Stabilizing the Kuramoto–Sivashinsky Equation Using Deep Reinforcement Learning with a DeepONet Prior

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Motivation

Controlling chaotic systems—such as fluid flows or flame fronts—remains a core challenge. The Kuramoto–Sivashinsky (KS) equation captures this spatiotemporal chaos, yet conventional linear or model-dependent controllers scale poorly. A data-driven, adaptive, and model-free strategy is therefore essential. Few study exist using either operator learning or RL.

Key Contribution

- Hybrid control combining Deep Operator Networks (DeepONet) with model-free RL for chaotic PDE stabilization.
- Two-stage framework: DeepONet learns a generalized operator offline; DDPG refines it online with trajectory-specific feedback.
- Achieves up to 85 % energy reduction, vastly outperforming traditional feedback control.

Methodology

KS equation: $$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial x^2} + \nu\frac{\partial^4 u}{\partial x^4} = f(x,t)$$

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Methodology

The control objective is to design $f(x,t)$ to minimize the system energy: $E(t) = \frac{1}{n_x}\sum_{j=1}^{n_x} u(x_j, t)^2$

- **Numerical Solution Using Spectral Methods**

$$\frac{\partial \hat{u}}{\partial t} = (k^2 - \nu k^4)\hat{u} - \frac{ik}{2}\mathcal{F}\{u^2\} + \hat{f}$$

- **Offline Operator Learning:**

  $u(x, t) \rightarrow f_{prior}(x, t)$ from 2000 noisy trajectories.

- **Online Adaptive Control:**

DDPG adds residual action a_t to refine the prior:

$c\,(x, t) = f_{prior}(x, t) + a_t$

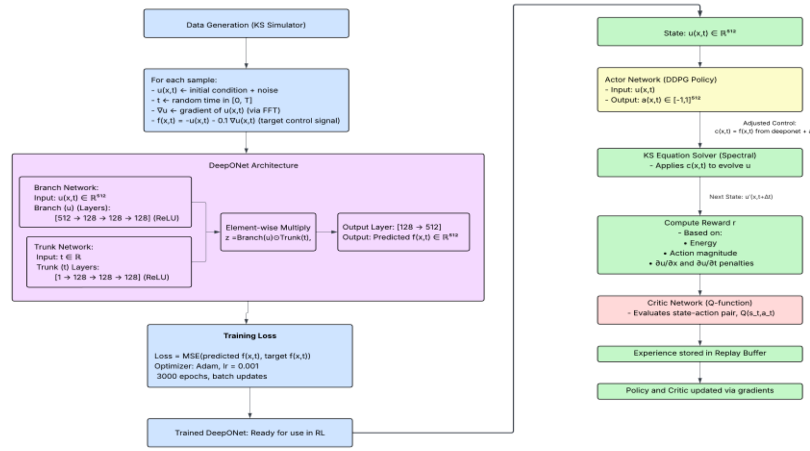

Fig 1. Model Architecture

Result

- 55 % energy drop in the first 0.2 time units; 5 times overall reduction.
- DeepONet MSE ↓ 99.3 %; RL mean reward ↑ 59.3 %.
- RL maintains u ∈ [−8, 4] vs. [−16, 12] under classic feedback.

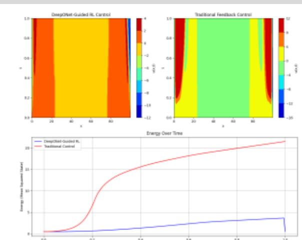

Fig 2. Spatio temporal plots and energy evolution.

Conclusion

DeepONet-guided RL delivers robust, low-energy stabilization of the KS system, showing a scalable path toward controlling 3-D turbulence and multi-physics PDEs.

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