Evaluating Bivariate Causal Statements based on Mutual Compatibility

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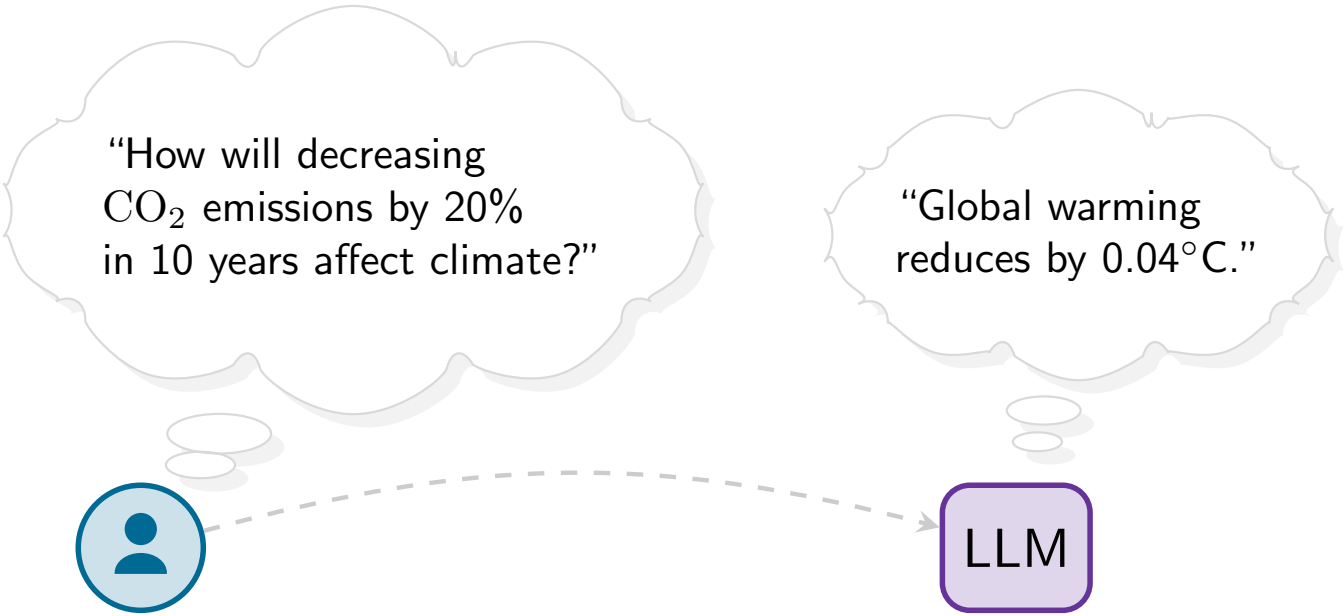
Motivation



Question

How can we falsify causal claims without experiments?

Idea

If we have many causal statements, we can try to check their compatibility!

Setup

We study **complete acyclic** lists of **linear bivariate** causal statements for n variables:

- **Bivariate:** Each statement is on a pair of variables (X, Y).
- **Linear:** Each statement specifies a causal direction $X \to Y$ and a linear causal coefficient α: If X is shifted by δ, then Y shifts by $\alpha\delta$.
- **Complete:** We have a causal statement for each of the $\binom{n}{2}$ possible pairs of variables.
- **Acyclic:** The causal directions are consistent with a total ordering of the variables.

We also have **observational data** for the n variables.

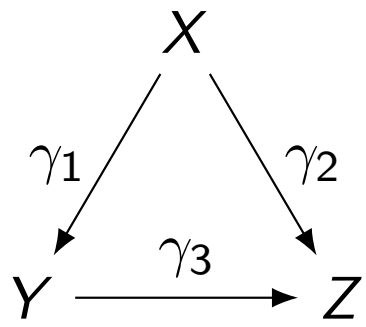
Induced Multivariate Model

Lemma

Each complete acyclic list of linear bivariate causal statements induces a unique multivariate linear causal model that fits the data and whose pairwise marginals coincide with the bivariate statements.

$$X \xrightarrow{\alpha_1} Y$$
$$X \xrightarrow{\alpha_2} Z$$
$$Y \xrightarrow{\alpha_3} Z$$

$\Longrightarrow$

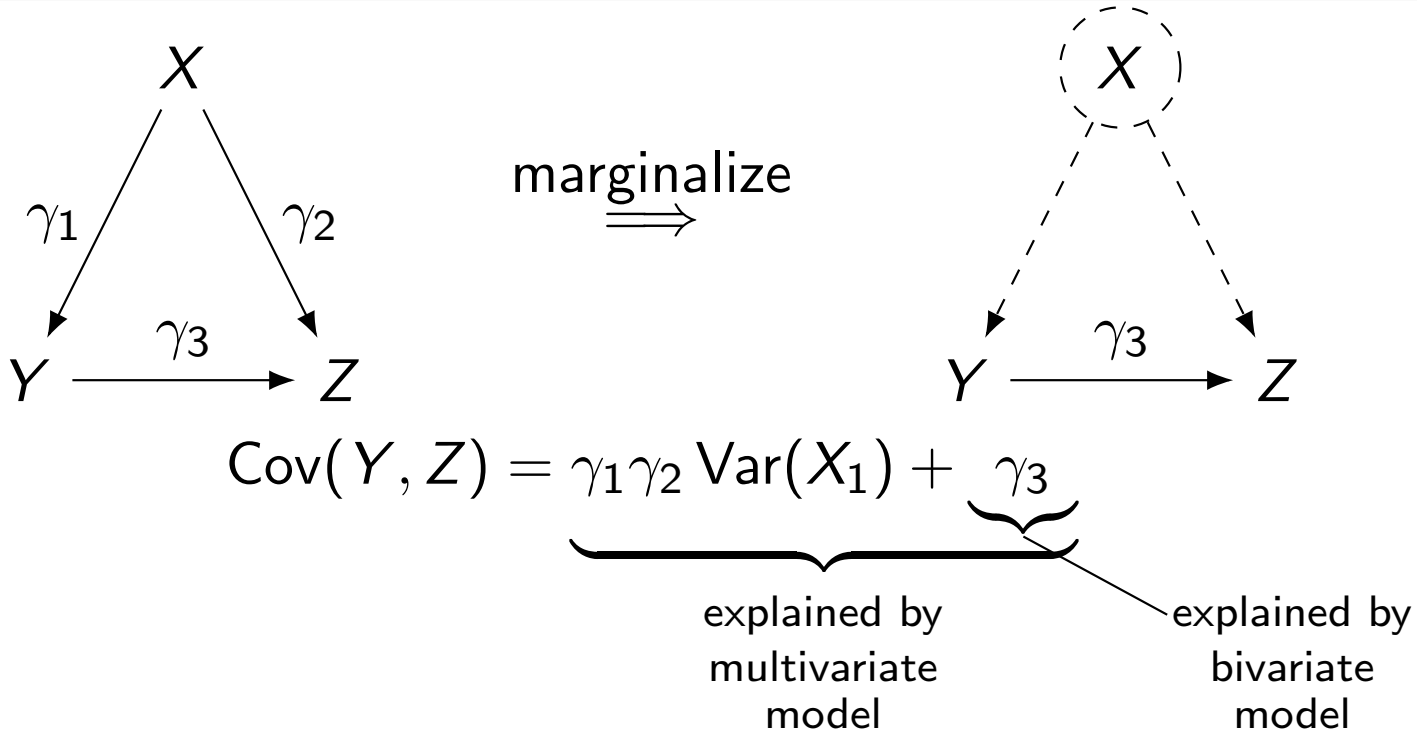


$$\alpha_1 = \gamma_1$$
$$\alpha_2 = \gamma_2 + \gamma_1\gamma_3$$
$$\alpha_3 = \gamma_3$$

Model Plausibility

Postulate

A plausible multivariate model should be able to explain pairwise covariances better than any individual bivariate model by itself.



Compatibility Score

For a list A of linear causal statements, we define:

- $\mathcal{C}^{\text{biv}}(X_i, X_j, A)$ = squared part of $\text{Cov}(X_i, X_j)$ that is unexplained by the bivariate model
- $\mathcal{C}^{\text{mult}}(X_i, X_j, A)$ = squared part of $\text{Cov}(X_i, X_j)$ that is unexplained by the multivariate model

Theorem

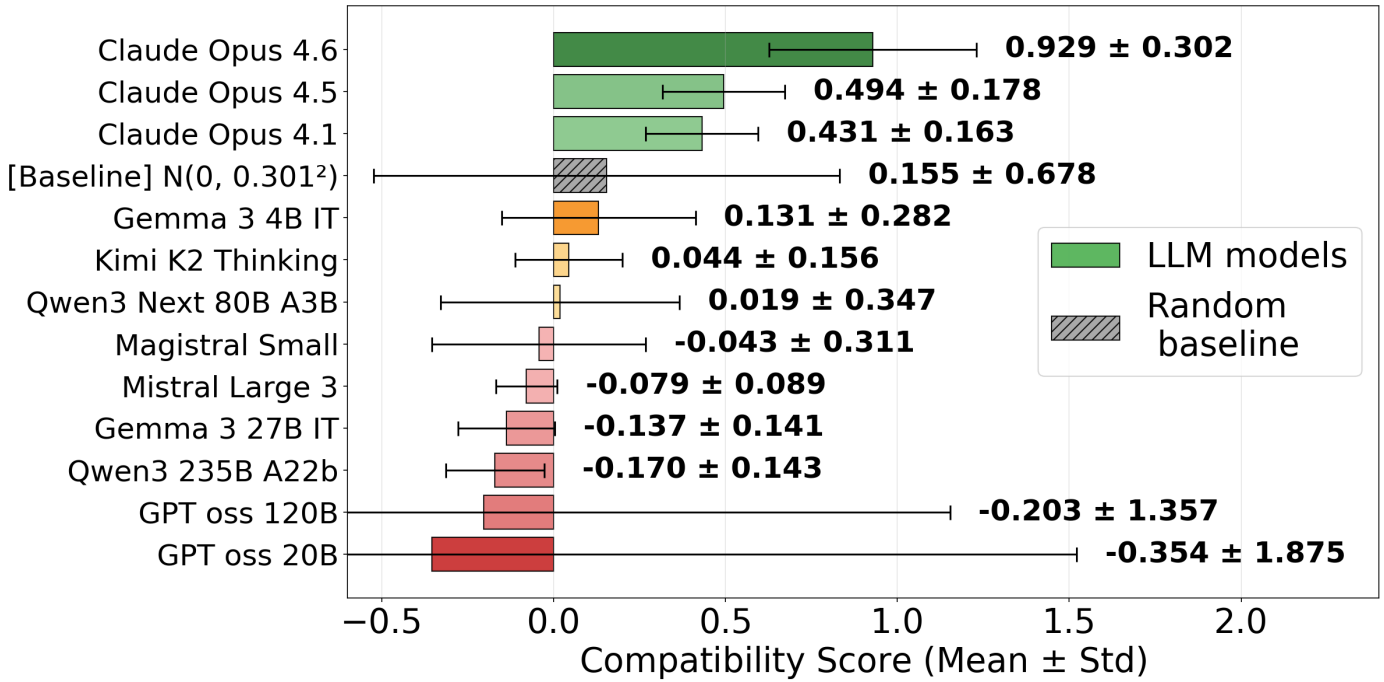
For random causal models we have $\sum_{i,j} \mathcal{C}^{biv}(X_i, X_j, A^*) > \sum_{i,j} \mathcal{C}^{mult}(X_i, X_j, A^*)$ for the true causal statements A^* in expectation (under an independence of mechanisms assumption).

Definition

We define the compatibility score for linear bivariate causal statements as

$$\text{comp}(X, A) := \sum_{i<j} \left(\mathcal{C}^{biv}(X_i, X_j, A) - \mathcal{C}^{mult}(X_i, X_j, A) \right).$$

Compatibility Scores of LLMs



Causal statements generated given data and variables from the Gapminder Foundation(`https://www.gapminder.org/data/`):

- population density,
- literacy rate,
- average daily income,
- access to basic sanitation,
- percentage of adults that smoke
- happiness score,
- average life expectancy.

Check out the paper!

https://arxiv.org/abs/2606.00278