

Minimizing Upper Confidence Bounds: A Data-Driven Framework for Stochastic Programming

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Data Scarcity

- **Problem:** Stochastic programming typically assumes the distribution P of uncertain parameters is known. In practice, we rely on limited data $\xi_{1:N}$, leading to **Epistemic Uncertainty** (risk stemming from finite samples).
- **Current Methods:**
 - **Sample Average Approximation (SAA)** yields overly optimistic decisions that fail in real-world deployment.
 - **Distributionally Robust Optimization (DRO)** incurs high complexity and requires sensitive parameter tuning to avoid over-conservatism.
- **Our Proposal:** Minimize a robust **Upper Confidence Bound (UCB)** to guarantee performance.

Average Percentile Upper Bound (APUB)

- **Major Challenge:** Traditional UCBs cannot preserve the convexity of objectives and thus have high complexity.

- Standard UCB: $\hat{\mu}_N + z_\alpha \hat{\sigma}_N / \sqrt{N}$

- Efron's Percentile UCB:

$$\mathbb{U}_{\text{Efron}}^\alpha[\mu | \hat{\mathbb{P}}_N] := \inf \left\{ t \in \mathbb{R} : \Pr \left(\frac{1}{N} \sum_{n=1}^N \zeta_n \leq t \mid \hat{\mathbb{P}}_N \right) \geq 1 - \alpha \right\}$$

- **APUB:**

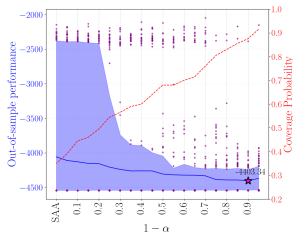
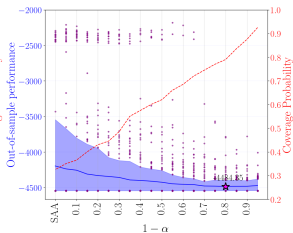
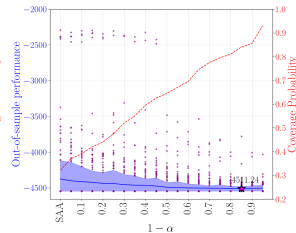
$$\mathbb{U}_{\text{APUB}}^\alpha[\mu | \hat{\mathbb{P}}_N] := \frac{1}{\alpha} \int_0^\alpha \mathbb{U}_{\text{Efron}}^\tau[\mu | \hat{\mathbb{P}}_N] d\tau$$

Advantage: Instead of picking a single bootstrap quantile, it averages the tail (worst scenarios) above Efron's bound. This smoothing operation preserves convexity while maintaining robust statistical guarantees.

APUB Optimization:

$$\min_{x \in \mathcal{X}} \mathbb{U}_{\text{APUB}}^\alpha[\mathbb{E}[F(x, \xi)] | \hat{\mathbb{P}}_N]$$

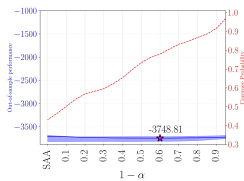
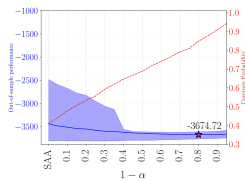
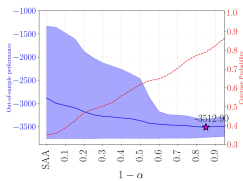
SAA vs. APUB (two-stage SP with random recourse)

(a) $N = 120$ (b) $N = 240$ (c) $N = 480$

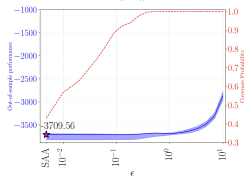
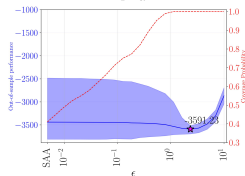
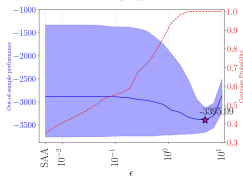
Feature	SAA	APUB
Stability ($N \leq 240$)	Unstable: High mean costs, wide dispersion, and frequent extreme outliers.	Robust: Lower mean costs and significantly reduced dispersion/worst-case risk.
Data Efficiency	Requires large N to achieve reliability.	High: $N = 120$ under APUB can outperform SAA with $N = 240$.
Convergence	Standard asymptotic improvement as $N \rightarrow \infty$.	Consistent: Smoothly converges to SAA performance as data grows; no over-conservatism.
Coverage Prob.	Very low empirical coverage even as N increases.	Reliable: Tracks nominal confidence ($1 - \alpha$); asymptotically correct.

DRO v.s. APUB (two-stage SP with fixed recourse)

APUB



DRO

(a) $N = 30$ (b) $N = 120$ (c) $N = 480$

Feature	DRO	APUB
Interpretation	Difficult to select meaningful uncertainty radius ϵ .	Tied to frequentist confidence $(1 - \alpha)$.
Data-Driven Nature	Requires manual tuning; radii decay must be hand-calibrated.	Contracts naturally with N ; no manual tuning.
Comp. Cost	Often NP-hard for random recourse matrices.	Efficient SGD and novel L-shaped decomposition method

Conclusions

- 1 **Unified Robust Framework:** Formalized APUB to rigorously quantify and mitigate epistemic uncertainty.
- 2 **Theory-Led Guarantees:** Established comprehensive asymptotic guarantees, bridging the interface between statistical estimation and robust optimization.