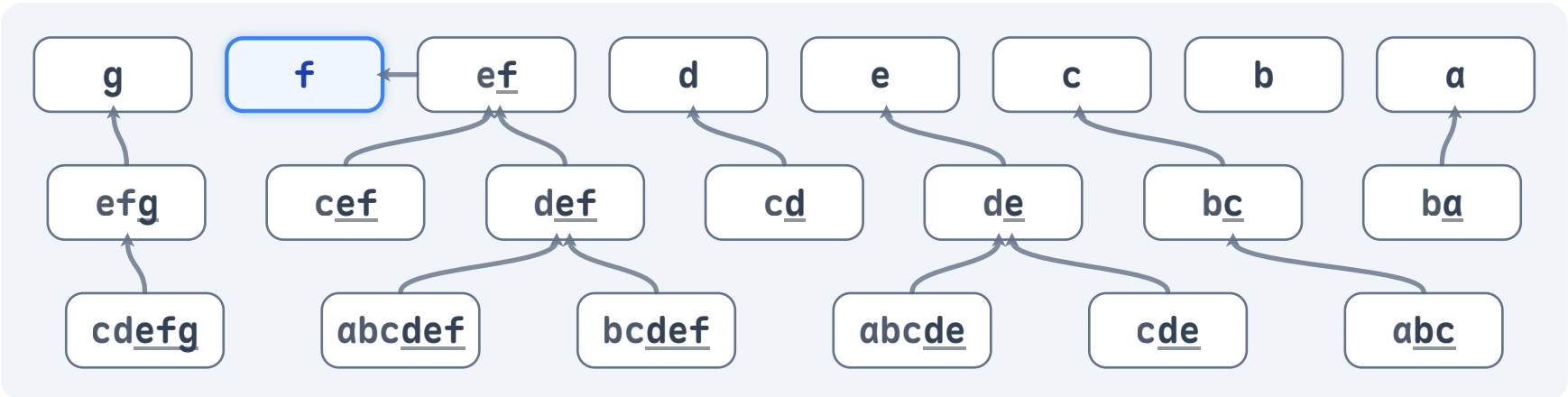

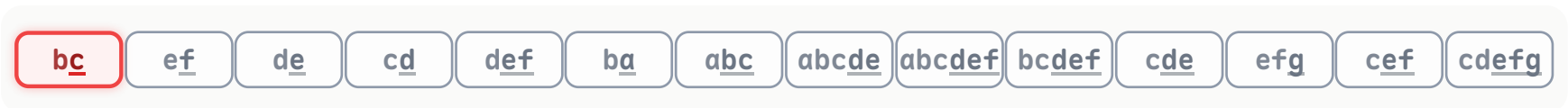

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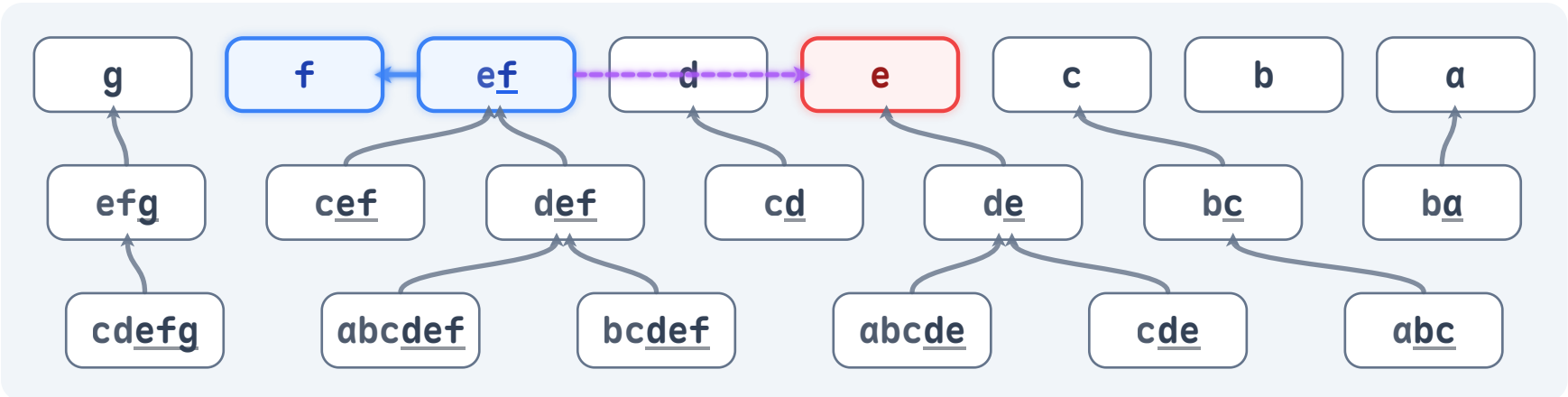

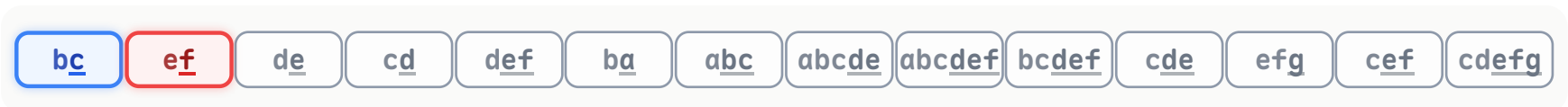

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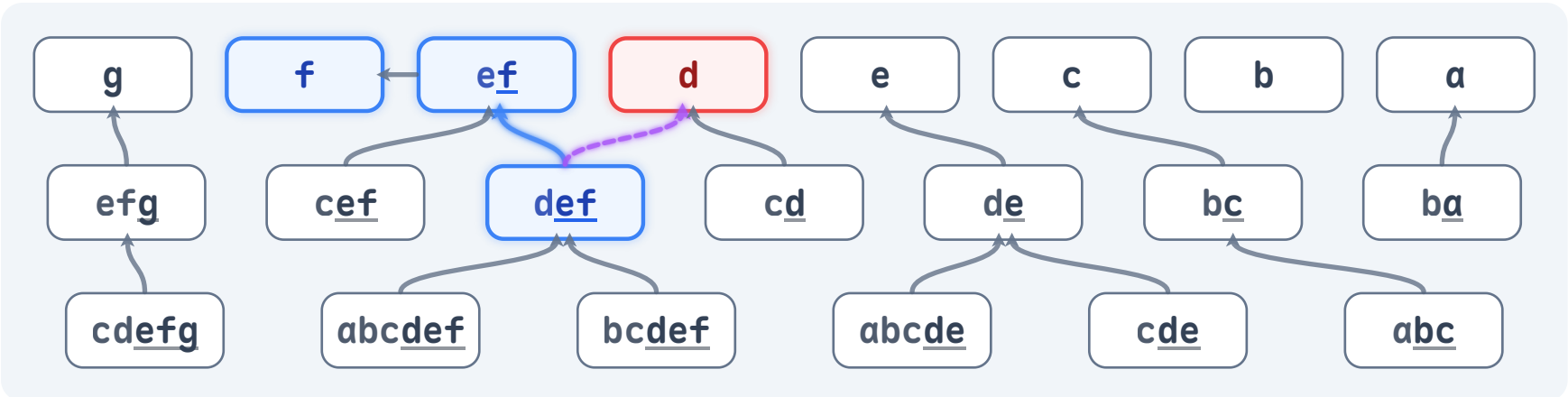

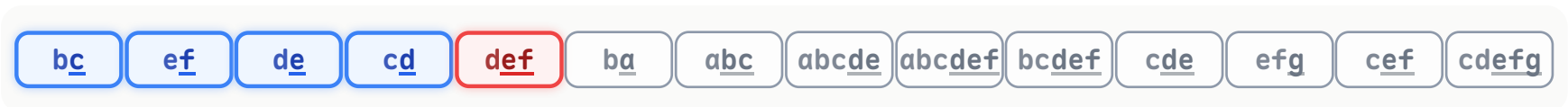

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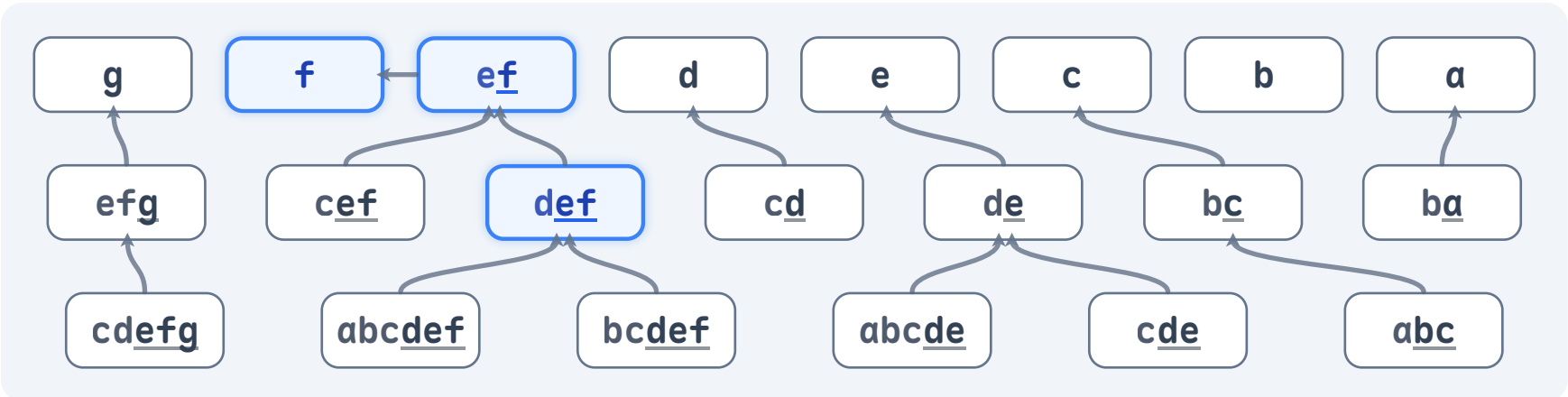

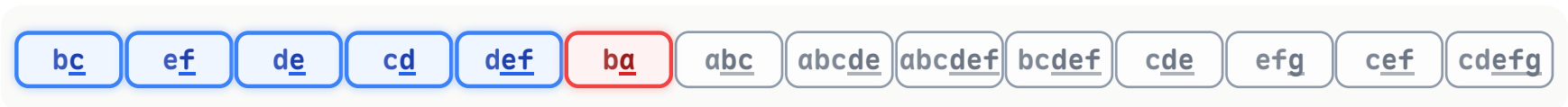

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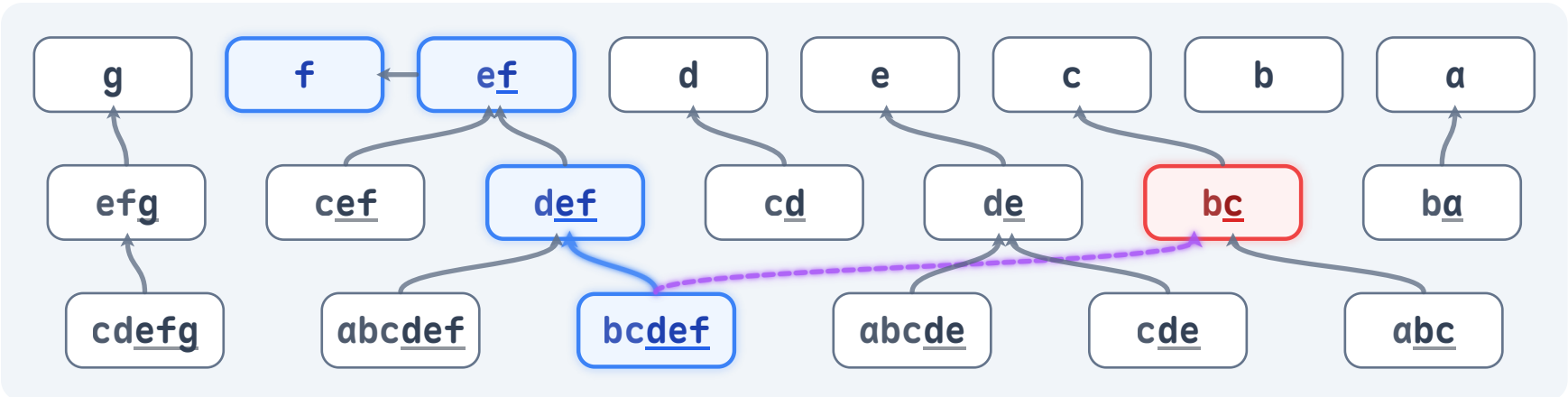

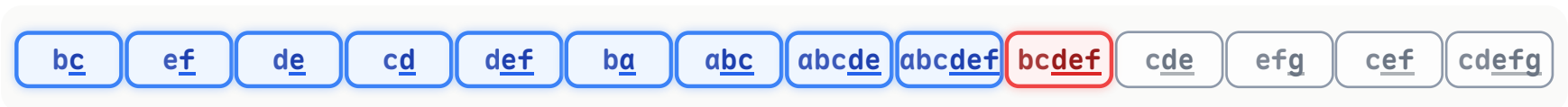

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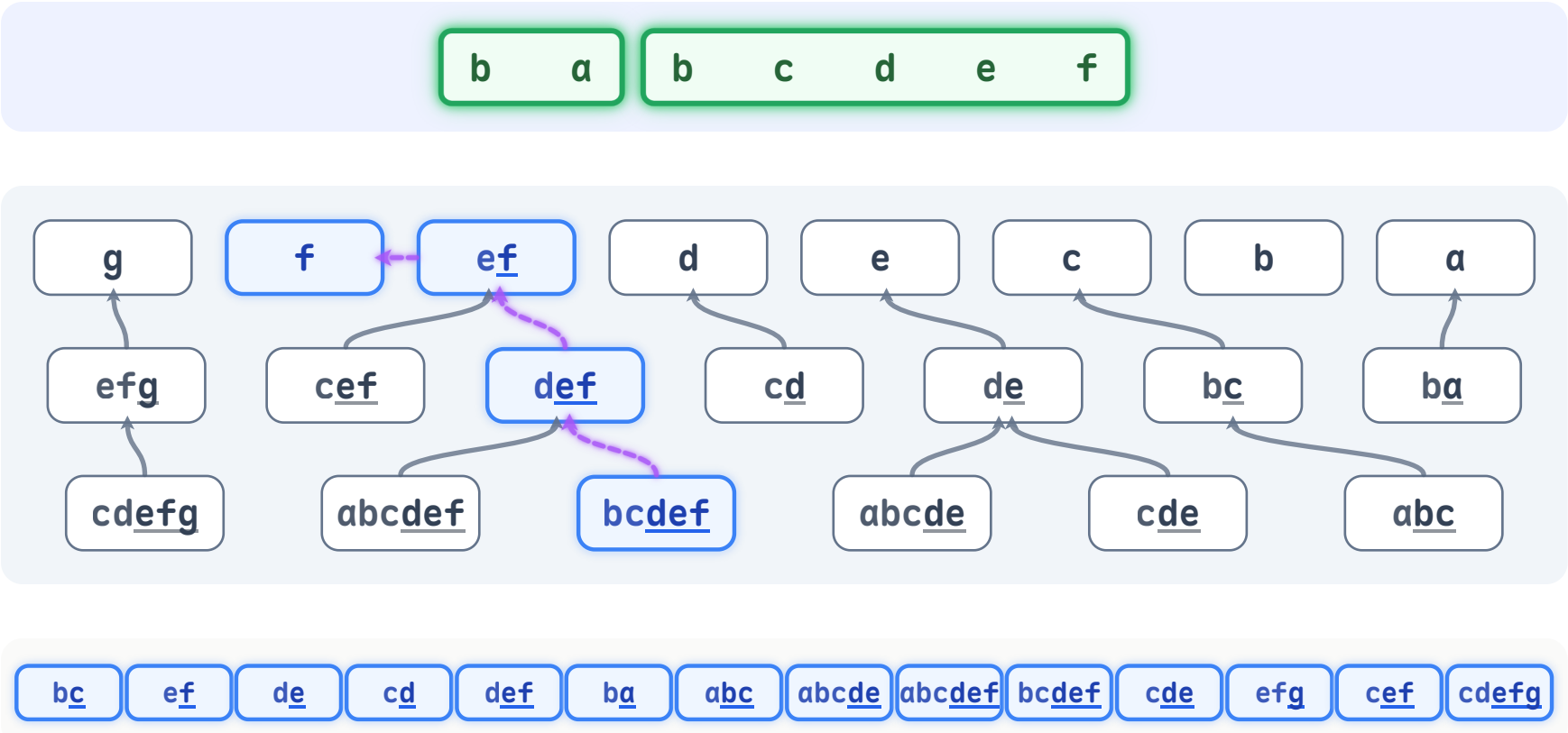
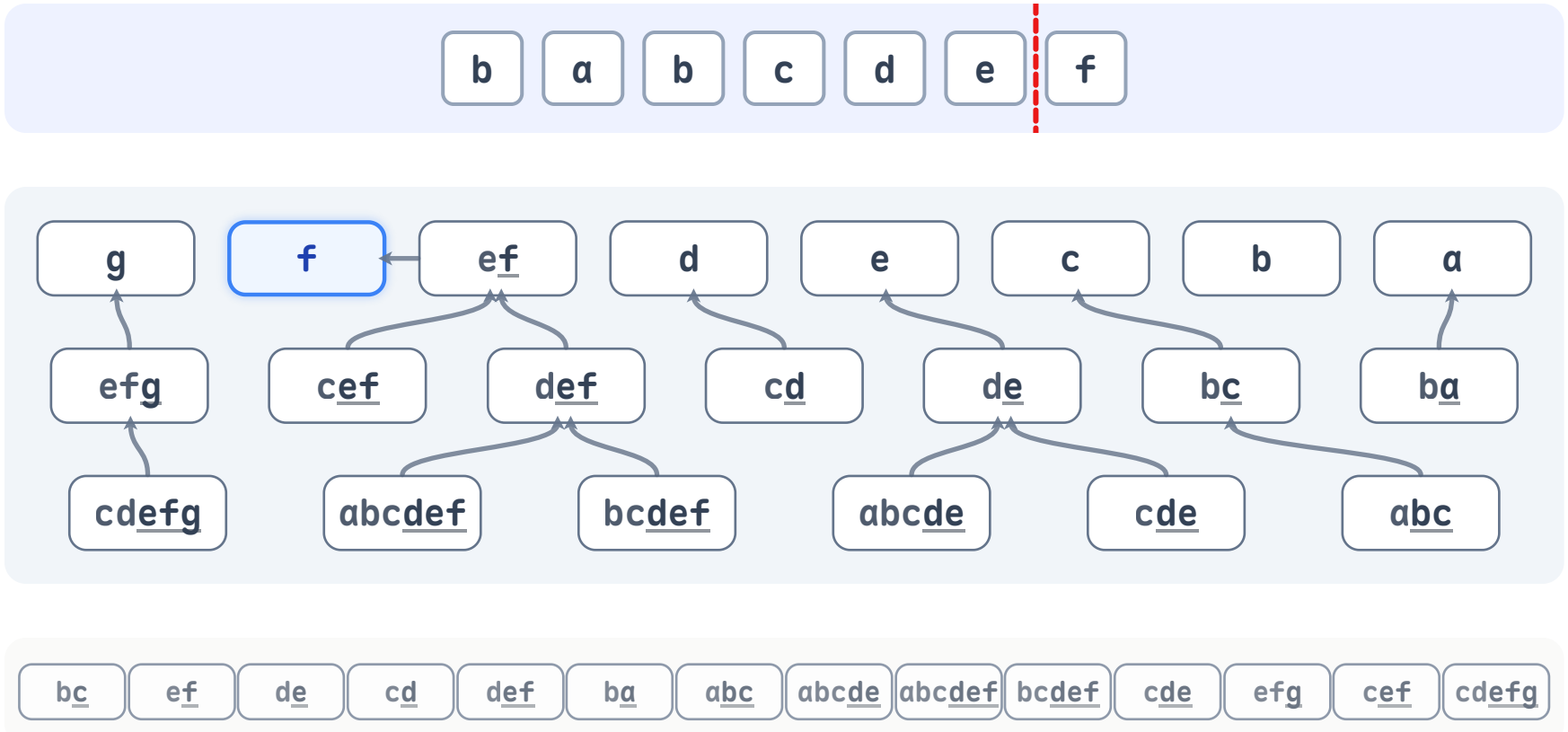


"Steal" the Tokens from the Previous States



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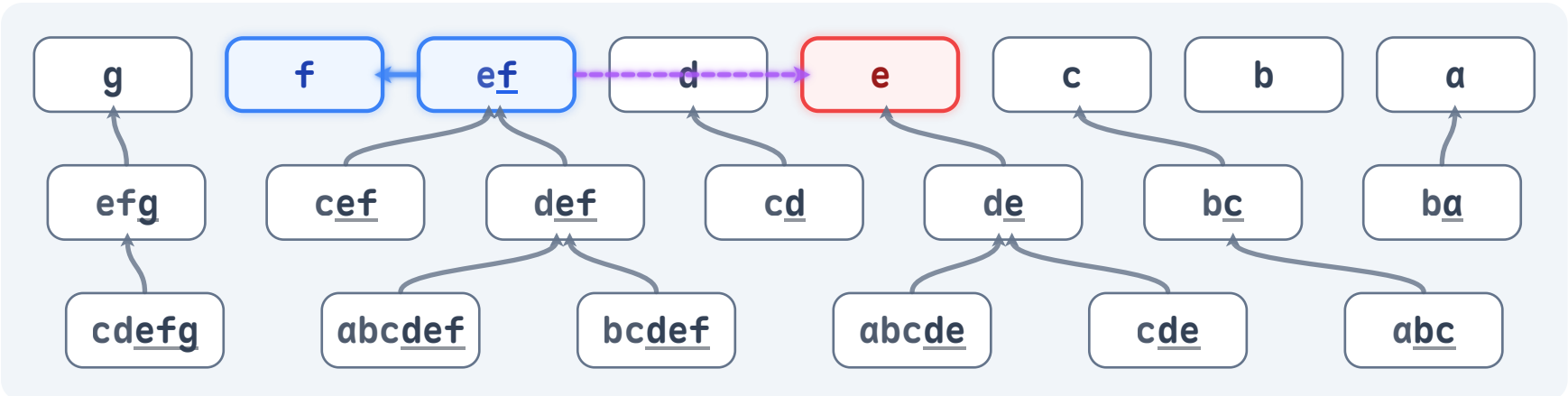

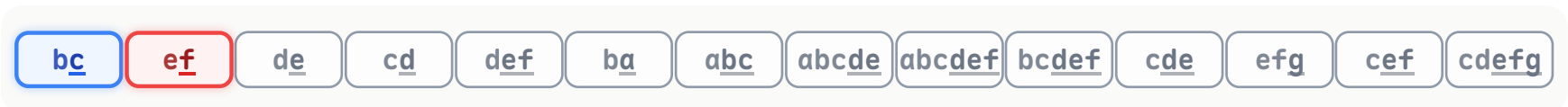

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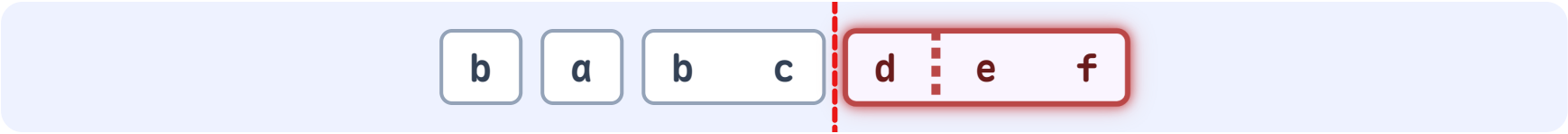

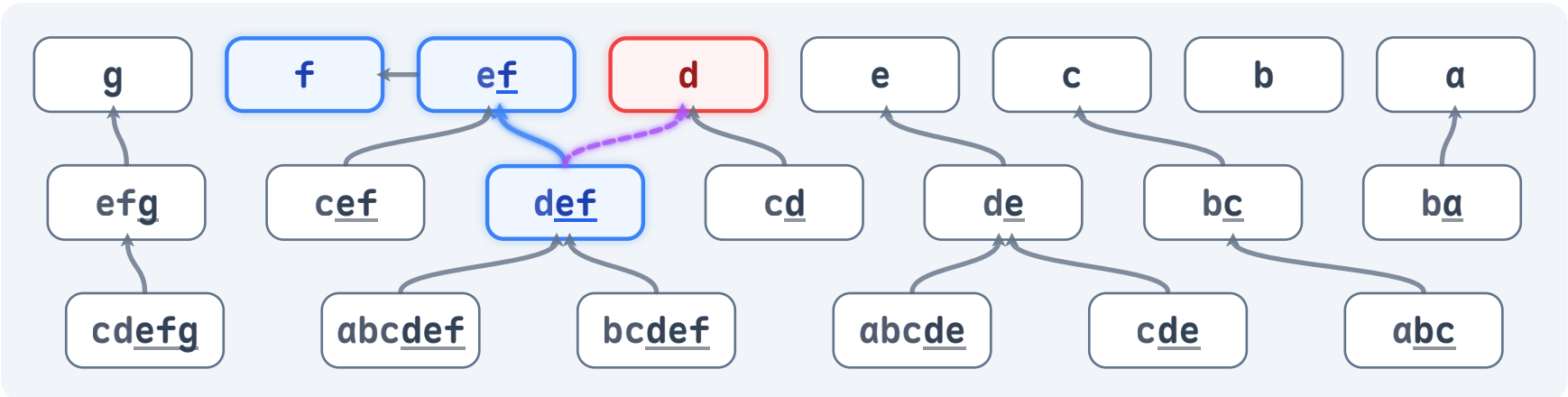

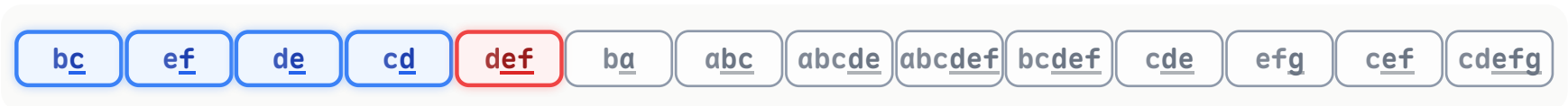

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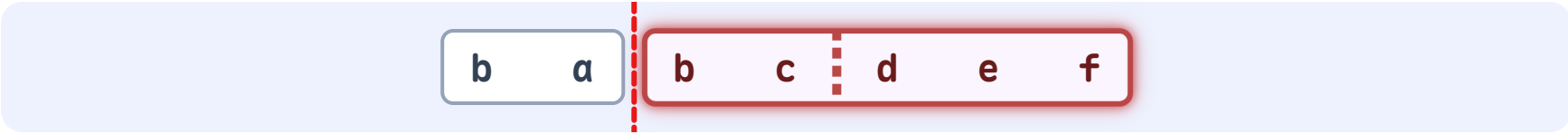

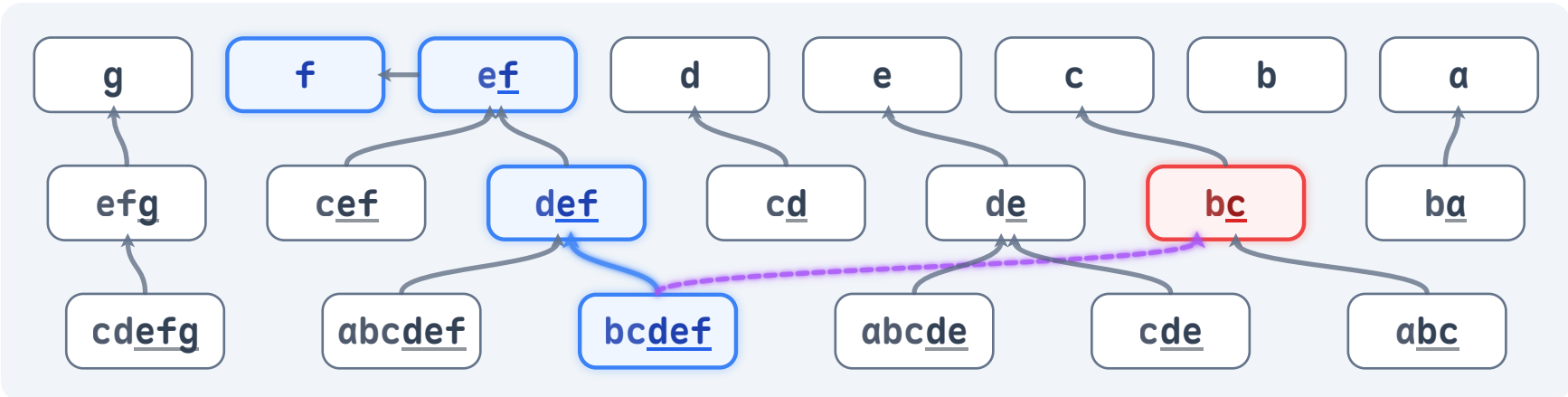

Which One is the Answer?

c d e

c d

c

bc ef de cd def ba abc abcde abcdef bcdef cde efg cef cdefg

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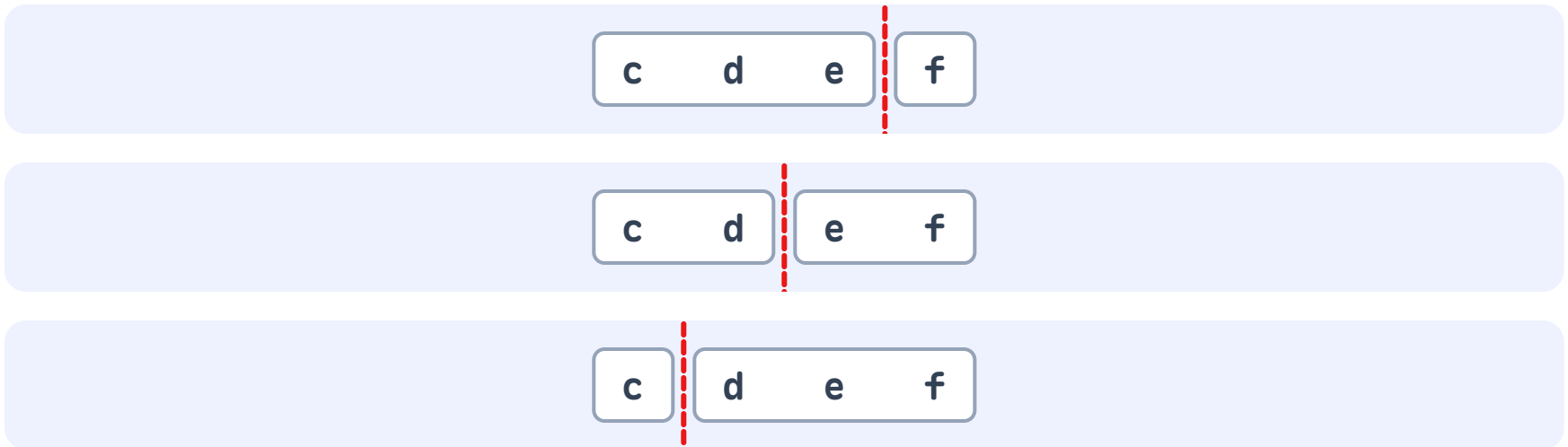

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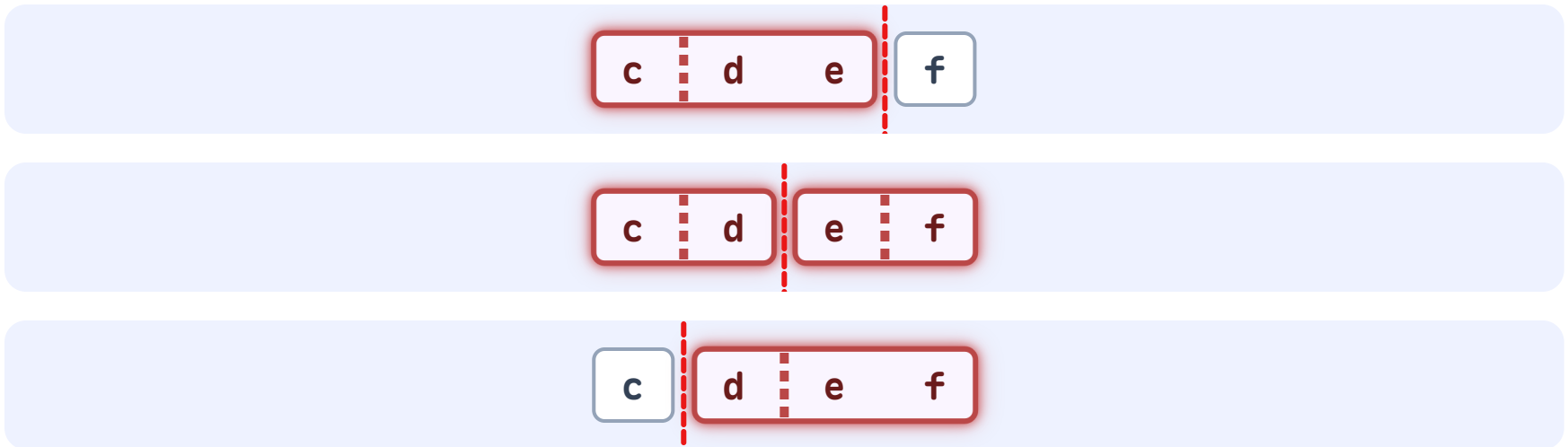

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Priorities Rule the Game

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- `ef` is capable of "stealing" `e` from `cde`, where `de` has a *lower* priority than `ef`

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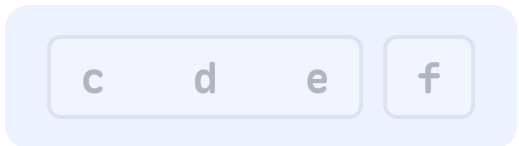
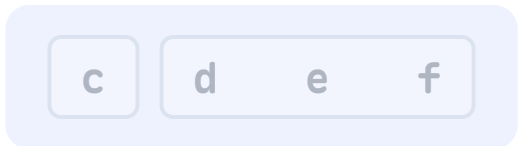
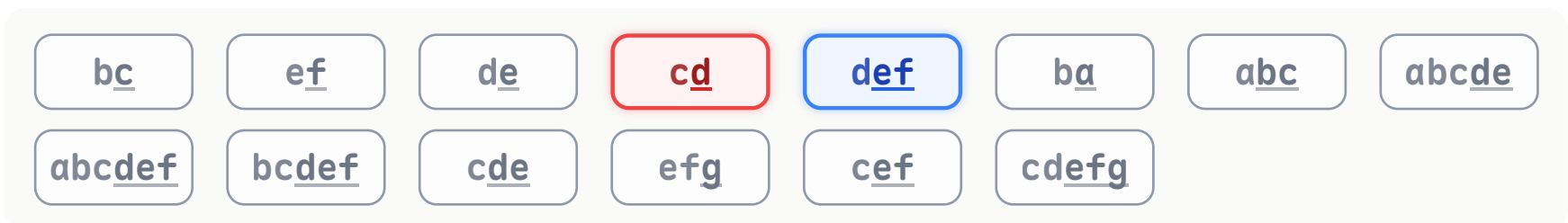


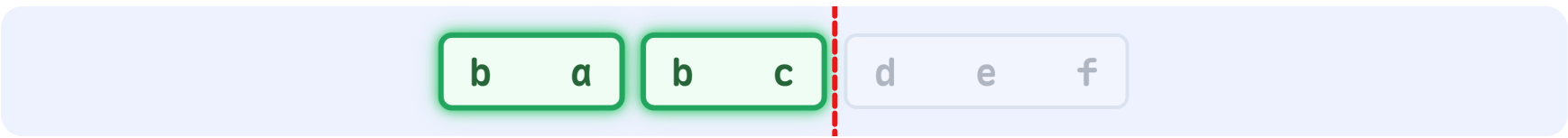


- `ef` is capable of "stealing" `e` from `cde`, where `de` has a *lower* priority than `ef`
- `def` is **not** capable of "stealing" `d` from `cd`, where `cd` has a *higher* priority than `def`



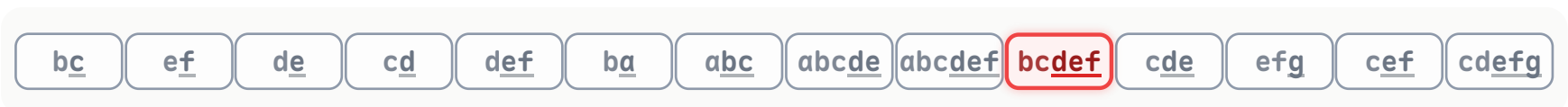
Prefix Last-Token Condition

Inspecting the token t = `bcdef`



The previous state k = the tokenization of `babc`

(Removing the successor of t from the suffix)

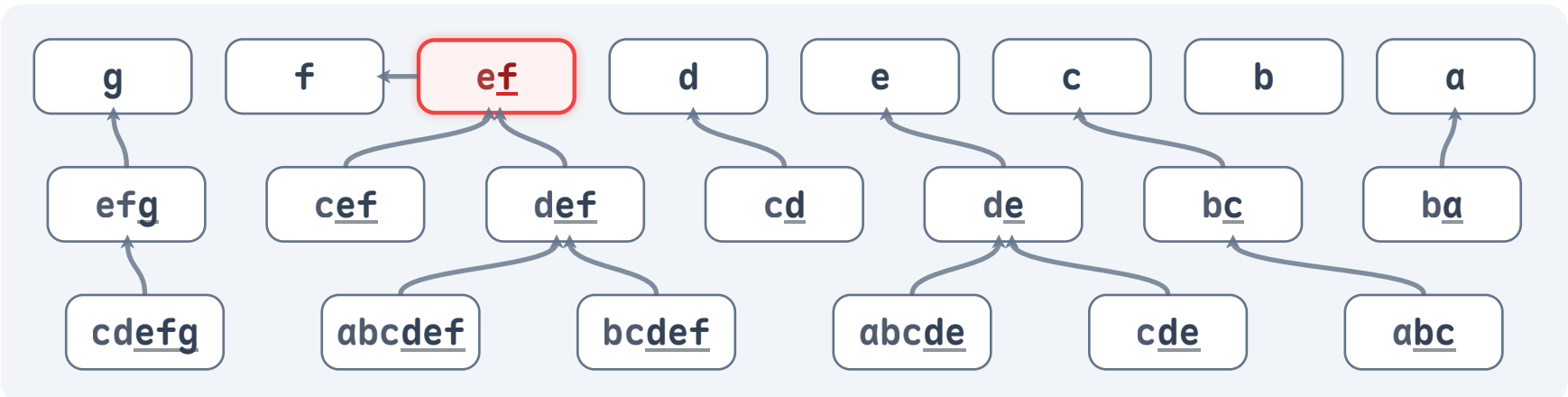


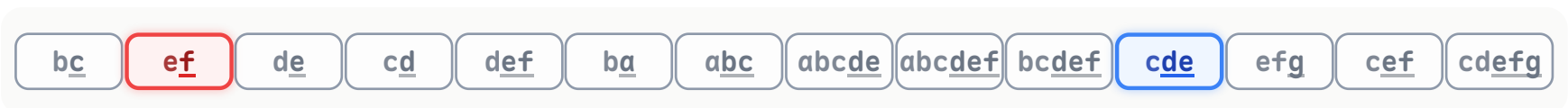
Prefix Last-Token Condition $\iff$ Monotonic Path

Theorem 4.2: Monotonic Path Property

With respect to the input string s, a token t is on the *Monotonic Path*, **if and only if** it satisfies the *Prefix Last-Token Condition*.

Prefix Last-Token Condition: t = ef





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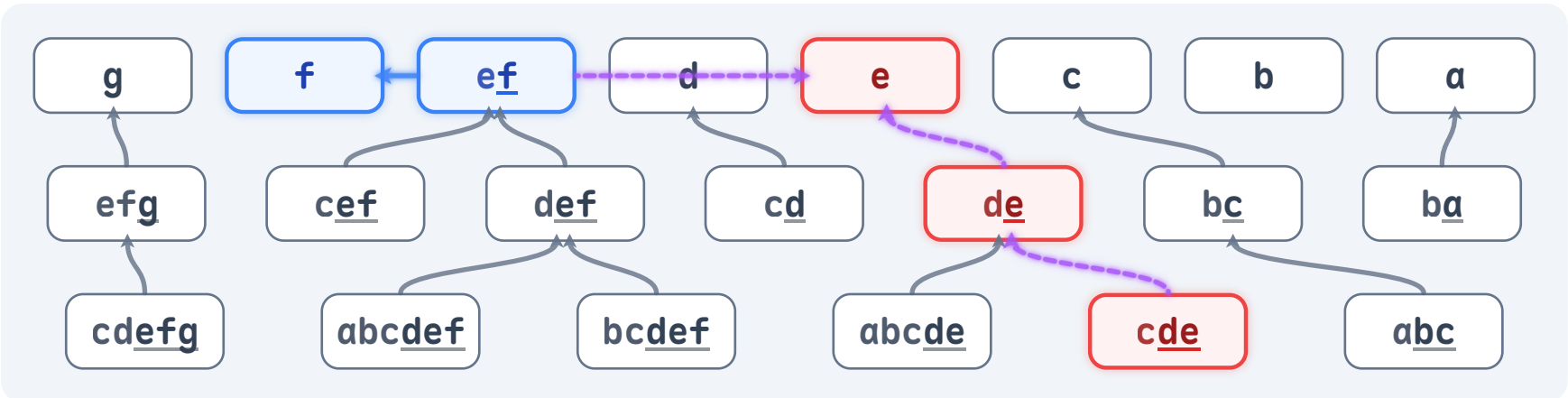
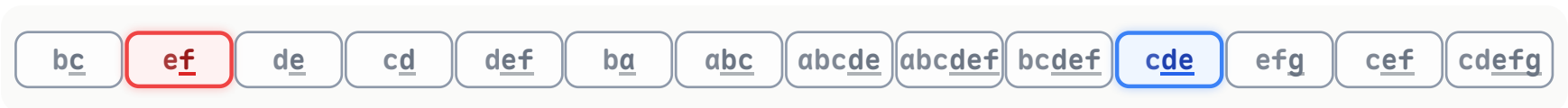
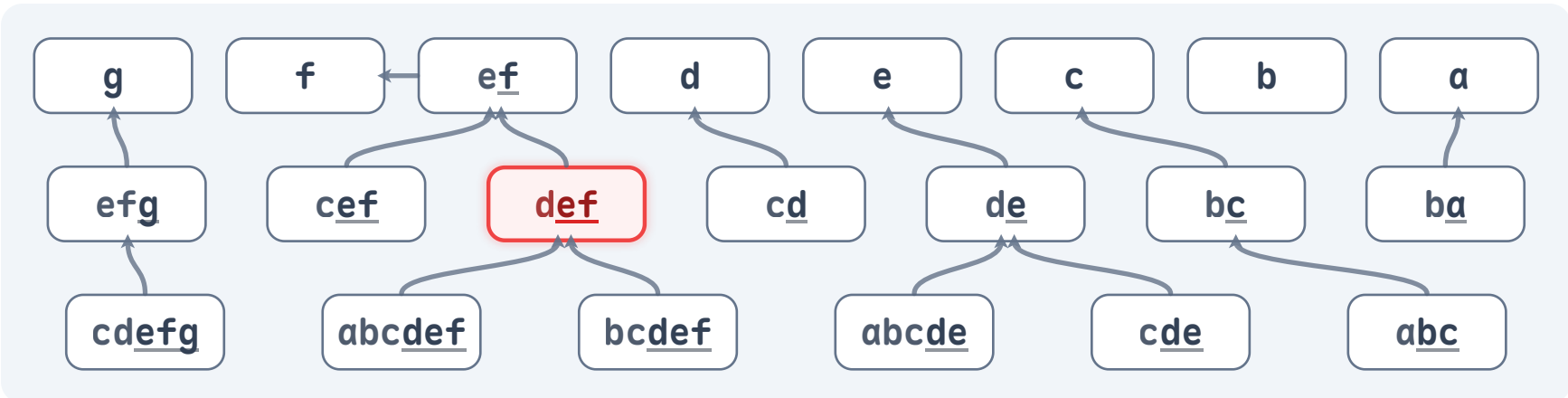
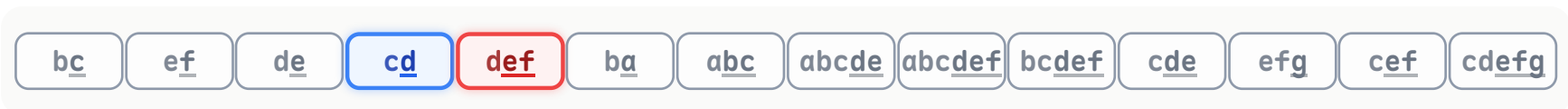




Prefix Last-Token Condition: t = def





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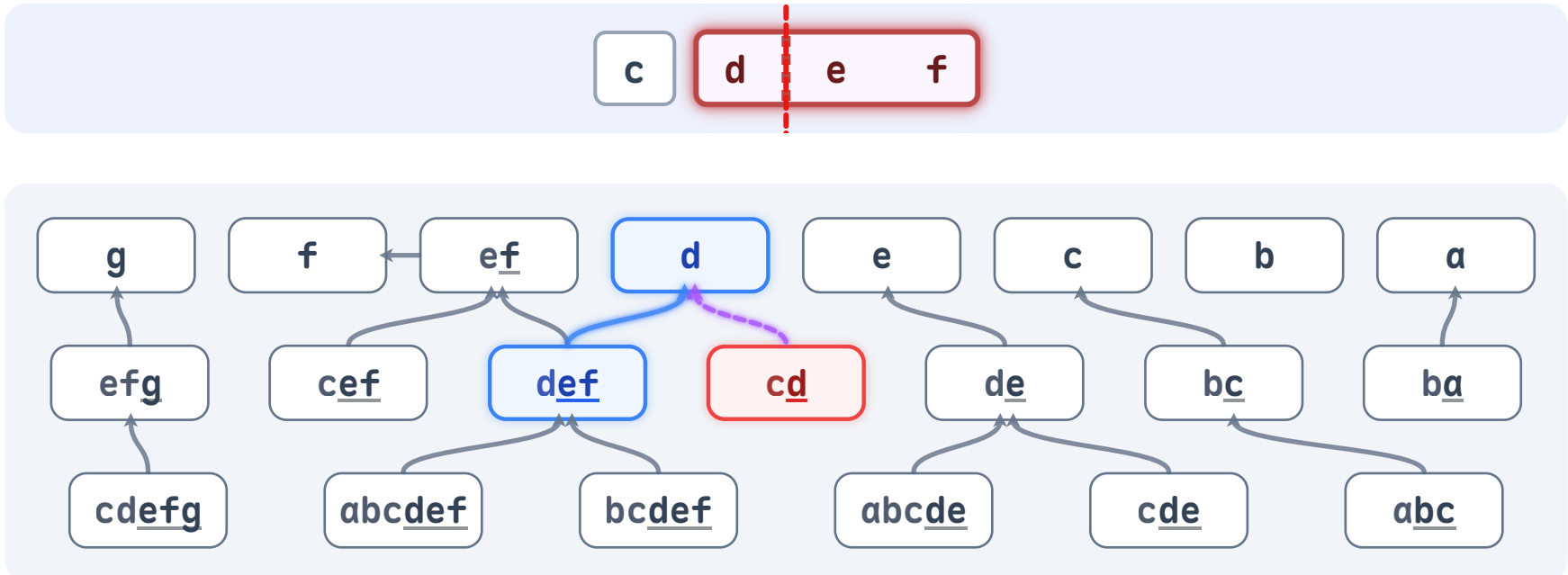
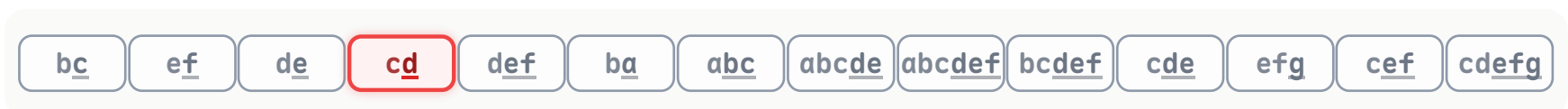
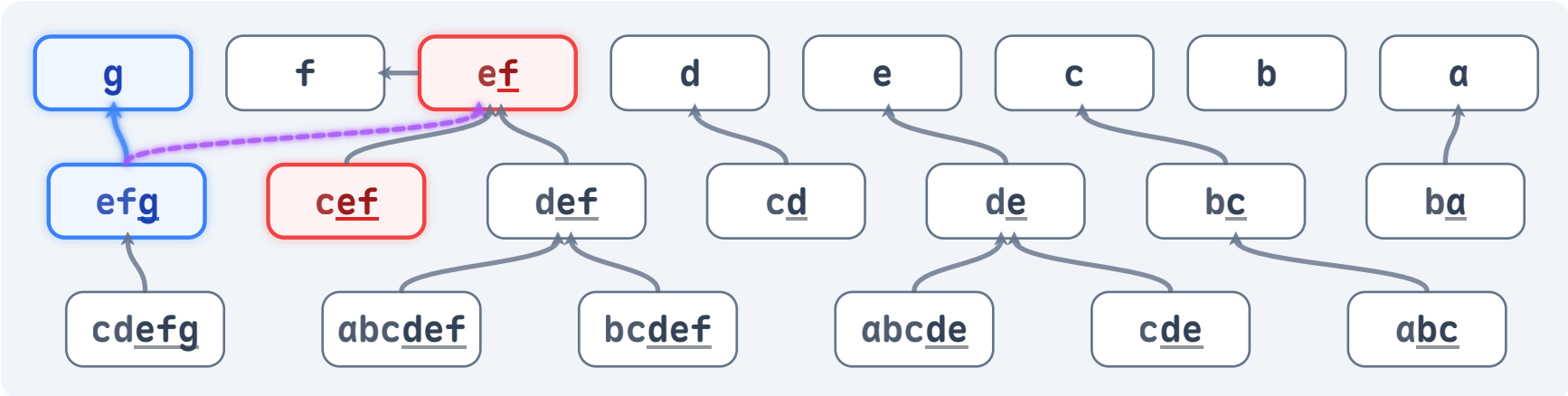
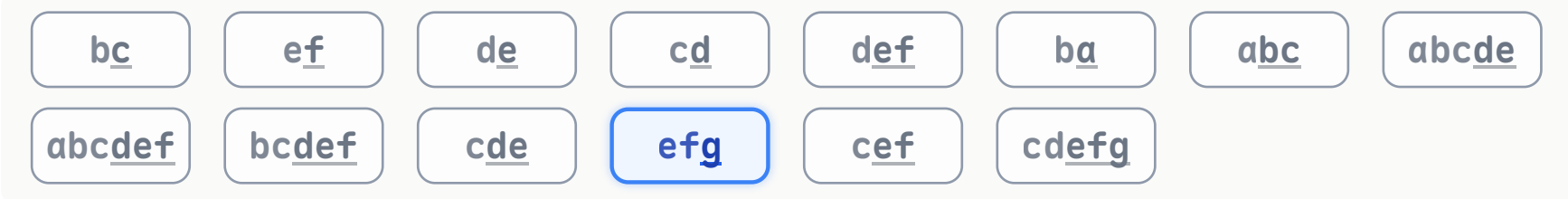




Valid "Last Tokens of Previous States": $\mathcal{C}_t$



$\mathcal{C}_t$: Candidates (in **red**) as the "last tokens" of the previous states for the token t



Merge Rules

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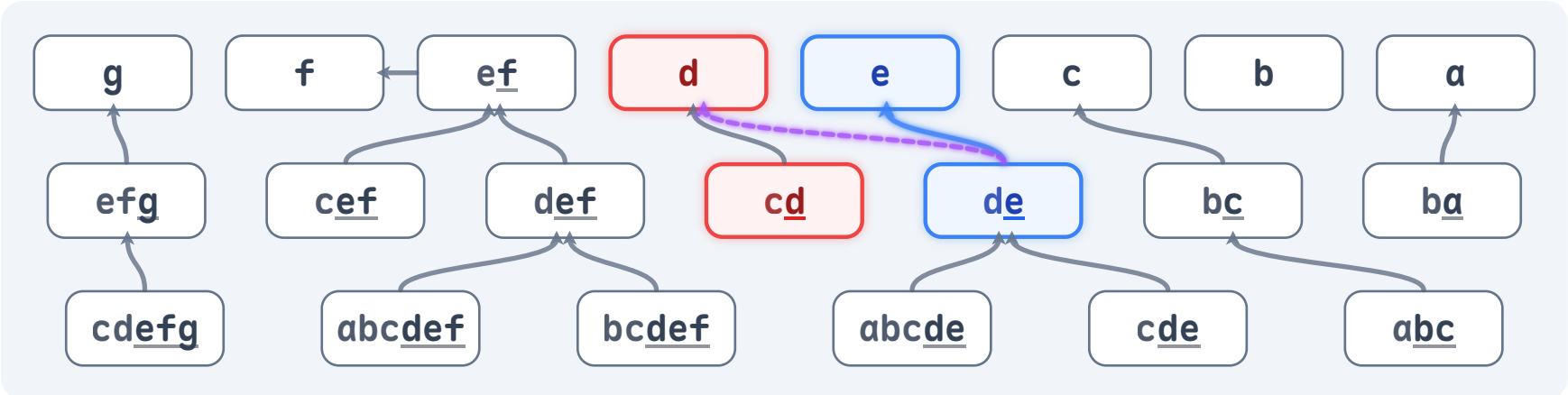

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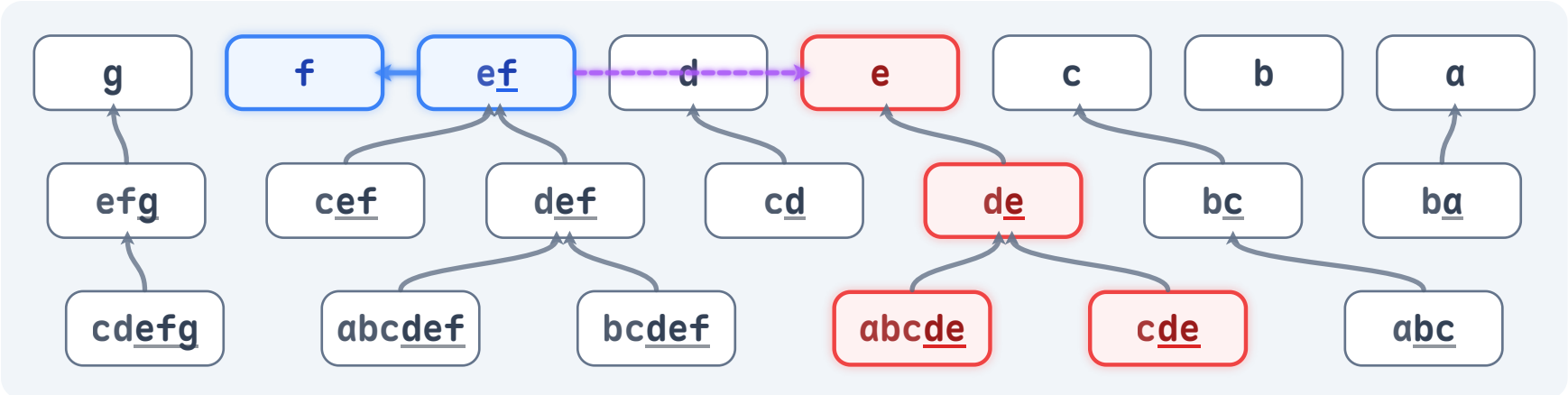

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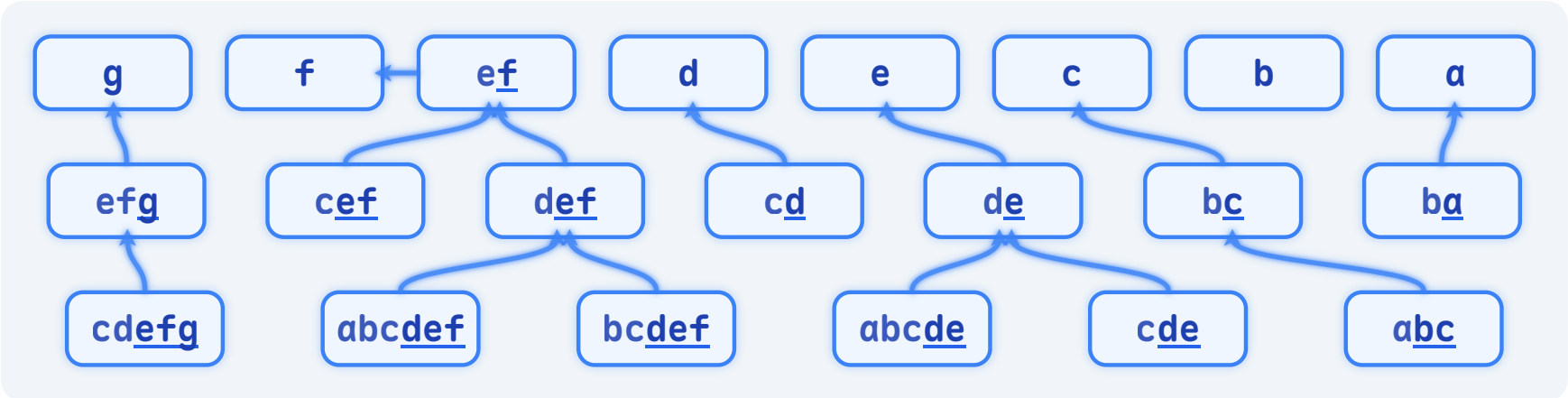
Merge Rules

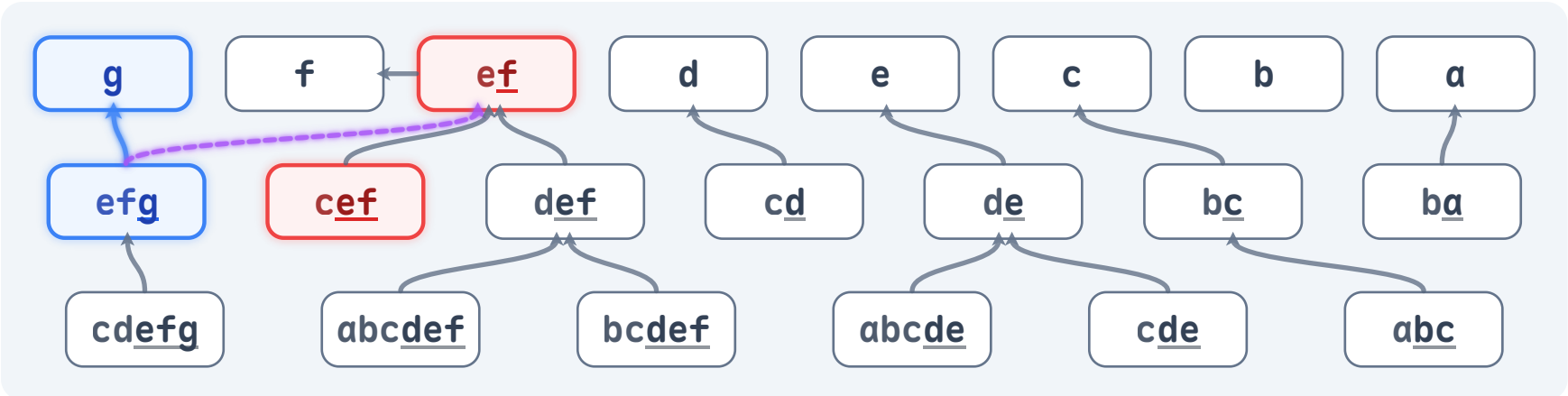
Linearization with Pre-order DFS



DFS the Successor Forest, visiting lower-priority children first

Pre-order DFS Linearization (DFS Numbering)

$\mathcal{C}_t \iff$ Intervals of Linearization



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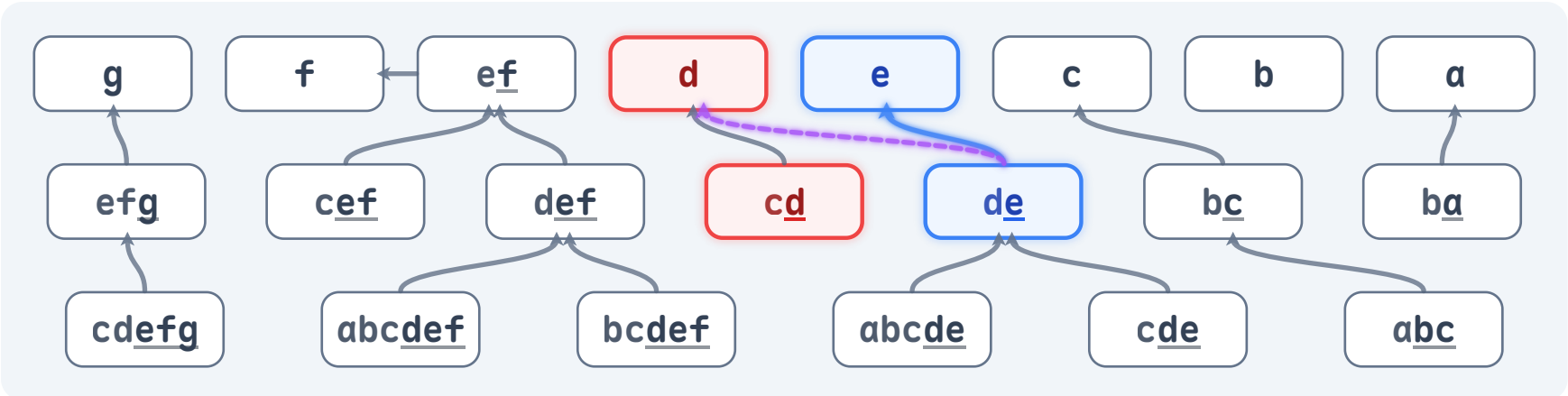
Pre-order DFS Linearization (DFS Numbering)

$\mathcal{C}_t \Longleftrightarrow$ Intervals of Linearization



$\mathcal{C}_t$: Candidates (in **red**) as the "last tokens" of the previous states for the token t

Pre-order DFS Linearization (DFS Numbering)

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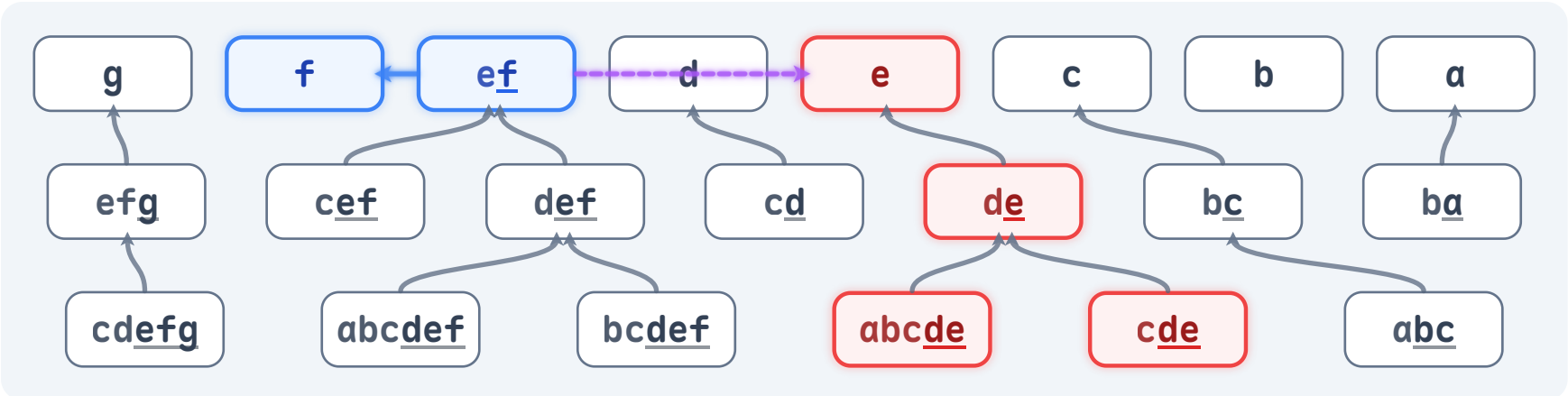

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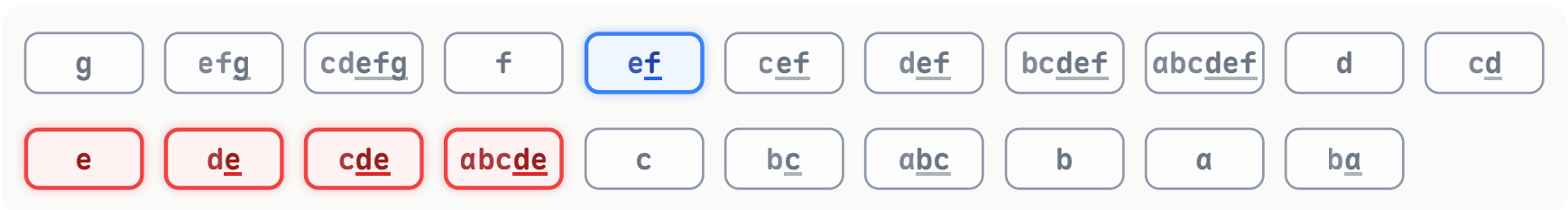
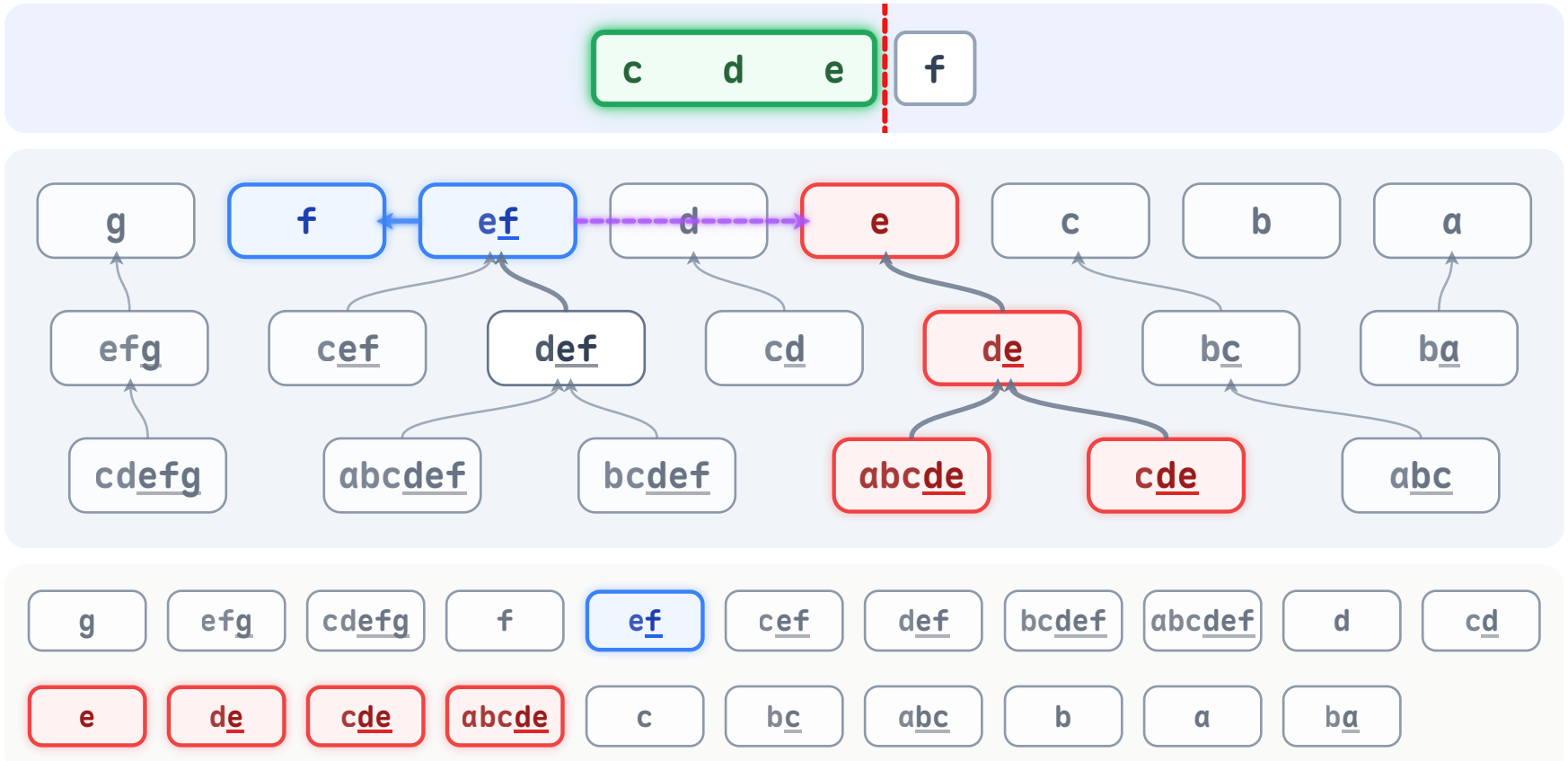


Pre-order DFS Linearization (DFS Numbering)

$\mathcal{C}_t \iff$ Intervals of Linearization: t = ef



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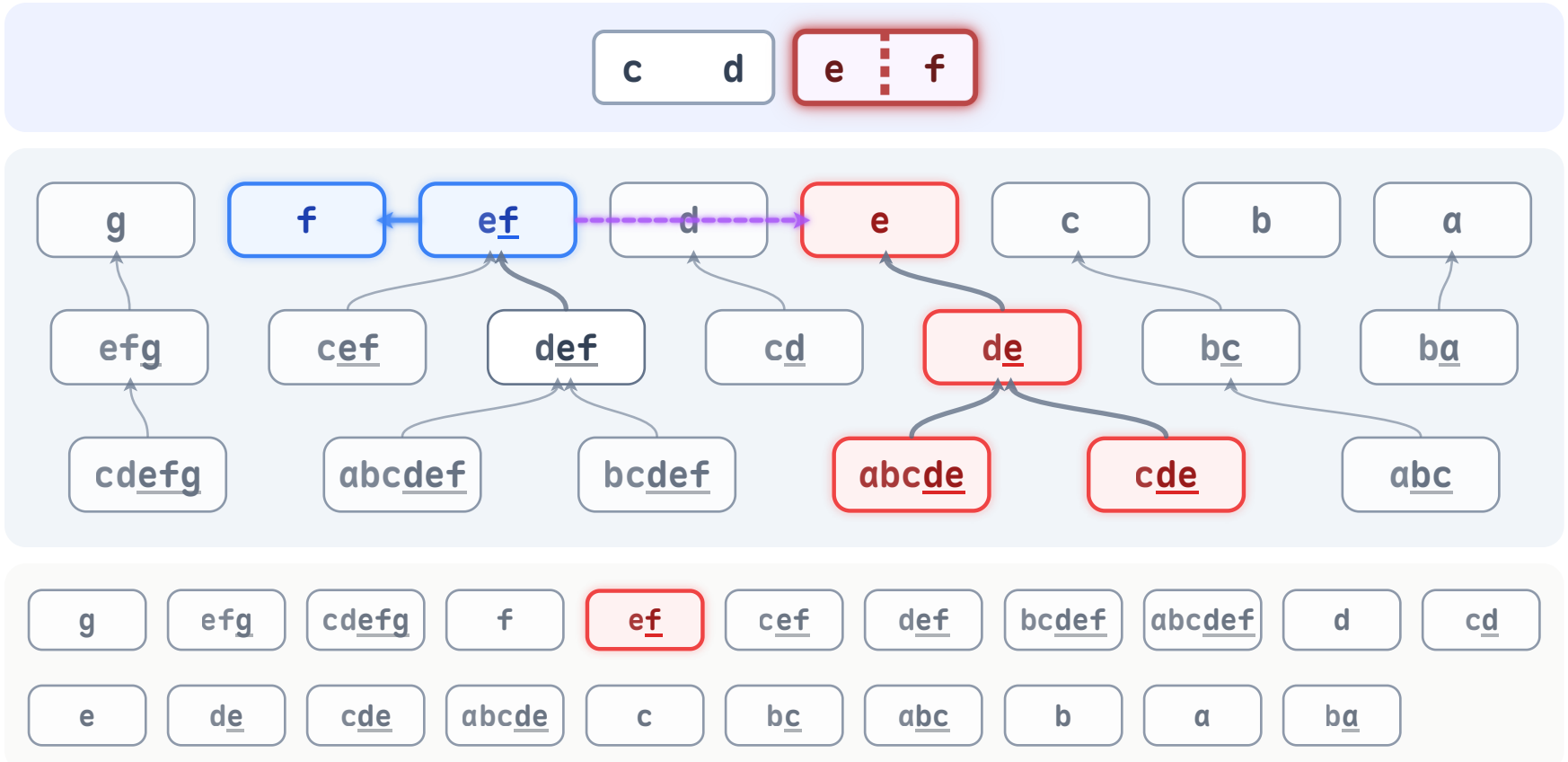

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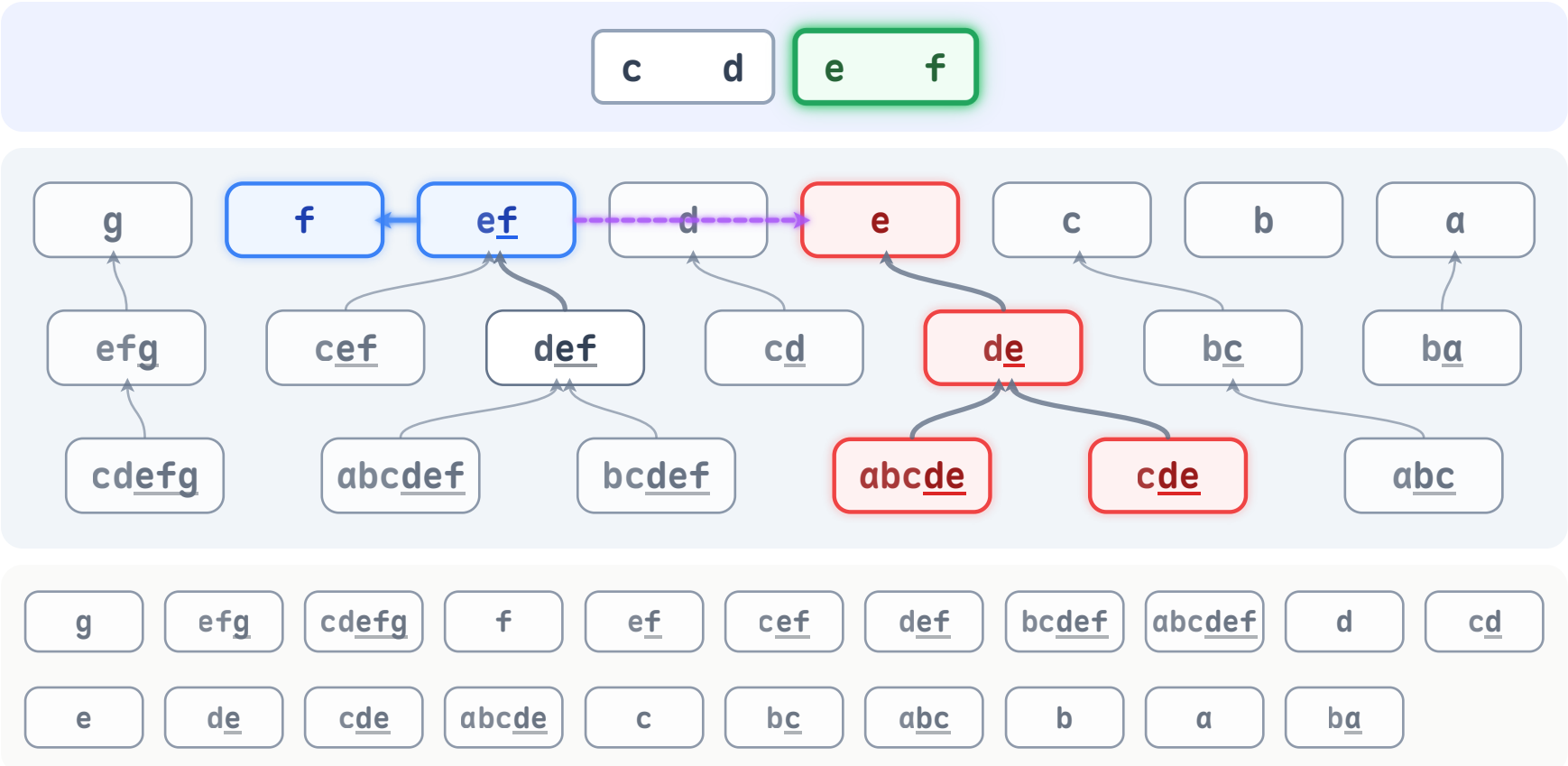
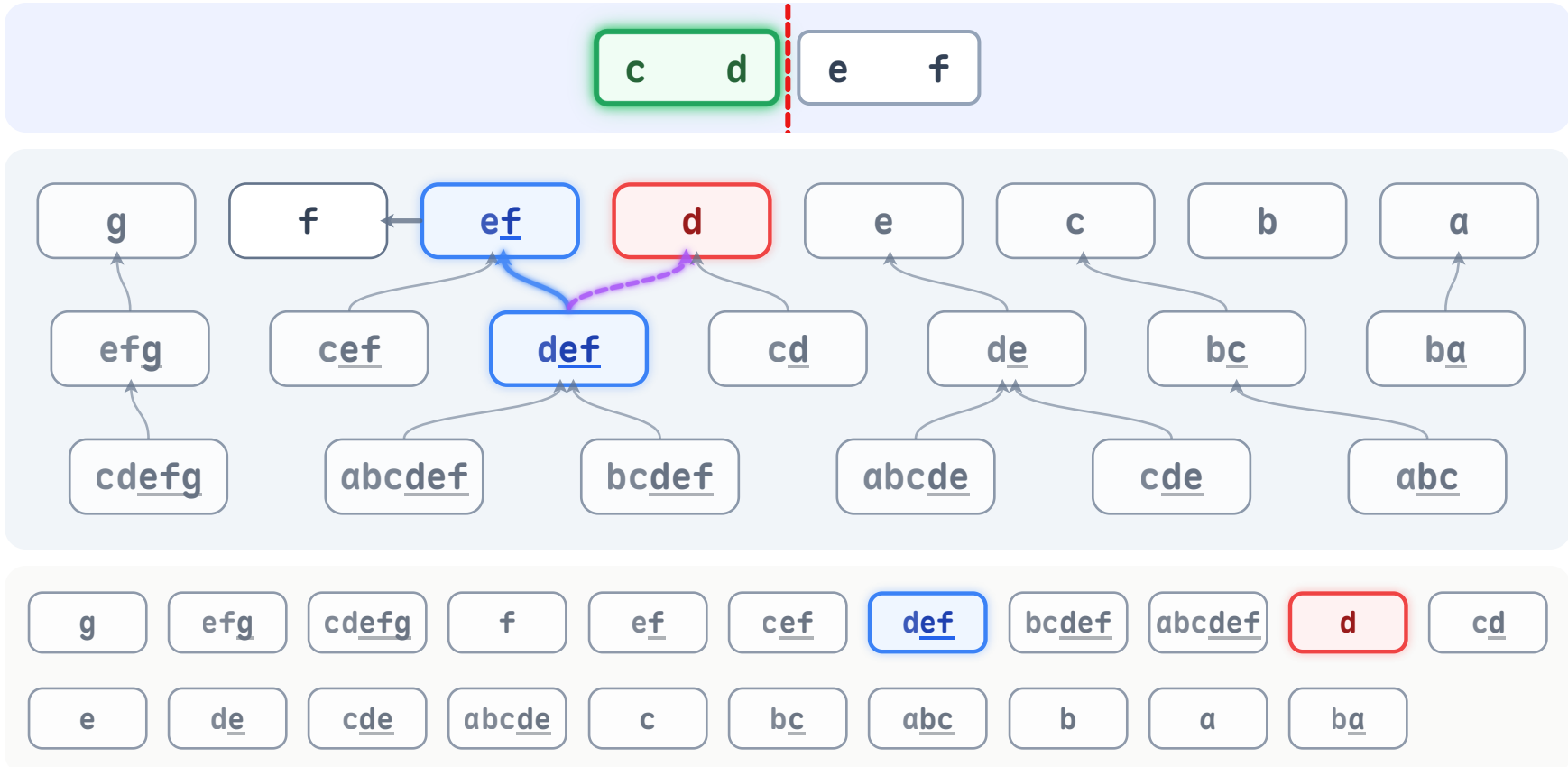


$\mathcal{C}_t \iff$ Intervals of Linearization: t = def



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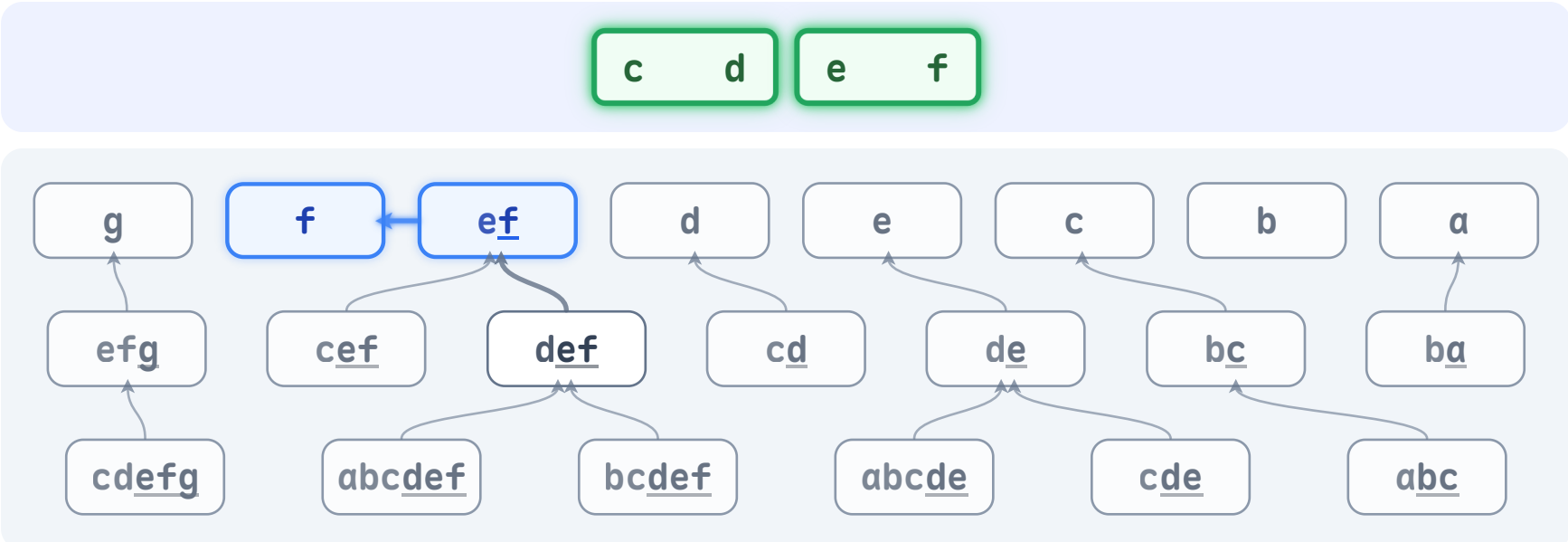
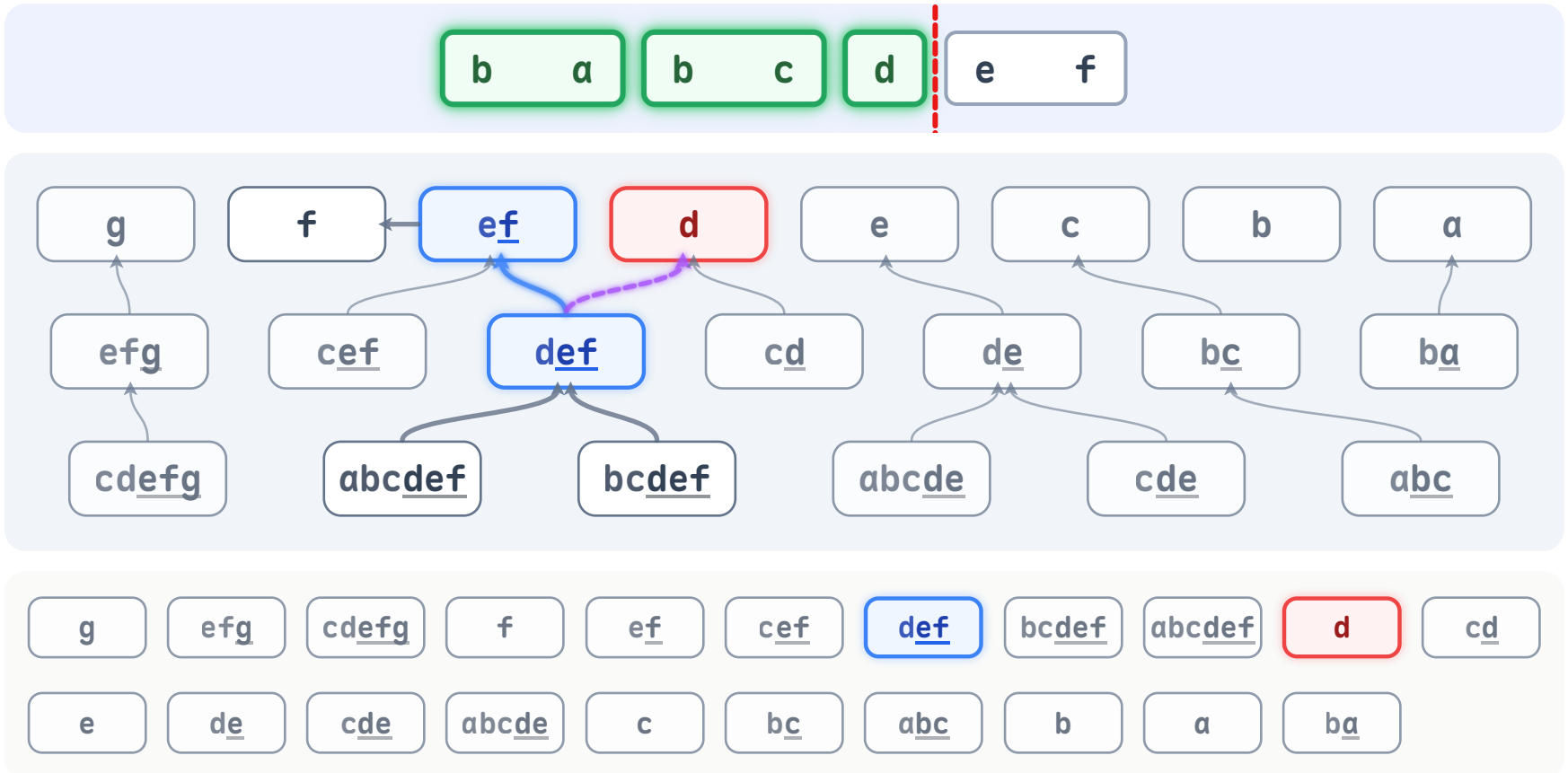


g efg cdefg f ef cef def bcdef abcdef d cd

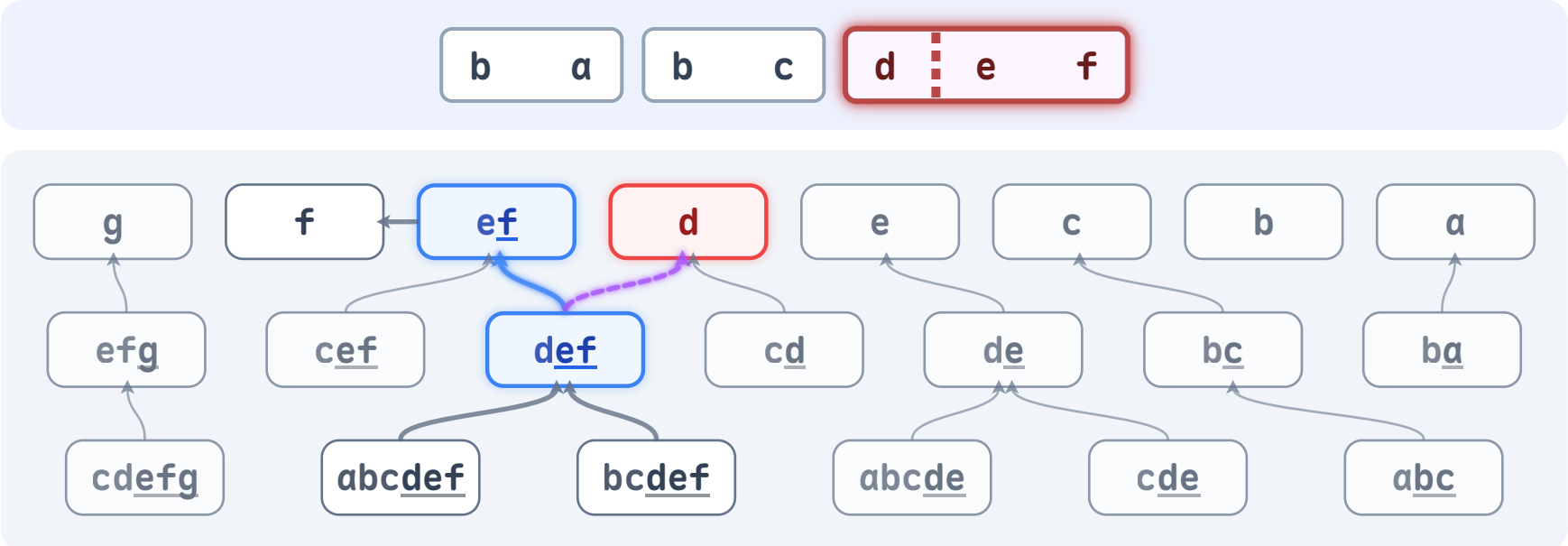
e de cde abcde c bc abc b a ba

Centroid Decomposition

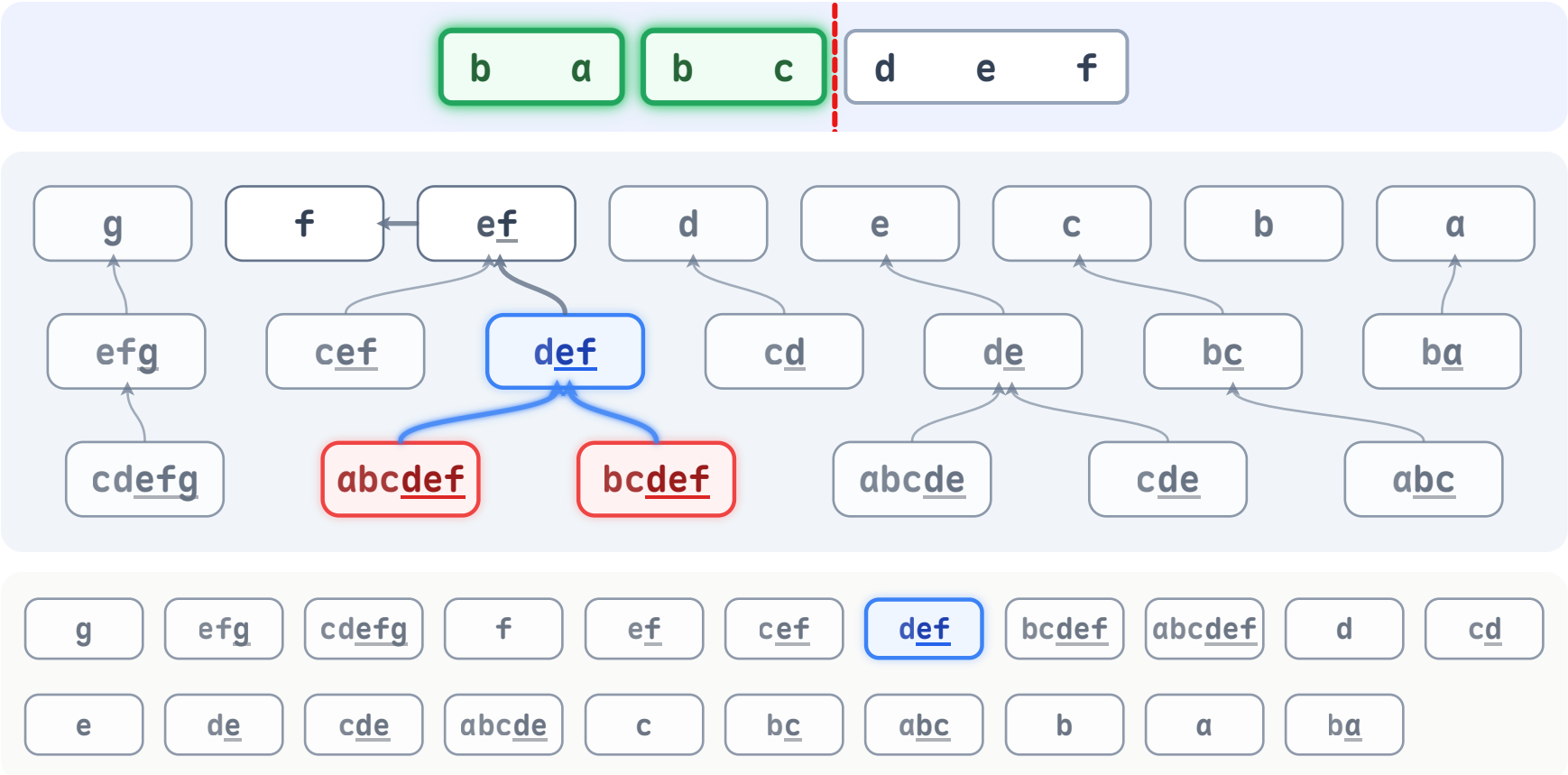
Centroid Decomposition



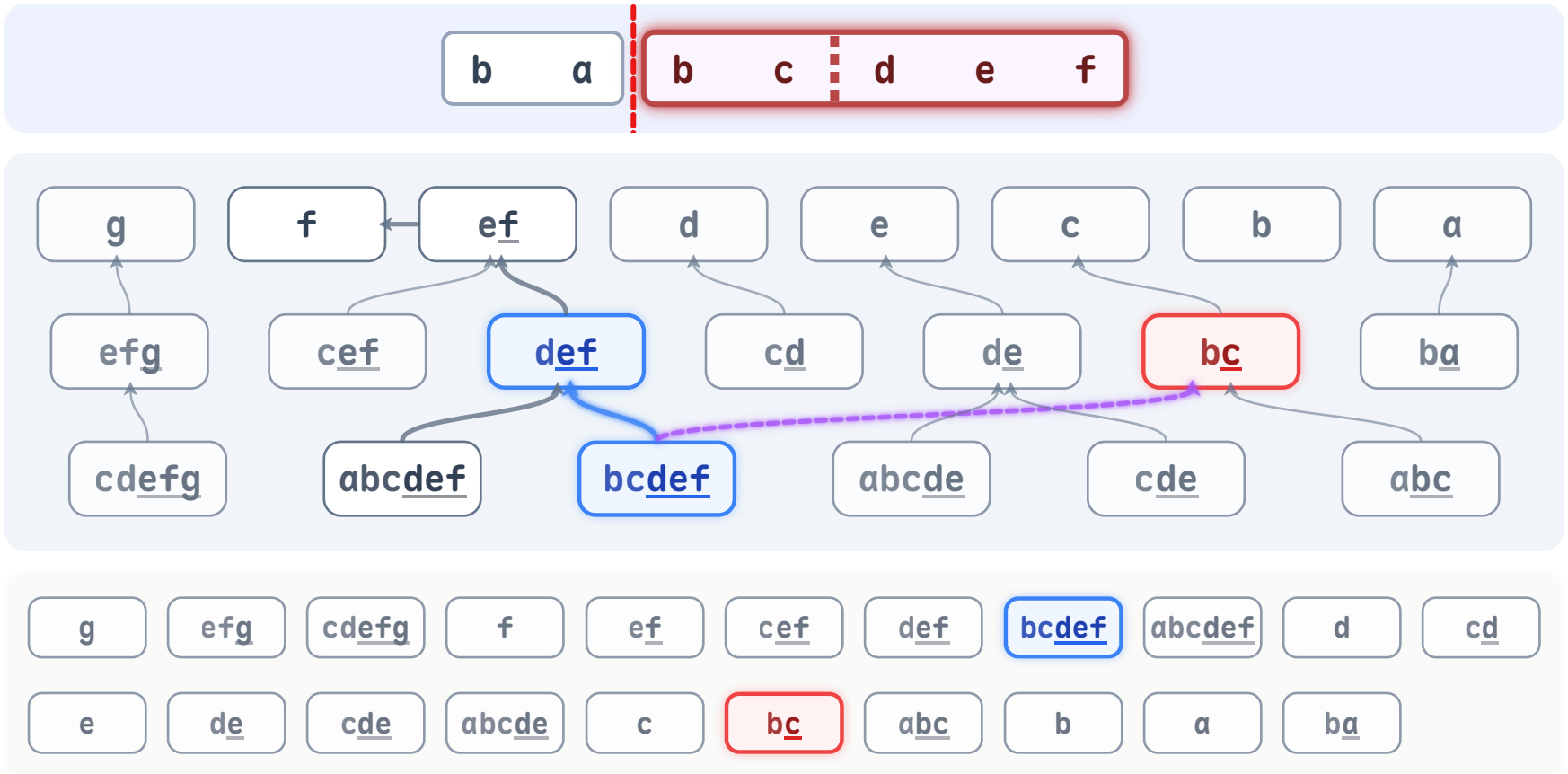
Centroid Decomposition



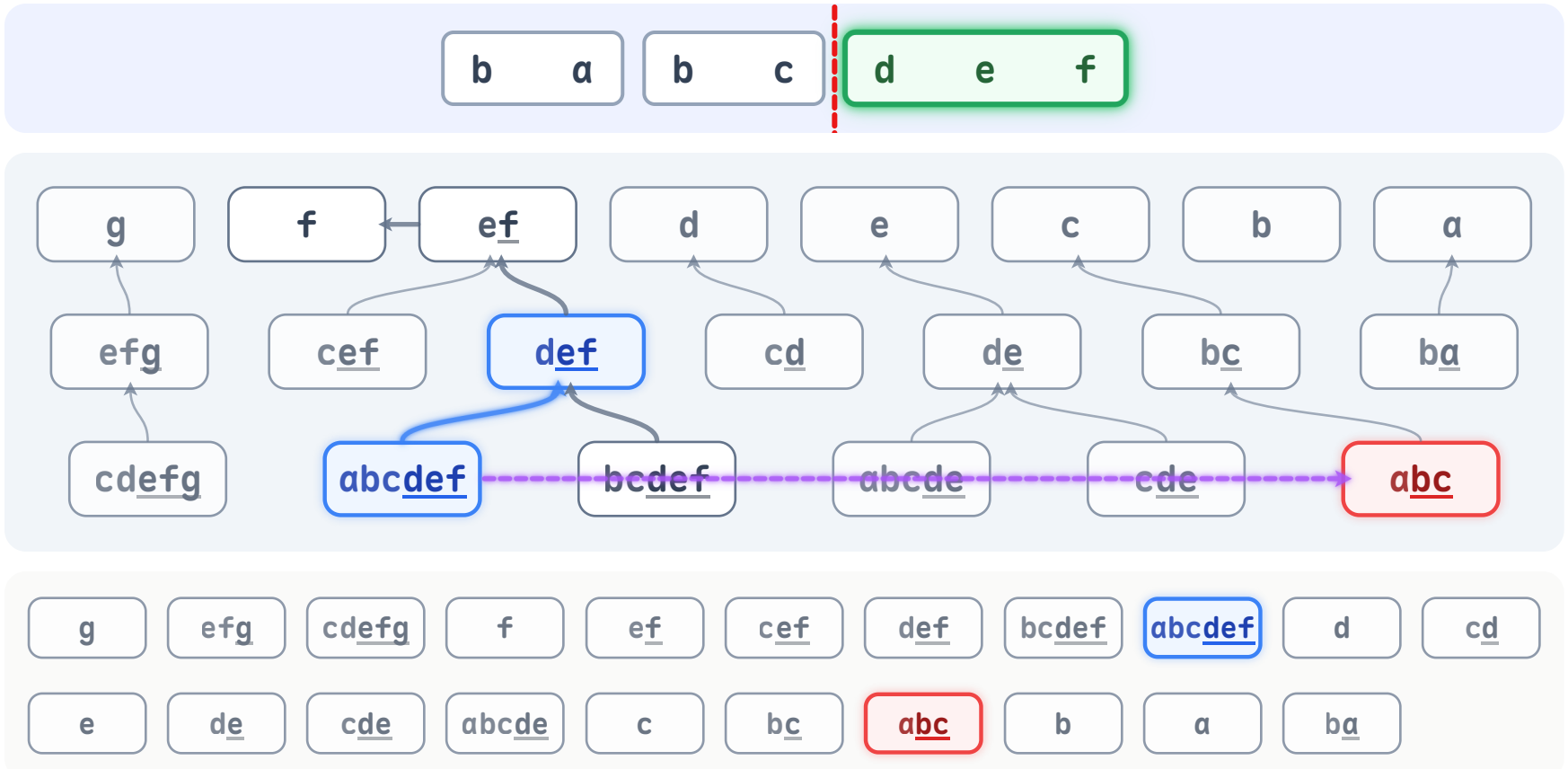
Centroid Decomposition



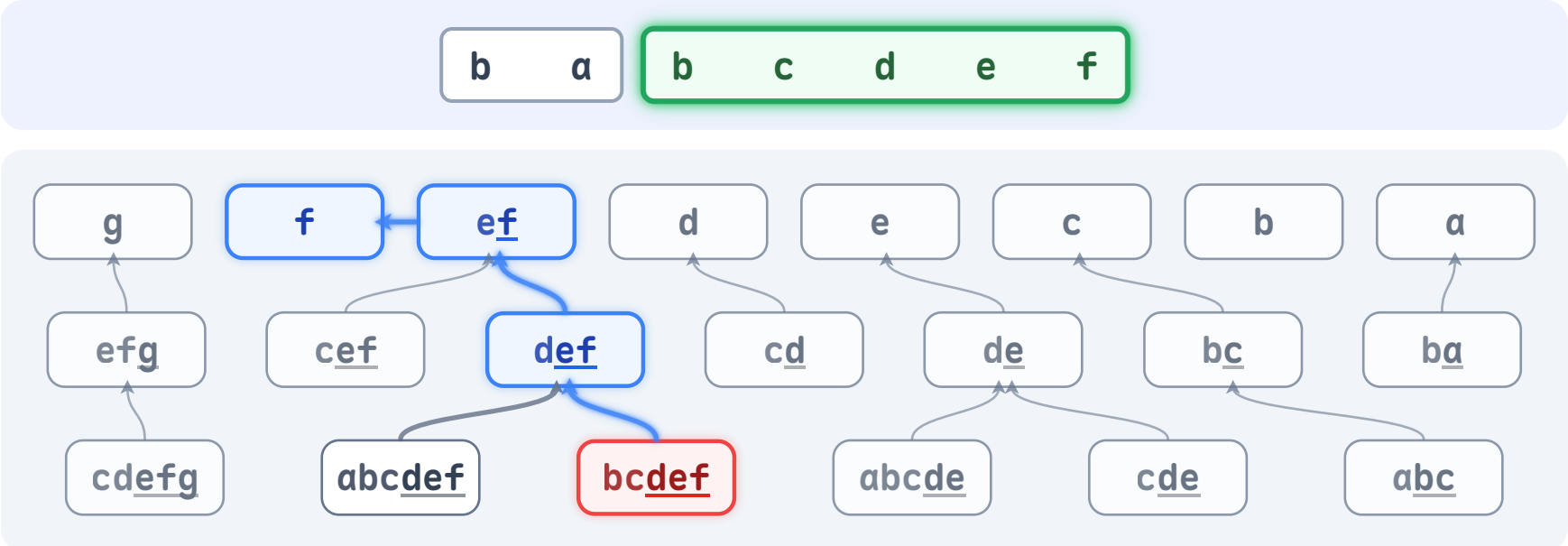
Centroid Decomposition



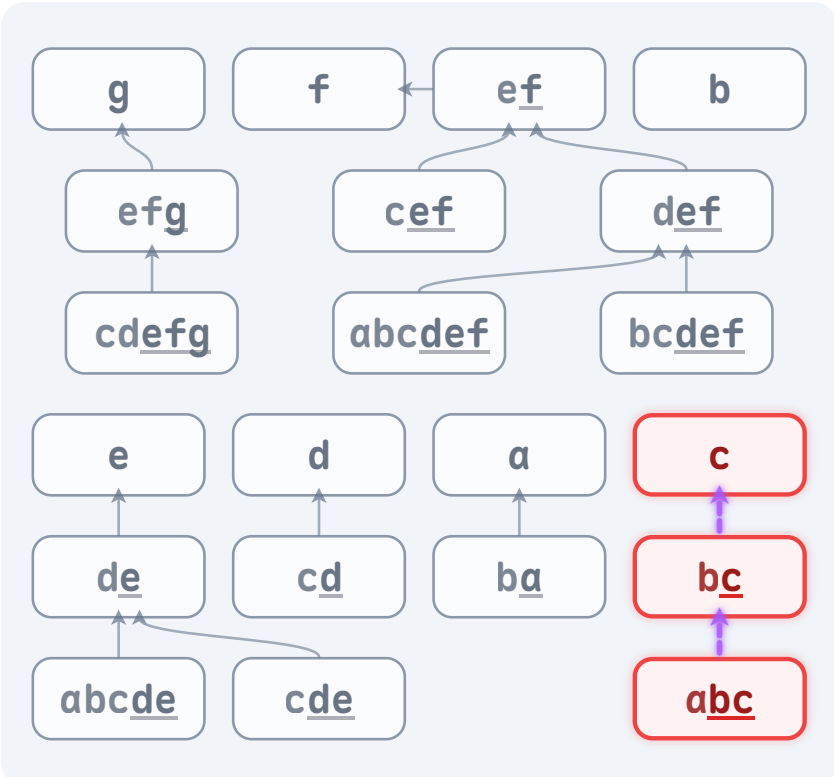
Centroid Decomposition



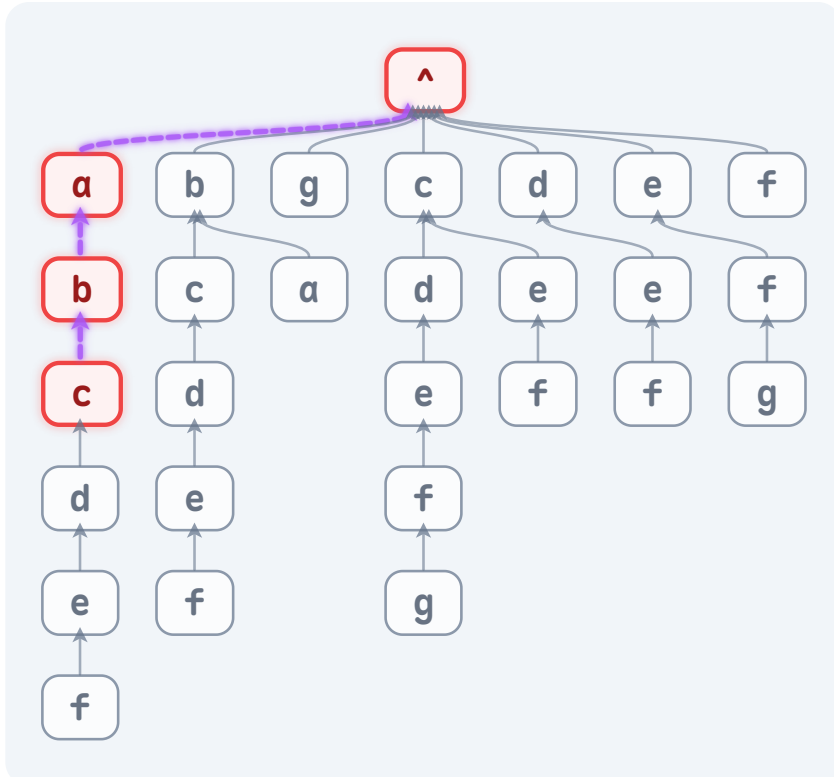
Centroid Decomposition



Aho–Corasick Automaton



Successor Forest



Aho–Corasick Automaton

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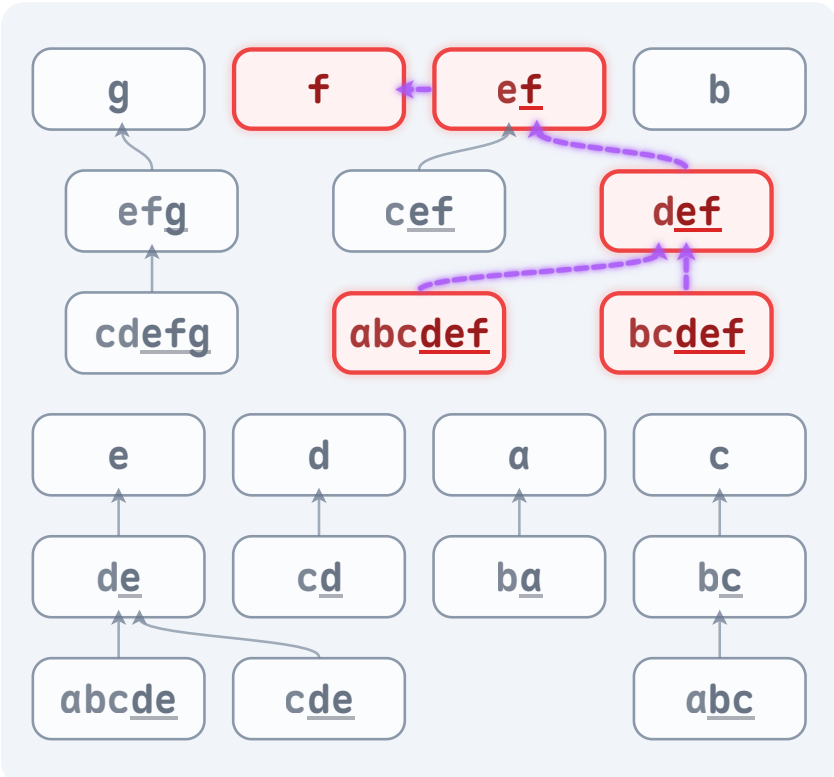
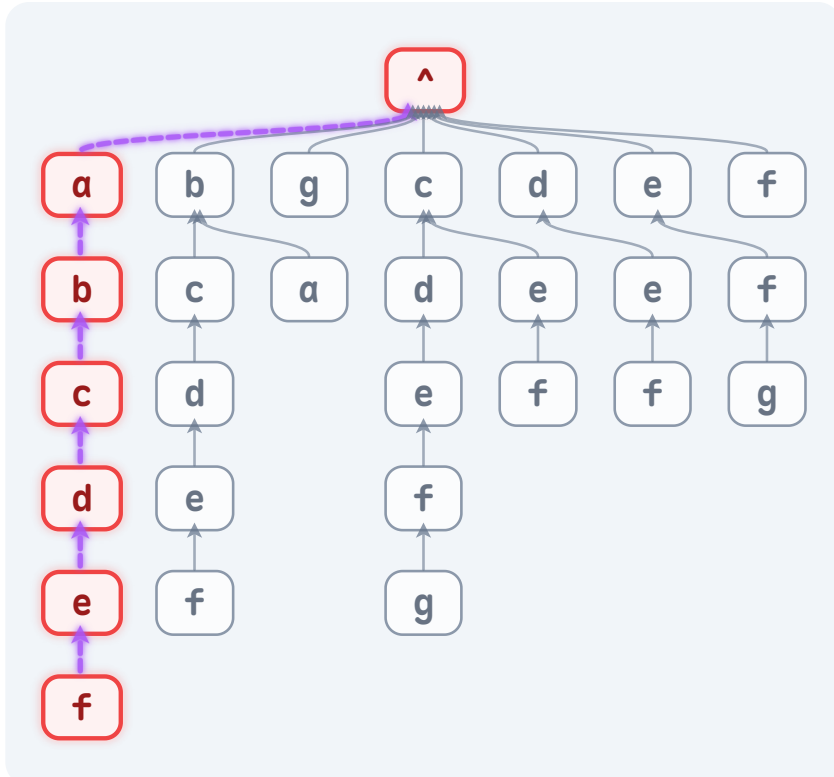


Successor Forest



Aho–Corasick Automaton

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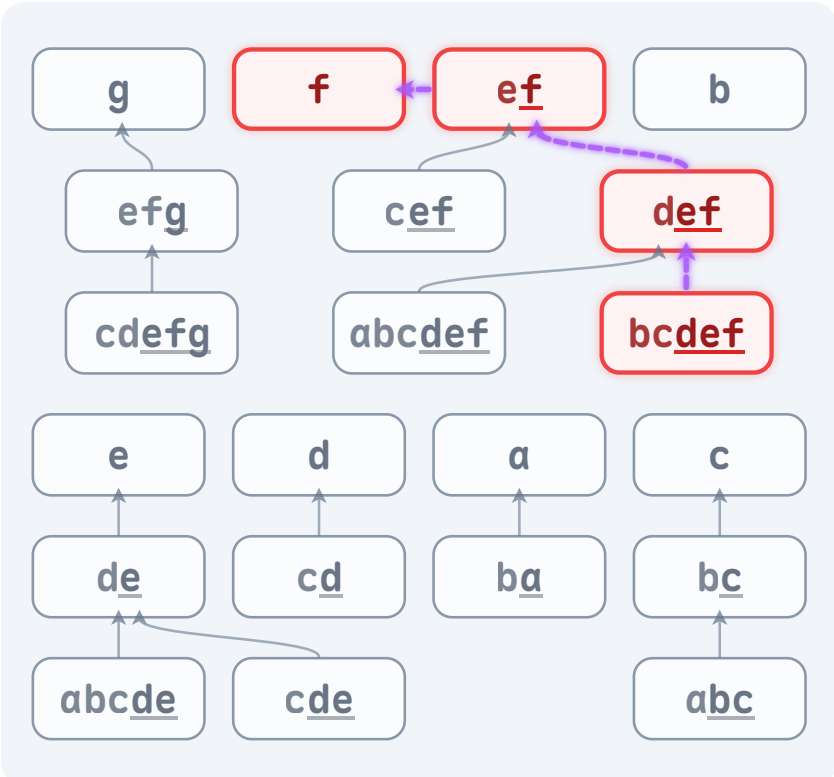
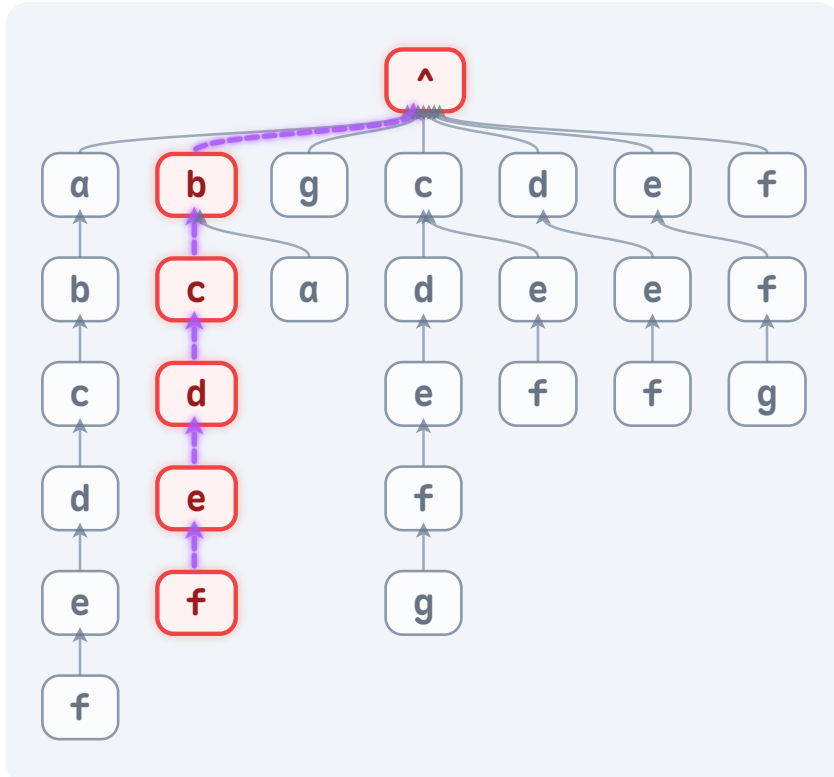


Successor Forest



Aho–Corasick Automaton

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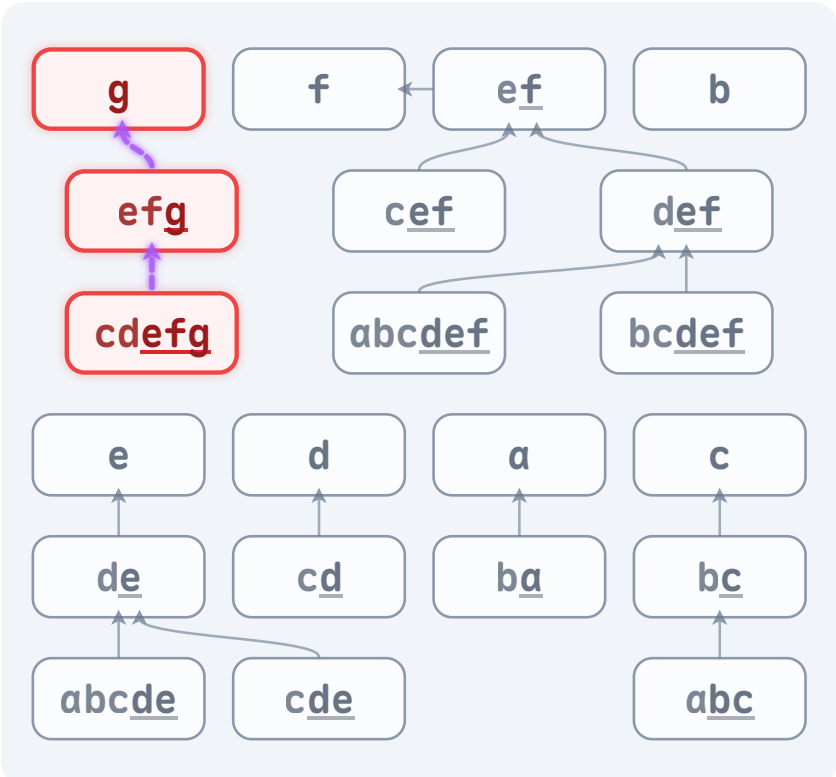

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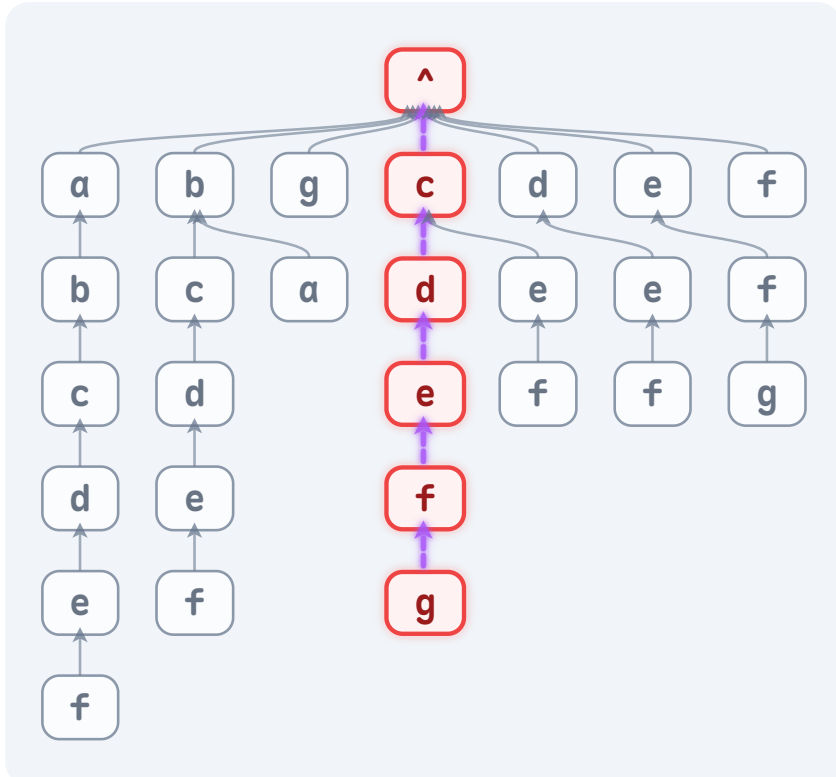

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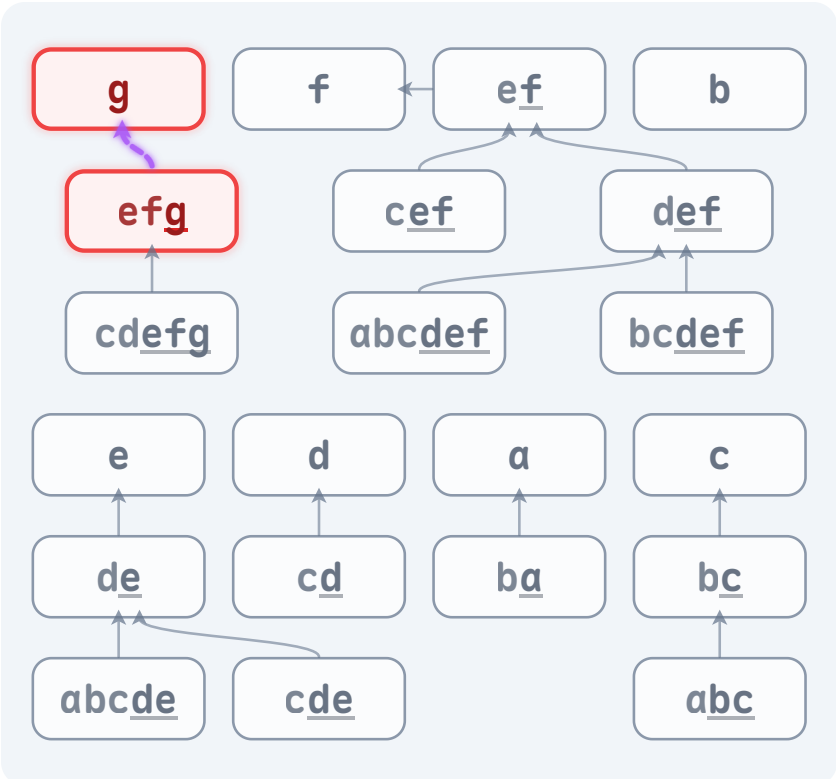

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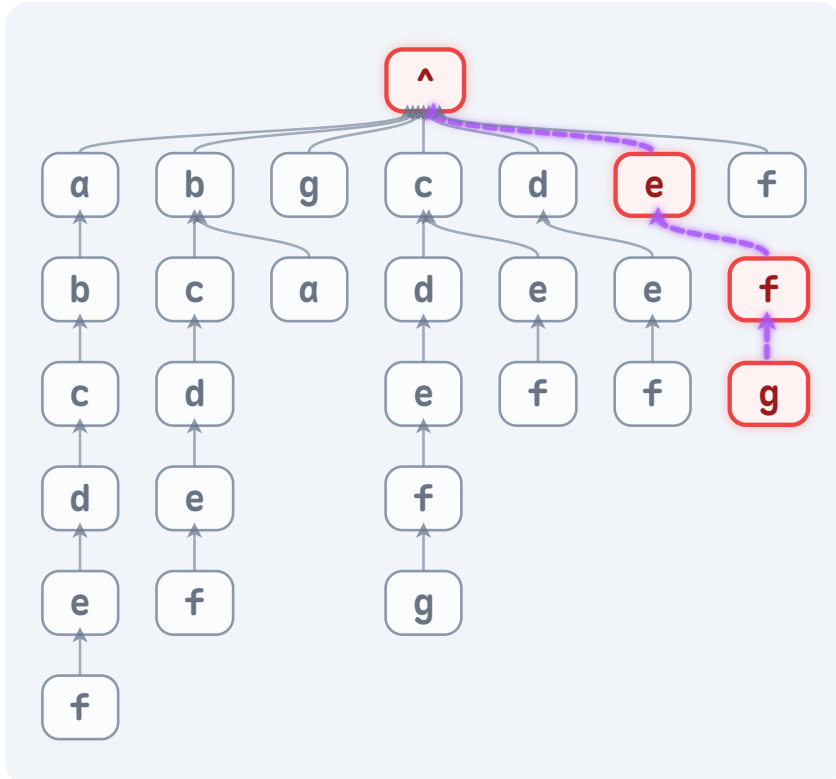

Aho–Corasick Automaton

Incremental BPE Algorithm

- **For each byte:**
  - **Transition on the Aho–Corasick automaton:** $\mathcal{O}(1)$
  - **Centroid Decomposition:** $\mathcal{O}(\log t)$
    - **Verification for the centroid:** $\mathcal{O}(1)$
    - **Binary search for its children:** $\mathcal{O}(\log t)$
- $\mathcal{O}(\log^2 t)$ **in total!** (t **is the length of the longest token)**
- **Check out the paper for more details!**

Thank you!

Incremental BPE Tokenization