Towards Robust Causal Effect Identification Beyond Markov Equivalence

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Keywords Causal Effect Identification, Observational Data, Graphical Models, Robustness

1. Motivation

Causal effect of $\boldsymbol{X}$ on $\boldsymbol{Y}$ depends on causal DAG $\mathcal{G}$!

$$\underbrace{p^{\mathrm{do}_{\mathcal{G}}(\boldsymbol{x})}(\boldsymbol{y})}_{\text{causal effect}} = \underbrace{\int \prod_{V_i \in \boldsymbol{V} \setminus \boldsymbol{X}} p_{V_i}(v_i \mid \mathrm{pa}_{\mathcal{G}}(v_i))\, d\overline{\boldsymbol{v}}}_{\text{truncated factorisation formula}}$$

(marginalised over $\overline{\mathbf{V}} = \mathbf{V} \setminus (\mathbf{X} \cup \mathbf{Y})$)

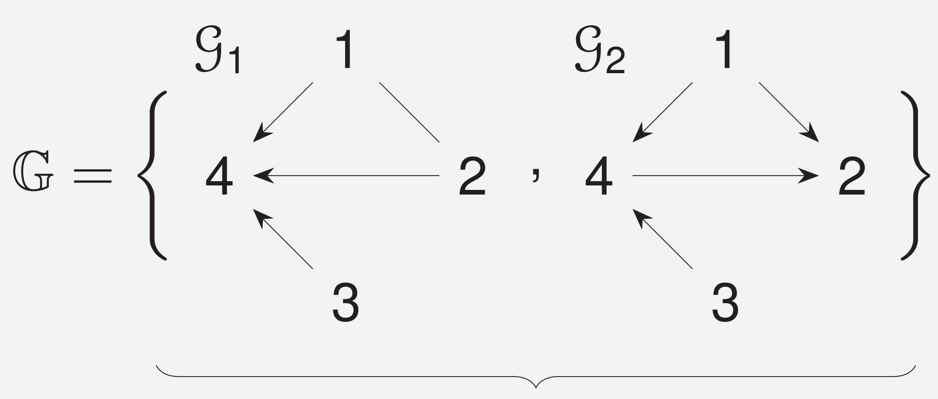
WHAT IF?? $\mathcal{G}$ is not specified up to Markov equivalence class? Can we still identify causal effects not knowing causal DAG $\mathcal{G}$??

Example †: Given distribution P with the conditional independencies:

$$\left\{ 1 \perp\!\!\!\perp 2, \quad 1 \perp\!\!\!\perp 2 \mid \{3,4\}, \quad 3 \perp\!\!\!\perp 4 \mid \{1,2\} \right\}$$

Running the Sparsest Permutation causal discovery algorithm (Raskutti and Uhler, 2018) with input P, does **not** return a unique output.

The set $\mathbb{G}$ of possible outputs is:



Non-Markov equivalent $\mathcal{G}_1$ and $\mathcal{G}_2$

and causal DAG $\mathcal{G}$ can be represented by either CPDAG $\mathcal{G}_1$ or $\mathcal{G}_2$ in $\mathbb{G}$.

Can happen in general when the faithfulness assumption is violated! (Teh, Sadeghi and Soo, 2024)

2. Problem Statement

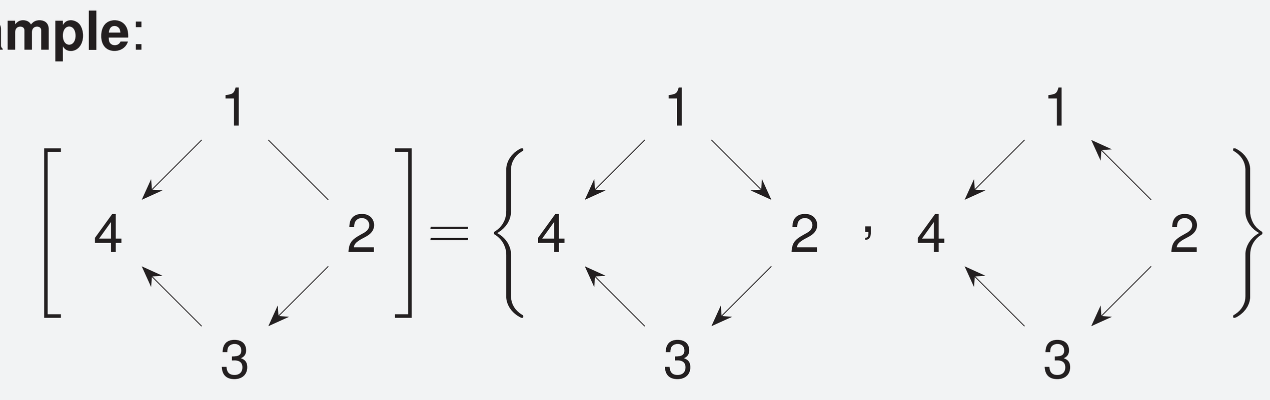
Given:
$\mathbb{G}$: a set of MPDAGs over nodes $\boldsymbol{V}$, and $\boldsymbol{X}, \boldsymbol{Y} \subseteq \boldsymbol{V}$.

- What are ***conditions*** on $\mathbb{G}$ such that

$$p^{\mathrm{do}_{\mathcal{G}_1}(\boldsymbol{x})}(\boldsymbol{y}) = p^{\mathrm{do}_{\mathcal{G}_2}(\boldsymbol{x})}(\boldsymbol{y})$$

  for every observational density p Markovian to all MPDAGs in $\mathbb{G}$, and every DAG $\mathcal{G}_1, \mathcal{G}_2 \in \bigcup_{\mathcal{G} \in \mathbb{G}} [\mathcal{G}]$?

  $[\mathcal{G}]$ is the set of DAGs represented by MPDAG $\mathcal{G}$.
  Example:



  MPDAG — Represented DAGs (obtained by directing undirected edges in the MPDAG.)

  i.e. for every possible observational density with $\mathbb{G}$, casual effect of $\boldsymbol{X}$ on $\boldsymbol{Y}$ does not depend on the choice of DAG chosen from ***all*** DAGs represented by ***all*** MPDAGs in $\mathbb{G}$; the causal effect of $\boldsymbol{X}$ on $\boldsymbol{Y}$ is ***simultaneously identifiable***.

- If simultaneously identifiable, ***formula*** for the causal effect?

3. Results

The **conditions** on $\mathbb{G}$:
For all $\mathcal{G} \in \mathbb{G}$,

1. there are no **semi-directed paths** from $\boldsymbol{X}$ to $\boldsymbol{Y}$ with only the first node in $\boldsymbol{X}$, that starts with $-$, **and**

$$\underbrace{X_1 - \cdot \rightarrow \cdot - \cdots \rightarrow Y_i}_{\text{semi-directed path (no } \leftarrow \text{ edges)}}$$

Note: same condition as Perkovic 2020

For all $\mathcal{G}_1, \mathcal{G}_2 \in \mathbb{G}$,

2a. for all $\boldsymbol{X}_j$ (maximal subsets (that are connected via $-$ in $\mathcal{G}_1$) of $\boldsymbol{X}$ that are also ancestors of $\boldsymbol{Y}$ in $\mathcal{G}_1$),

$$\text{parents of } X_j \text{ in } \mathcal{G}_1 \quad = \quad \text{parents of } X_j \text{ in } \mathcal{G}_2$$

and likewise with the roles of $\mathcal{G}_1$ and $\mathcal{G}_2$ switched.

Intuition: reweighting used to obtain the truncated factorisation formula is the same for both $\mathcal{G}_1$ and $\mathcal{G}_2$

or

2b. $RM(\mathcal{G}_1; \boldsymbol{X}, \boldsymbol{Y})$ and $RM(\mathcal{G}_2; \boldsymbol{X}, \boldsymbol{Y})$ are Markov equivalent

Intuition: both $\mathcal{G}_1$ and $\mathcal{G}_2$ implies the same truncated factorisation formula over $\boldsymbol{Y}$.

$RM(\mathcal{G}; \boldsymbol{X}, \boldsymbol{Y})$: ancestral margin of $\boldsymbol{Y}$ in graph $\mathcal{G}$ after intervening on $\boldsymbol{X}$.

Theorem

Simultaneous identifiability if 1 + 2a **or** 1 + 2b holds.

The **formula** is the same as Perkovic 2020, since the **conditions** ensure equal evaluation of the causal effect across $\mathbb{G}$.

TL; DR

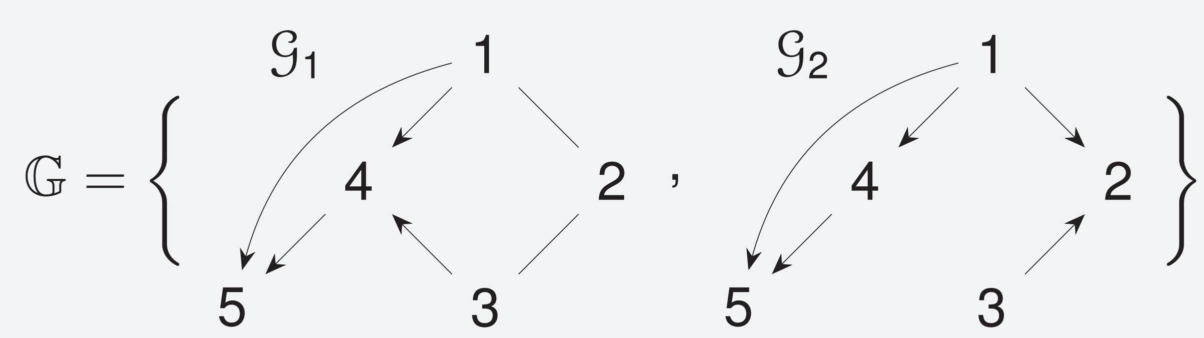
We propose sufficient conditions for **identifying causal effects** when the **Markov equivalence class of the causal graph cannot be uniquely identified**.

This can happen during causal discovery when faithfulness is violated or when edge detection lacks power.

Faithfulness is violated

Consider the set $\mathbb{G}$ in Example † from Motivation. The causal effect of 3 on 2 is simultaneously identifiable since 1 + 2a holds.

Uncertain edge 3 → 4



$RM(\mathcal{G}_1; \{4\}, \{5\}) = RM(\mathcal{G}_2; \{4\}, \{5\}) = 5 \leftarrow 4 \quad 1$

Condition 1 + 2b holds, the causal effect of 4 on 5 is simultaneously identifiable.

References
1. Perkovic, E. Identifying causal effects in maximally oriented partially directed acyclic graphs, 2020.
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3. Teh, K. Z., Sadeghi, K., and Soo, T. A general framework on conditions for constraint-based causal learning, 2024.

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