Evidential Copula Concept Embedding Models

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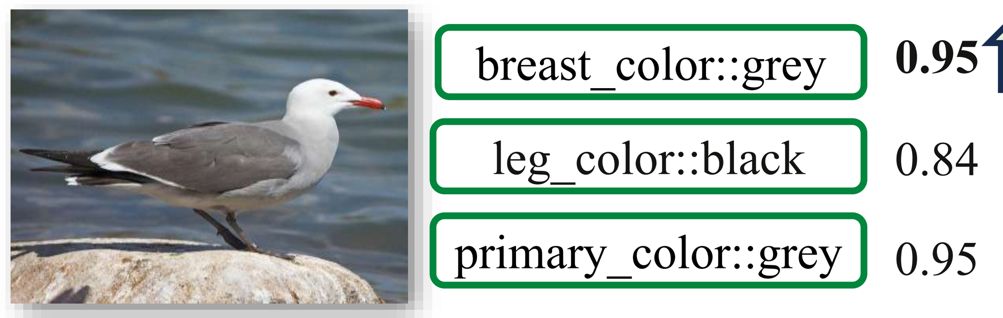
Motivation

Independence Modeling

Dependence Modeling(Ours)



- In real data, **concepts strongly correlated**: co-occurrence patterns can be essential for correct decisions, especially when visual evidence is ambiguous or concept supervision is sparse.
- Existing Concept Modeling Method **entangle Concept Uncertainty with Concept Correlation** and affect each other.

Our Contribution

- **Decoupled probabilistic framework:** EC-CEM separates per-concept uncertainty from inter-concept dependency.
- **Evidential Copula construction:** each concept is modeled by a Beta distribution, then linked through a Gaussian Copula.
- **Variational joint optimization:** concept inference and downstream classification are trained end to end.

Variational Loss

Concept Optimization

$$\mathbb{E}_{q(\mu|h)}[\log p(c|\mu)] = \sum_{k=1}^{K}(1-c_k)\psi(\beta_k) - \psi(\alpha_k+\beta_k) + c_k\psi(\alpha_k)$$

Regularization

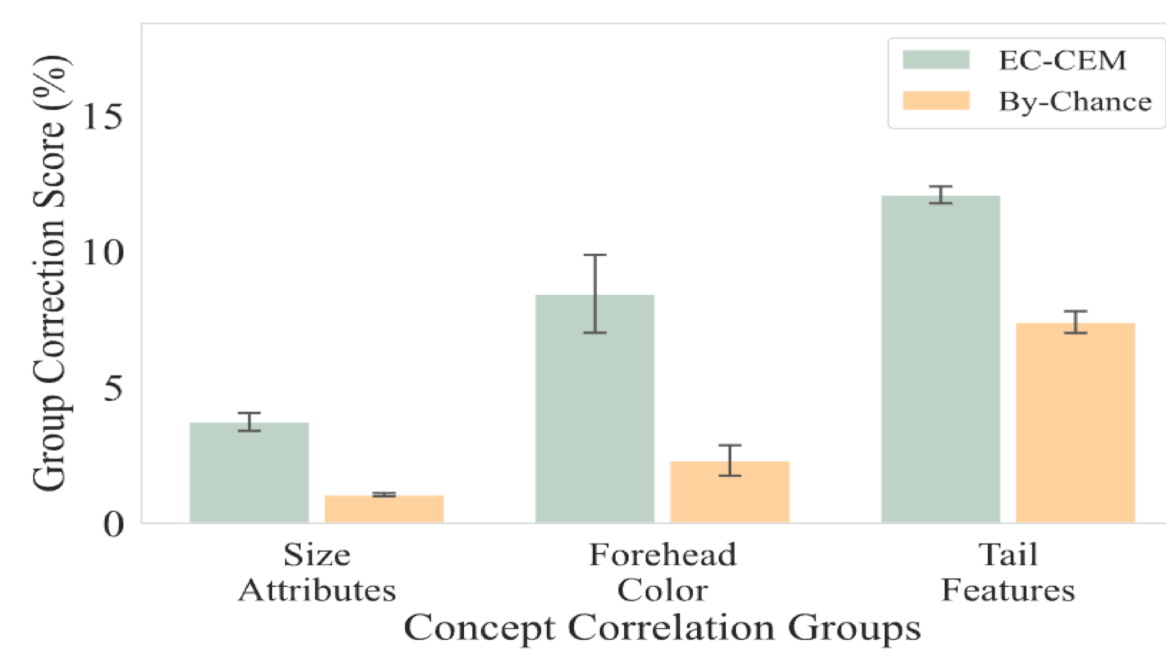
$$\mathrm{KL}[q(\mu|h)||p(\mu)] = \mathrm{KL}[N(0,I)||N(0,R)] + \sum_{k=1}^{K}\mathrm{KL}[\mathrm{Beta}(\mu_k;\alpha_k,\beta_k)\ ||\mathrm{Beta}(\mu_k;1,1)]$$

Classification

$$\mathbb{E}_{q(\mu|h)}[\log p(y|\mu,h)]$$

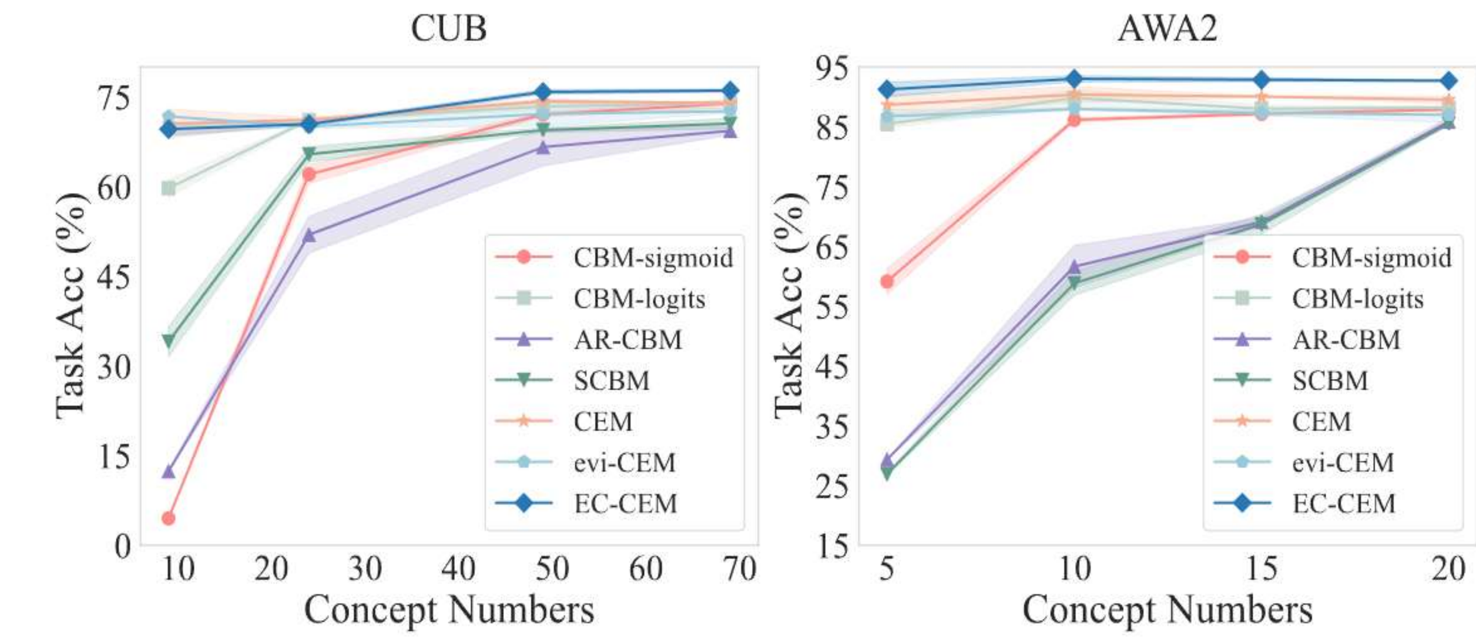
Result

Correlated Concept Correction

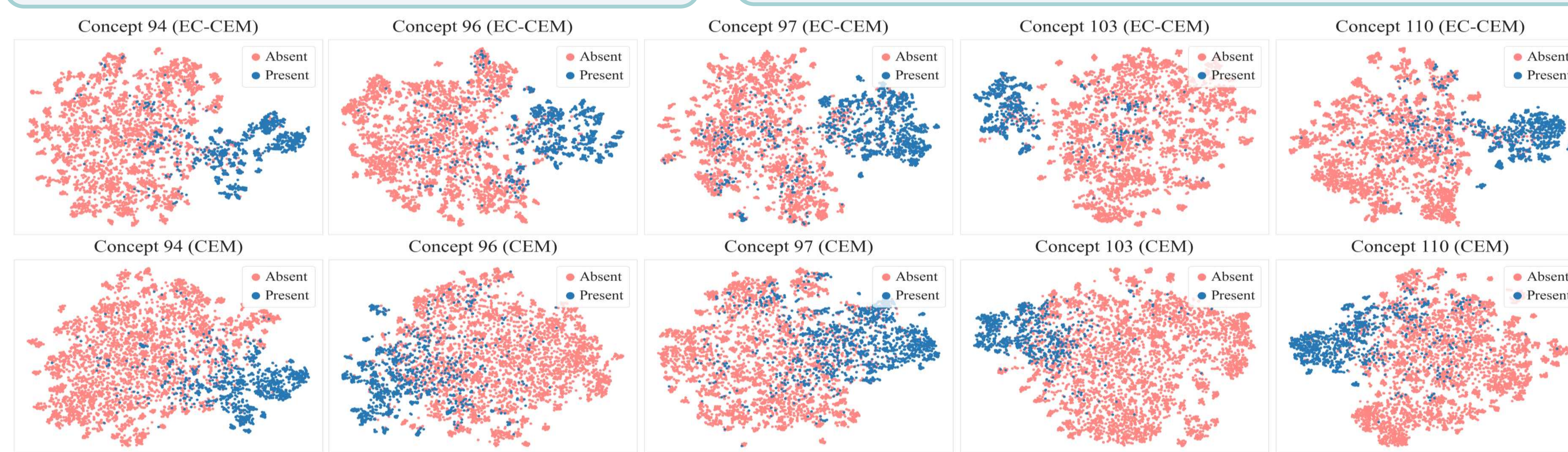


Learned dependencies enable meaningful co-correction within semantic groups.

Stronger Under Sparsity

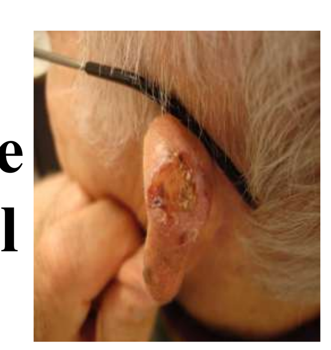
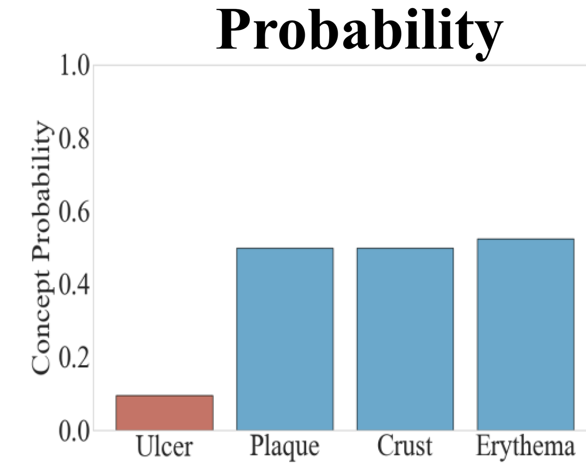
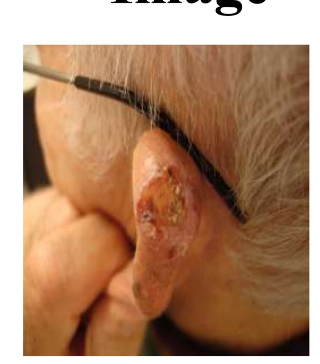
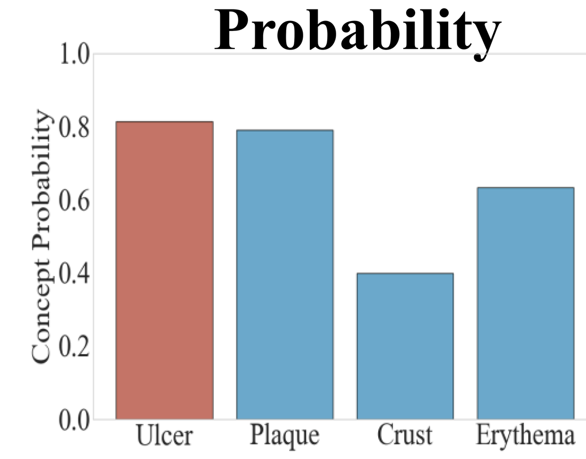


Lead over AR-CBM expands from 3.08% to 57.24% on CUB, and from 2.6% to 31.96% on AwA2, as concepts become sparse.



EC-CEM learns more semantically aligned concept embeddings.

Case Study

	Image	Probability	Prediction
Concept Independence Model			Non-neoplastic (Wrong)
EC-CEM			Malignant (Correct)

VS

Correlation Matrix