LiMuon: Light and Fast Muon Optimizer for Large Models

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Roadmap

- Background
- LiMuon Optimizer
- Convergence Properties
- Experimental Results
- Conclusions

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Background

- Over the past decade, training large models has been dominated by the vector-wise optimization methods including SGD, momentum-based SGD, Adam and AdamW algorithms.

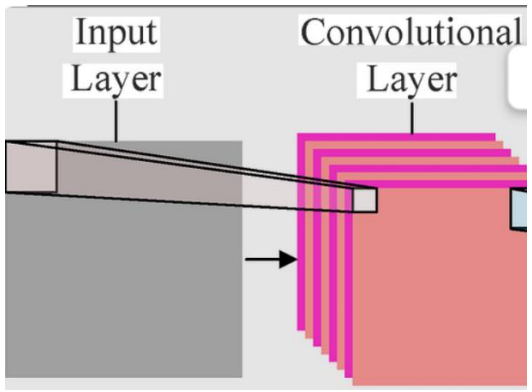
ChatGPT

deepseek

Background

- These vector-wise algorithms ignore the inherent matrix structure of parameters in AI models such as the convolutional layers in CNNs , and the query, key, and value matrices in transformers, which results in suboptimal convergence efficiency.

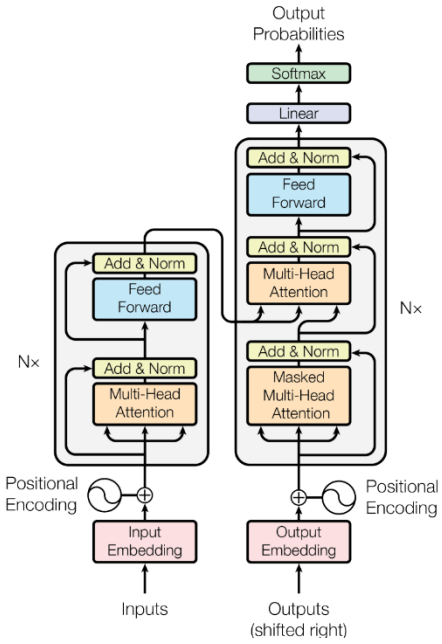
CNNs



$$Z^{(t)} = W^{(t)} * X^{(t-1)} + b^{(t)}$$

$$X^{(t)} = a(Z^{(t)})$$

Transformers



$$\text{Attention}(Q, K, V) = \text{softmax}(\frac{QK^T}{\sqrt{d_k}})V$$

Background

- More recently, a useful matrix-wise optimizer(i.e., Muon) has been proposed to train the large matrix-structured models, which shows markedly faster convergence than the vector-wise algorithms.

$$\min_{W \in \mathbb{R}^{m \times n}} F(W) = \mathbb{E}_{\xi \sim \mathcal{D}}[f(W; \xi)]$$

Algorithm 1 Muon Optimizer Jordan et al., 2024

Input: $\eta > 0$ and $\mu > 0$;
Initialize: $W_1 \in \mathbb{R}^{m \times n}$;
for $t = 1, 2, \ldots, T$ **do**
  Draw a sample $\xi_t \sim \mathcal{D}$;
  $G_t = \nabla f(W_t; \xi_t)$;
  $M_t = \mu M_{t-1} + G_t$;
  $O_t = \text{Newton-Schulz}(B_t)$;
  $W_{t+1} = W_t - \eta O_t$;
end for
Output: W_T.

Jordan, K., Jin, Y., Boza, V., Jiacheng, Y., Cesista, F., Newhouse, L., and Bernstein, J. Muon: An optimizer for hidden layers in neural networks. URL https://kellerjordan. github. io/posts/muon, 2024

Background

- More recently, the Muon optimizer has been used to train some typical large models such as e.g., DeepSeek V4, Kimi K2 and YOLO26.

Background

- However, the Muon optimizer still requires high sample complexity and high memory cost in training large models.

more Compute/GPU
more Electricity

more Storage

Expensive!

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LiMuon Optimizer

In our paper, we propose a light and fast Muon (called as LiMuon) optimizer, which simultaneously has a lower memory cost and lower sample complexity than the Muon and its variants.

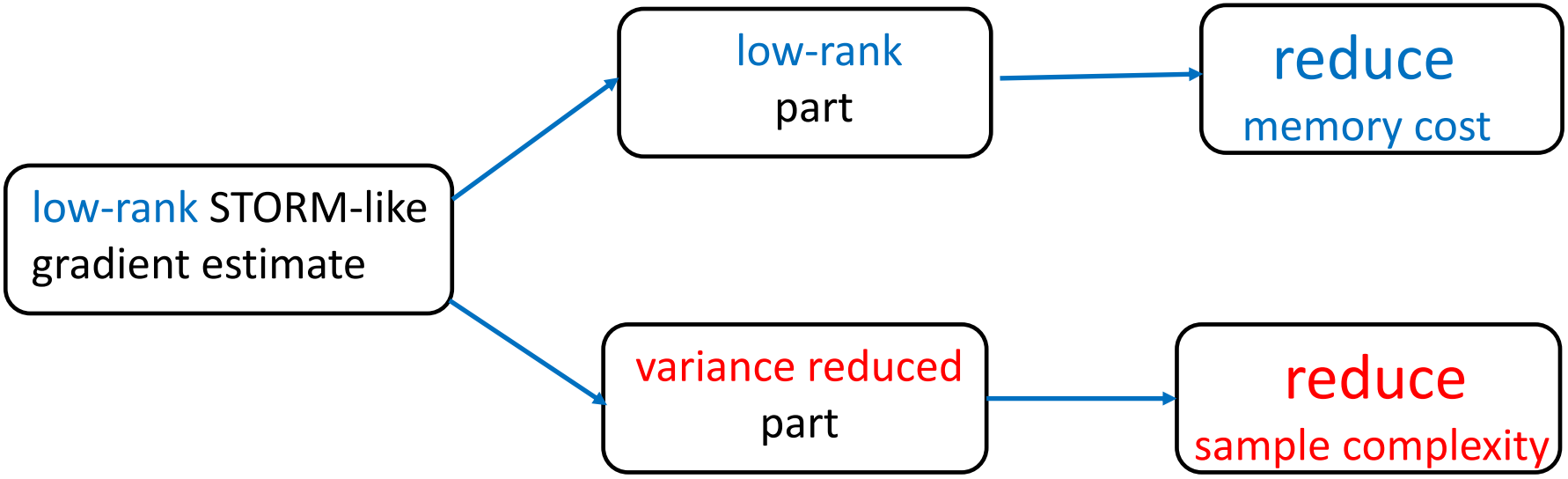
$$\min_{W \in \mathbb{R}^{m \times n}} F(W) = \mathbb{E}_{\xi \sim \mathcal{D}}[f(W; \xi)]$$

LiMuon Optimizer

Here, we first introduce a new low-rank STORM-like gradient estimate:

$$M_{t+1} = \nabla f(W_{t+1}; \xi_{t+1}) + (1 - \beta_{t+1})(\hat{M}_t - \nabla f(W_t; \xi_{t+1}))$$

where $\hat{M}_t = \hat{U}_t \hat{S}_t \hat{V}_t^\top$ is a low-rank approximated estimate of momentum M_t.



LiMuon Optimizer

Algorithm 1 LiMuon Optimizer

Input: $\eta_t > 0$, $\beta_t \in [0, 1)$, $0 < \hat{r} < r = \min(m, n)$ and $s \geq 2$;
Initialize: $W_0 \in \mathbb{R}^{m \times n}$, $\xi_0 \sim \mathcal{D}$ and $M_0 = \nabla f(W_0; \xi_0)$;
for $t = 0, 1, \ldots, T-1$ **do**
  $(U_t, \Sigma_t, V_t) = \text{SVD}(M_t)$;
  $W_{t+1} = W_t - \eta_t U_t V_t^\top$;
  Draw a sample $\xi_{t+1} \sim \mathcal{D}$;
  $M_{t+1} = \nabla f(W_{t+1}; \xi_{t+1}) + (1 - \beta_{t+1})(M_t - \nabla f(W_t; \xi_{t+1}))$ (**Option** #1);
  $(\hat{U}_t, \hat{S}_t, \hat{V}_t) = \text{RSVD}(M_t, \hat{r}, s)$ (from Algorithm 2), and $\hat{M}_t = \hat{U}_t \hat{S}_t \hat{V}_t^T$;
  $M_{t+1} = \nabla f(W_{t+1}; \xi_{t+1}) + (1 - \beta_{t+1})(\hat{M}_t - \nabla f(W_t; \xi_{t+1}))$ (**Option** #2).
end for
Output: W_T.

Randomized SVD

Algorithm 2 RSVD$(A, \hat{r}, s)$

Input: Matrix $A \in \mathbb{R}^{m \times n}$, target rank $\hat{r} > 0$, oversampling parameter $s > 0$;
Output: U, Σ, V;
$l = \hat{r} + s$, and generate a random Gaussian matrix $\Omega \in \mathbb{R}^{n \times l}$;
$Y = A\Omega \in \mathbb{R}^{m \times l}$, and compute the QR decomposition $Y = QR$;
$B = Q^\top A \in \mathbb{R}^{l \times n}$, and compute SVD of the small matrix: $(\tilde{U}, \Sigma, V) = \text{SVD}(B)$, and $U = Q\tilde{U}$.

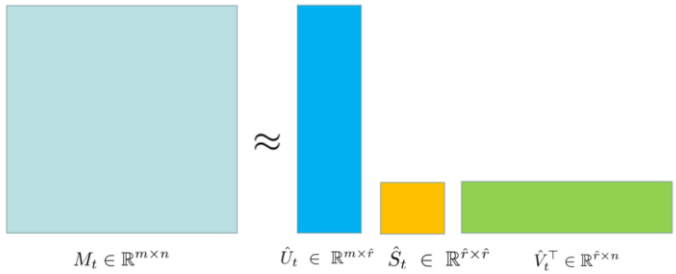


Figure 1. Low Rank Compression of Momentum M_t.

LiMuon Optimizer

Algorithm 3 LiMuon Optimizer with Newton-Schulz

Input: $\eta_t > 0$, $\beta_t \in [0, 1)$, $0 < \hat{r} < r = \min(m, n)$, $s \geq 2$ and $q \geq 1$;
Initialize: $W_0 \in \mathbb{R}^{m \times n}$, $\xi_0 \sim \mathcal{D}$ and $M_0 = \nabla f(W_0; \xi_0)$;
for $t = 0, 1, \ldots, T-1$ **do**
  $O_t =$ Newton-Schulz(M_t, q) (from Algorithm 4);
  $W_{t+1} = W_t - \eta_t O_t$;
  Draw a sample $\xi_{t+1} \sim \mathcal{D}$;
  $M_{t+1} = \nabla f(W_{t+1}; \xi_{t+1}) + (1 - \beta_{t+1})(M_t - \nabla f(W_t; \xi_{t+1}))$ (**Option** #1);
  $(\hat{U}_t, \hat{S}_t, \hat{V}_t) = \text{RSVD}(M_t, \hat{r}, s)$ (from Algorithm 2), and $\hat{M}_t = \hat{U}_t \hat{S}_t \hat{V}_t^T$;
  $M_{t+1} = \nabla f(W_{t+1}; \xi_{t+1}) + (1 - \beta_{t+1})(\hat{M}_t - \nabla f(W_t; \xi_{t+1}))$ (**Option** #2).
end for
Output: W_T.

Algorithm 4 Newton-Schulz(M, q)

Input: Matrix $M \in \mathbb{R}^{m \times n}$, step size $q \geq 1$, and polynomial $p_\kappa(\cdot)$, where κ is degree;
Initialize: $X_0 = M/\alpha$ with $\alpha = \max(1, \|M\|_{\text{F}})$;
for $j = 1, 2, \ldots, q$ **do**
  $X_j = p_\kappa(X_{j-1} X_{j-1}^\top) X_{j-1}$;
end for
$O = X_q$;
Output: O.

We use a small and fixed number of Newton-Schulz steps to approximate the orthogonalization process instead of computing exact SVD.

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Convergence Properties

In our paper, we studied the convergence properties of our LiMuon optimizer with exact SVD and Newton-Schulz steps, respectively, under **the generalized smooth condition**.

Table 1. Comparison of the Muon optimizer and its variants for finding an ϵ-stationary point ($\mathbb{E}\|\nabla F(W)\|_{\mathrm{F}} \leq \epsilon$) of nonconvex stochastic optimization. Here $\hat{r} \ll \min(m,n)$ and $\chi_q = \frac{1}{1-\varepsilon_q} > 1$, where $\varepsilon_q \in (0,1)$ is a polar approximation error.

Algorithm	Reference	SFO Complexity	State Memory	Generalized Smooth	Newton-Schulz
Muon	(Shen et al., 2025)	$O(\epsilon^{-4})$	mn		
SCG	(Pethick et al., 2025a)	$O(\epsilon^{-4})$	mn		
Gluon	(Riabinin et al., 2025)	$O(\epsilon^{-4})$	mn	$\surd$	
GGNC	(Pethick et al., 2025b)	$O(\epsilon^{-4})$	mn	$\surd$	
SUMO	(Refael et al., 2025)	$O(\epsilon^{-4})$	$(m+n)\hat{r}$		
Muon^{++}	(Sfyraki & Wang, 2025)	$O(\epsilon^{-3})$	mn		
LiMuon	Ours	$O(\epsilon^{-3})$	$(m+n)\hat{r}$	$\surd$	
Muon	(Kim & Oh, 2026)	$O(\chi_q^4\epsilon^{-4})$	mn		$\surd$
LiMuon	Ours	$O(\chi_q^3\epsilon^{-3})$	$(m+n)\hat{r}$	$\surd$	$\surd$

Convergence Properties

Assumption 4.4 (Frobenius-Norm Generalized Smoothness). Functions $F(W)$ and $f(W;\xi)$ for all $\xi \sim \mathcal{D}$ are (L_0, L_1)-Frobenius norm smooth, if for any $W, W' \in \mathbb{R}^{m\times n}$, we have

$$\begin{aligned}
&\|\nabla F(W) - \nabla F(W')\|_{\mathrm{F}}^2 \\
&\le (L_0^2 + L_1^2\|\nabla F(W)\|_{\mathrm{F}}^2)\|W - W'\|_{\mathrm{F}}^2, \\
&\mathbb{E}\|\nabla f(W;\xi) - \nabla f(W';\xi)\|_{\mathrm{F}}^2 \\
&\le (L_0^2 + L_1^2(\mathbb{E}\|\nabla F(W)\|_{\mathrm{F}})^2)\|W - W'\|_{\mathrm{F}}^2.
\end{aligned}$$

Theorem 4.13. *Under the Assumptions 4.4, 4.5, 4.6, the sequence $\{W_t\}_{t=0}^T$ be generated from Algorithm 1 with **Option #2**. Let $\beta_t = \beta \in (0,1)$ and $\eta_t = \eta \le \min\left(\frac{1}{4L_1 r}, \frac{\beta}{12L_1 r(1-\beta)}\right)$ for all $t \ge 0$, and $s \ge 2$, $\hat{r} \ge 2$, $s + \hat{r} \le r$, and $\gamma \le \frac{\beta}{3\sqrt{r}(1-\beta)}$, we have*

$$\begin{aligned}
\frac{1}{T+1}\sum_{t=0}^{T}\mathbb{E}\|\nabla F(W_t)\|_* &\le \frac{4(F(W_0) - F^*)}{T\eta} + 2rL_0\eta \\
&+ \frac{6\sqrt{r}\sigma}{T\beta} + 6\sqrt{r}\sqrt{\frac{\beta}{2-\beta}}\sigma + 6L_0\eta r\sqrt{\frac{(1-\beta)^2}{(2-\beta)\beta}},
\end{aligned}$$

where $\gamma = \tau\left(1 + \frac{\hat{r}}{s-1}\right)^{\frac{1}{2}}$ and $\tau \in (0,1)$.

Remark 4.14. From the above Theorem 4.13, let $\eta = O(\frac{1}{T^{2/3}})$ and $\beta = O(\frac{1}{T^{2/3}})$, we have

$$\frac{1}{T+1}\sum_{t=0}^{T}\mathbb{E}\|\nabla F(W_t)\|_* \le O\left(\frac{1}{T^{1/3}}\right) \le \epsilon.$$

Our LiMuon algorithm with lower memory cost still obtains a lower sample complexity of $O(\epsilon^{-3})$ under generalized smooth condition.

Convergence Properties

Assumption 4.16 (Nuclear-Norm Generalized Smoothness). Functions $F(W)$ and $f(W;\xi)$ for all $\xi \sim \mathcal{D}$ are (L_0, L_1)-nuclear norm smooth, if for any $W, W' \in \mathbb{R}^{m\times n}$, we have

$$\begin{aligned}
&\|\nabla F(W) - \nabla F(W')\|_*^2 \\
&\le (L_0^2 + L_1^2\|\nabla F(W)\|_*^2)\|W - W'\|_{op}^2, \\
&\mathbb{E}\|\nabla f(W;\xi) - \nabla f(W';\xi)\|_*^2 \\
&\le (L_0^2 + L_1^2(\mathbb{E}\|\nabla F(W)\|_*)^2)\|W - W'\|_{op}^2.
\end{aligned}$$

Theorem 4.23. *Under the Assumptions 4.16, 4.5, 4.6, the sequence $\{W_t\}_{t=0}^T$ be generated from Algorithm 3 with **Option** #2. Let $\beta_t = \beta \in (0,1)$, and $\eta_t = \eta \le \min\left(\frac{1-\varepsilon_q}{4L_1(1+\varepsilon_q)^2}, \frac{(1-\varepsilon_q)\beta}{4L_1\sqrt{r}(1-\beta)(1+\varepsilon_q)(3+\varepsilon_q)}\right)$ for all $t \ge 0$, $s \ge 2$, $\hat{r} \ge 2$, $\hat{r} + s \le r$ and $\gamma \le \frac{(1-\varepsilon_q)\beta}{(3+\varepsilon_q)\sqrt{r}(1-\beta)}$, we have*

$$\begin{aligned}
&\frac{1}{T+1}\sum_{t=0}^{T}\mathbb{E}\|\nabla F(W_t)\|_* \le \frac{4(F(W_0) - F^*)}{(T+1)\eta\chi_q^{-1}} \\
&+ \frac{2L_0\eta(1+\varepsilon_q)^2}{\chi_q^{-1}} + \frac{2(3+\varepsilon_q)}{\chi_q^{-1}}\Big(\frac{\sqrt{r}\sigma}{(T+1)\beta} + \sqrt{r}\sqrt{\frac{\beta}{2-\beta}}\sigma \\
&+ \sqrt{r}\sqrt{\frac{(1-\beta)^2}{(2-\beta)\beta}}L_0\eta(1+\varepsilon_q)\Big),
\end{aligned}$$

where $\gamma = \tau\left(1 + \frac{\hat{r}}{s-1}\right)^{\frac{1}{2}}$ with $\tau \in (0,1)$, and $\chi_q = \frac{1}{1-\varepsilon_q} > 1$.

Remark 4.24. From the above Theorem 4.23, let $\eta = O(\frac{1}{T^{2/3}})$ and $\beta = O(\frac{1}{T^{2/3}})$, we have

$$\frac{1}{T+1}\sum_{t=0}^{T}\mathbb{E}\|\nabla F(W_t)\|_* \le O\Big(\frac{\chi_q}{T^{1/3}}\Big) \le \epsilon.$$

Our LiMuon algorithm with lower memory cost still obtains a lower sample complexity of $O(\chi_q^3\epsilon^{-3})$ under generalized smooth condition than $O(\chi_q^4\epsilon^{-4})$ of the Muon optimizer with Newton-Schulz (Kim & Oh, 2026).

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Experimental Results

1) Training Mamba-130M Model

Table 2. Performance comparison at Mamba-130M

Method	Memory (GB) ↓	Training PPL ↓	Validation PPL ↓
Adam	22.92	480.33	526.19
AdamW	22.92	253.75	266.43
Lion	22.56	75.25	83.82
SUMO	20.10	111.56	118.52
Muon	22.20	62.49	71.27
Muon++	22.35	51.97	56.79
LiMuon(rank=4)	**18.86**	71.46	79.12
LiMuon(rank=8)	**20.25**	59.07	62.23
LiMuon(rank=16)	22.31	**50.26**	**53.84**
LiMuon(full rank)	22.80	**47.08**	**47.78**

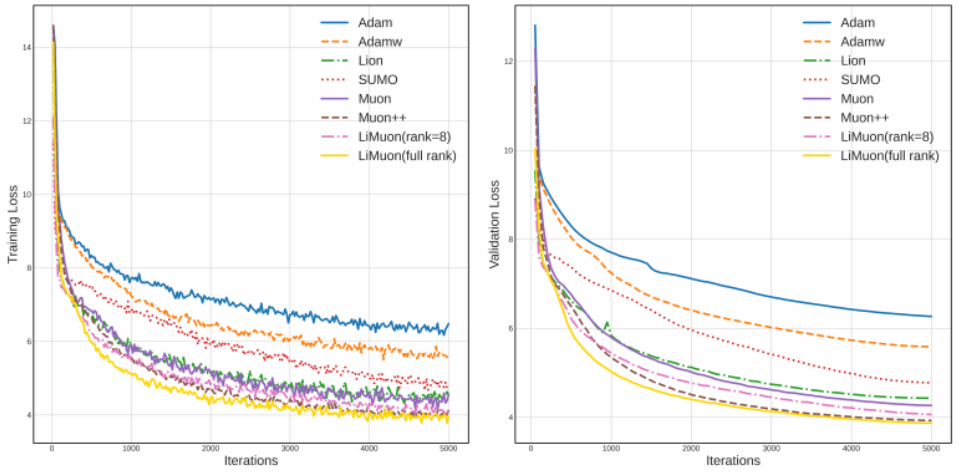

Figure 2. Performance comparison at Mamba-130M

Experimental Results

2) Training Qwen2.5-0.5B Model

From these results, the full-rank LiMuon achieves the lowest perplexity (PPL), while providing more stable training dynamics. Notably, our low-rank LiMuon variant (rank= 8) achieve competitive performance with substantially reduced memory cost, making our LiMuon suitable for training larger models under memory constraints.

Table 3. Performance comparison at Qwen-2.5-0.5B

Method	Memory (GB) ↓	Training PPL ↓	Validation PPL ↓
Adam	55.24	91.84	181.27
AdamW	55.26	57.97	113.25
Lion	54.08	48.42	113.92
SUMO	52.73	41.26	88.73
Muon	54.14	35.52	67.60
Muon++	54.30	35.16	82.26
LiMuon(rank=4)	**50.86**	41.46	92.06
LiMuon(rank=8)	**52.55**	38.86	**54.87**
LiMuon(rank=16)	54.21	**33.43**	**46.77**
LiMuon(full rank)	55.15	**30.87**	**40.83**

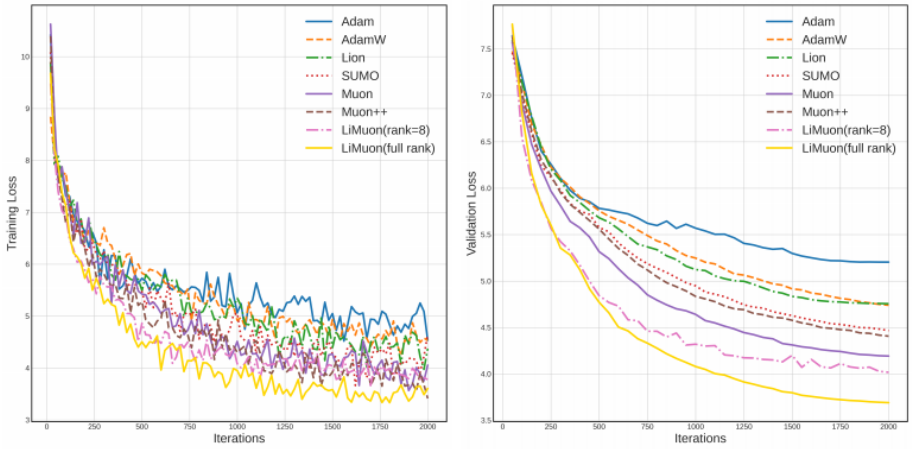

Figure 3. Performance comparison at Qwen-2.5-0.5B.

Experimental Results

3) Training ViT Model

Table 4. Performance comparison at ViT

Method	Memory (GB) ↓	Training Top-1 Accuracy(%) ↑	Training Top-5 Accuracy(%) ↑	Validation Top-1 Accuracy(%) ↑	Validation Top-5 Accuracy(%) ↑
Adam	5.57	37.47	65.45	32.91	59.96
AdamW	5.58	38.41	65.56	35.85	63.96
Lion	5.44	40.97	68.55	40.21	68.02
SUMO	5.31	51.23	78.08	44.23	70.19
Muon	5.50	78.96	93.64	47.87	73.17
Muon++	5.52	60.44	83.52	43.62	69.94
LiMuon (rank=4)	**5.10**	64.73	85.19	44.35	70.86
LiMuon (rank=8)	**5.28**	71.94	90.06	46.75	72.17
LiMuon (rank=16)	5.44	**80.58**	**93.91**	**47.98**	**73.22**
LiMuon (full rank)	5.53	**81.99**	**94.64**	**48.04**	**73.58**

Experimental Results

3) Training ViT Model

In Table 5, Time. Step is the wall-clock time of updating variable, which does not include time of computing gradients via backpropagation. Time to Target is the wall-clock time to first reach a target loss, which include time of computing gradients via backpropagation. Here we give a target loss $L^* = 3.70$ (an example, the other target losses have similar conclusions). From Table 5, our full rank LiMuon uses the shortest amount of time to reach the target loss. Meanwhile, our low-rank LiMuon (rank=8) uses a comparable time to reach the target loss under less memory usage.

Table 5. Runtime analysis on ViT.

Optimizer	**Time. Step ↓**	**Time to Target ↓**
Baseline optimizers		
Adam	$5.58ms$	$15min15s$
AdamW	$5.66ms$	$13min11s$
Lion	$4.81ms$	$12min37s$
Muon	$23.43ms$	$7min6s$
Muon++	$24.50ms$	$9min54s$
SUMO	$22.08ms$	$11min20s$
LiMuon (ours)		
LiMuon ($\hat{r}$=4)	$24.75ms$	$12min4s$
LiMuon ($\hat{r}$=8)	$25.49ms$	$10min29s$
LiMuon ($\hat{r}$=16)	$26.87ms$	$10min51s$
LiMuon (full rank)	$23.37ms$	**7min 1s**

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Conclusions

- 1) We proposed a LiMuon optimizer, which simultaneously has a lower memory cost and lower sample complexity than the Muon and its variants.

- 2) We studied the convergence properties of our LiMuon optimizer with exact SVD and Newton-Schulz steps, respectively, under the generalized smooth condition.

- 3) In particular, we proved that our LiMuon with Newton-Schulz obtains a lower sample complexity under a lower memory cost than the Muon with Newton-Schulz.

Thanks!

Q&A