

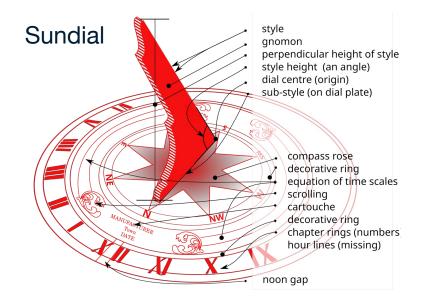


Sundial: A Family of Highly Capable Time Series Foundation Models

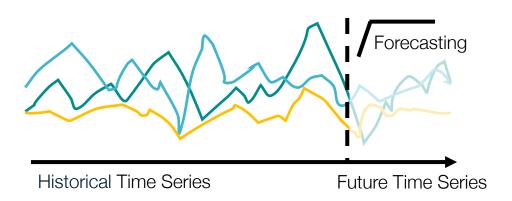
Yong Liu*, Guo Qin*, Zhiyuan Shi, Zhi Chen, Caiyin Yang, Xiangdong Huang, Jianmin Wang, Mingsheng Long

School of Software, Tsinghua University July 16th, 2025

Time Series Forecasting



Predict future values based on historical data





Weather Forecasting



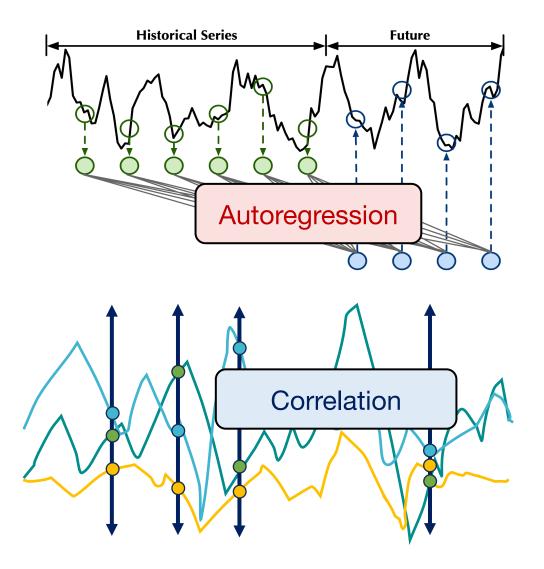
Assets Trading



Supply Chain



The Golden Rule: Learn from History



Temporal Dependency (Univariate)

Auto-Regression, Moving-Average

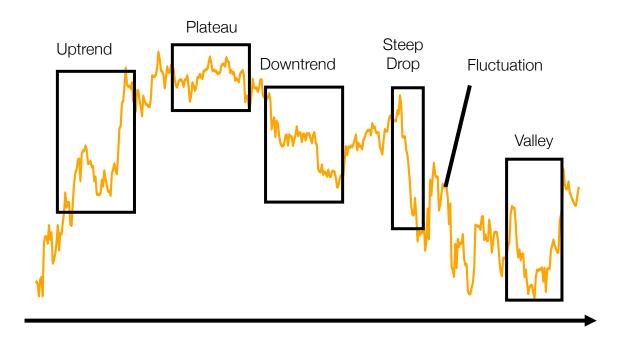
Variable Correlation (Multivariate)

VAR, Cointegration



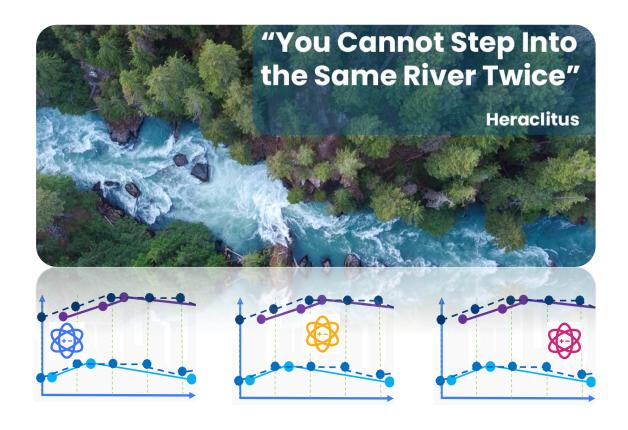
Nobel Prize in Economics

Challenges



Nonlinearity: The linear assumption may not hold for complicated variations

-> Require Good Factors (Features)



Adaptiveness: Statistical methods need to be fitted separately at different periods

-> Need Large Model Capacity

Deep Learning for Forecasting

Model Architecture

(Linear Layer; MLP; CNN; RNN; GNN Transformer; State-Space Model...)



Statistics & Theory

(Autoregression; Moving-Average; Stationary: Normalization, Differencing, Decomposition: STL; Spectral Analysis: Auto-Correlation, FFT, Wavelets...)

Challenges: Data Hungry...

- Hard to train with insufficient data
- Fail to generalize to different datasets

Challenges: Nonlinearity...

- Rely on good features
- Small model capacity

Time Series Foundation Models

Model Architecture

(Linear Layer; MLP; CNN; RNN; GNN Transformer; State-Space Model...)



Statistics & Theory

(Autoregression; Moving-Average; Stationary: Normalization, Differencing, Decomposition: STL; Spectral Analysis: Auto-Correlation, FFT, Wavelets...)

Challenges: Nonlinearity...

Era of Large Models

Challenges: Data Hungry...





Time Series

Foundation Models



Generality & Scalability

(Architecture: Transformer, Tokenization: Point/Patch, Discrete/Continuous; Pre-training: Next-Token Prediction, Masked Modeling, Generative Modeling; Data Curation...)

- ✓ Generalizable: zero-shot forecasting
- ✓ Good Factors: feature extractor of time series

Challenges: Heterogeneity, ...

Timeline

Stage 1: From Statistics/Theories to Deep Models

Holt-Winter	ARIMA	DeepAR	TFT	Prophet	N-BEATS	Informer	
1960	1982	2017	2018	2018	2019	2021	

Stage 2: Unlocking the Capability of Deep Models

Autoformer	RevIN	PatchTST	DLinear	TiDE	TimesNet	iTransformer	
2021	2022	2022	2022	2023	2024	2024	

Stage 3: Towards Time Series Foundation Models

LagLlama	TimesFM	Time-LLM	Moment	Moirai	Chronos	Sundial	
2024	2024	2024	2024	2024	2024	2025	

Not exhaustive. Additional important works, while not enumerated, remain integral to the process.

Heterogeneity

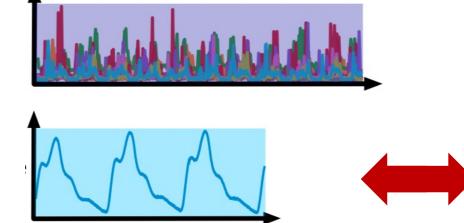
Time Series Are Highly Unstructured Compared To Natural Language

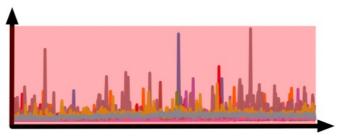
Diverse Shape/Freq./Scale

Dataset	Dim	Frequency
ETTh1, ETTh2	7	Hourly
ETTm1, ETTm2	7	15min
Exchange	8	Daily
Weather	21	10min
ECL	321	Hourly
Traffic	862	Hourly
Solar-Energy	137	10min
PEMS03	358	5min
PEMS04	307	5min
PEMS07	883	5min

Hard for unified pre-training







1D discrete token sequences

Natural language processing (NLP) is a subfield of computer science and especially artificial intelligence. It is primarily concerned with providing computers with the ability to process data encoded in natural language and is thus closely related to information retrieval, knowledge representation and computational linguistics, a subfield of linguistics.

Major tasks in natural language processing are speech recognition, text classification, natural language understanding, and natural language generation.

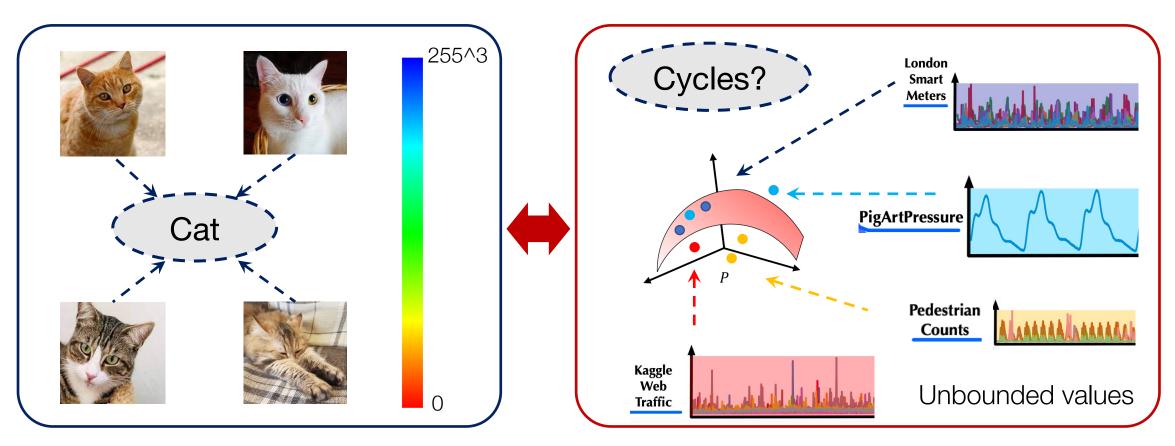
History [edit]

Further information: History of natural language processing

Natural language processing has its roots in the 1950s.^[1]
Already in 1950, Alan Turing published an article titled
"Computing Machinery and Intelligence" which proposed

Heterogeneity

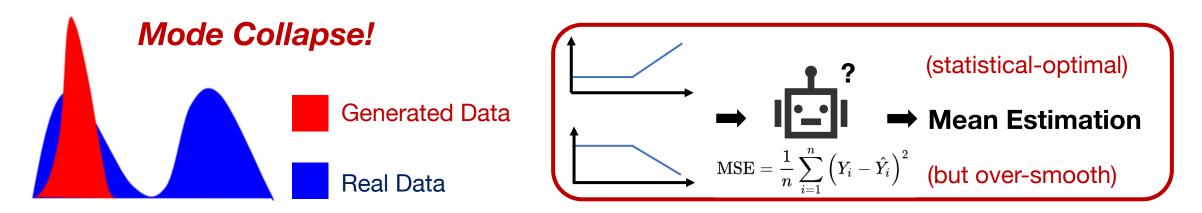
Time Series Can Be More Ambiguous (Distributed) Compared To Images



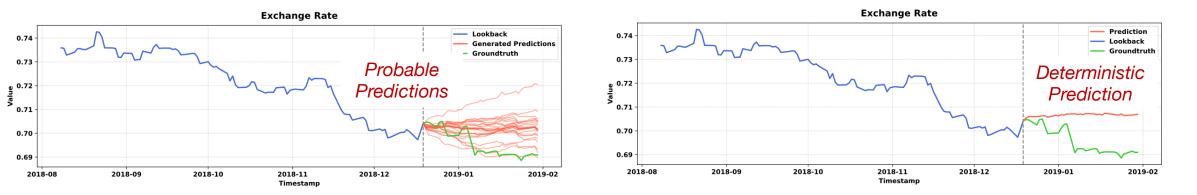
Align well with semantics

Hard to unify the semantics

Forecasting With Uncertainty



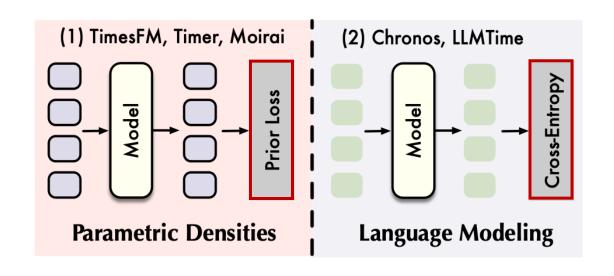
A Simple Prior Is The Scaling Bottleneck of Time Series Foundation Models



Real-World Observation is also a 'sampling' result

How: Can we use a more Flexible prior?

A Dilemma of Flexible Prior and Native Tokenization

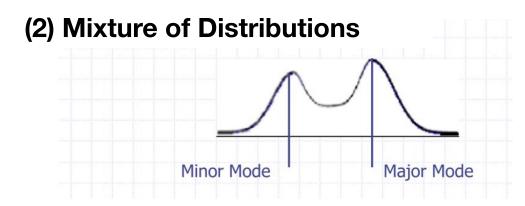


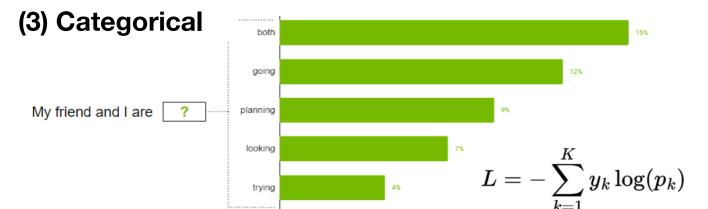
(1) Unimodal

- MSE Mean Values
- MAE Median Value
- Pinball Loss Quantiles...



TSFM cannot presuppose its objective







A Dilemma of Flexible Prior and Native Tokenization

Time Series Is A Foreign Language To LLM

-8.12, 2.33, 2.47, 3.40, 5.66, ...

81 23 24 34 56 ...

Discrete Tokenization (Foreign)

Embedding like LLM

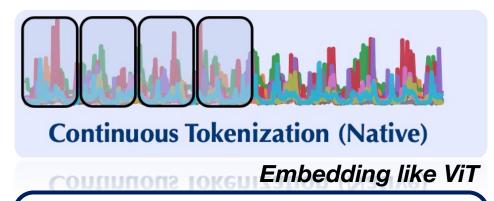
Discrete



- Two-stage round-off
- Out-of-vocabulary

Point-wise

- Long-context input
- Multi-step autoregression



Continuous



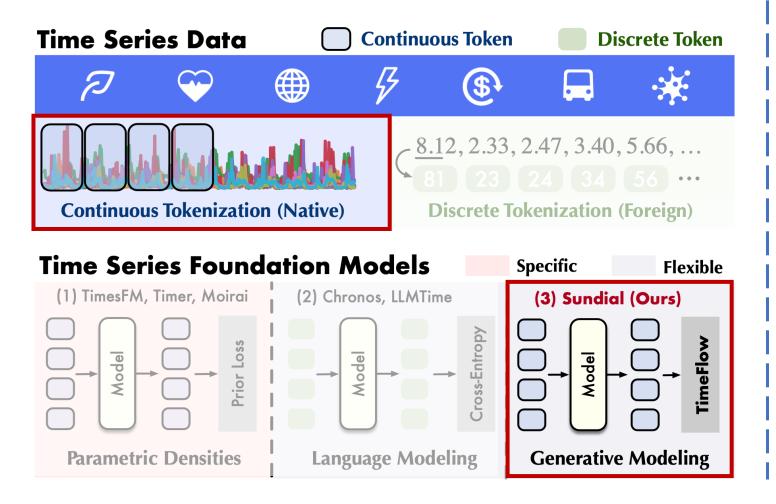
- Lossless embedding
- Prior-free optimization?

Patch-wise

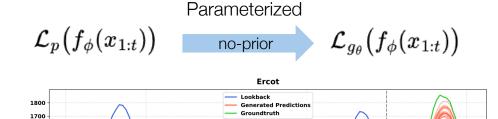
- More semantic tokens
- Less computation costs

Learning Native and Flexible Language of Time Series

Native: Transformer without Discrete Tokenization



| Flexible: Generative Forecasting



9 1500 8 1400

> 1300 1200

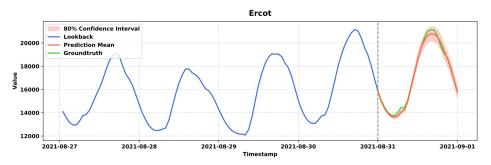
> > 2021-08-27

(1) Generate Samples (Pre-training)

2021-09-01



(2) Get Desired Statistics (Inference)



The First-Principle Design

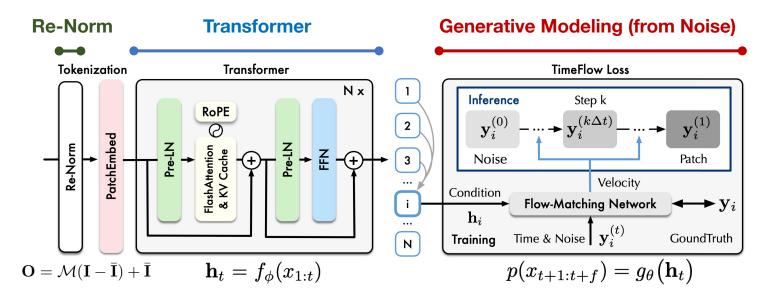
ARMA (Theoretically Complete)

$$y_t' = c + \phi_1 y_{t-1}' + \cdots + \phi_p y_{t-p}' + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q} + \varepsilon_t$$
Mean Autoregression Moving Average (from Noise)

Statistics Model

- Mean Estimation
- Autoregressive (Point)
- Moving-Average
- Separately Fitting...

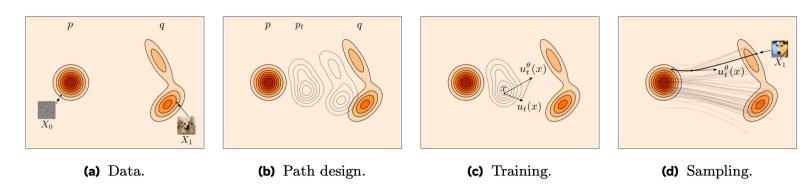
Sundial (Nonlinear and Adaptive)



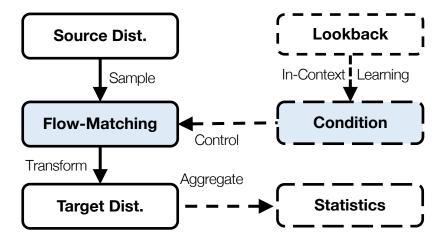
Deep Model

- Instance Re-Norm
- Decoder-Only (Patch)
- Conditional Generation
- In-Context Learning...

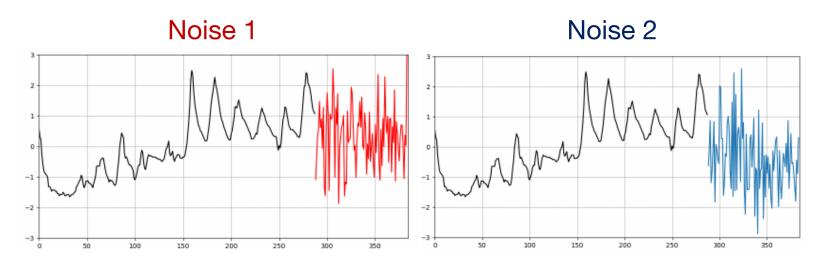
TimeFlow Loss



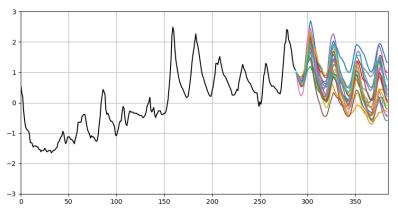
Pipeline of TimeFlow



Conditional Flow-Matching

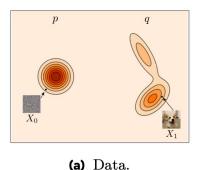


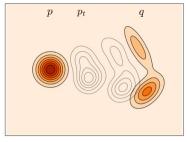




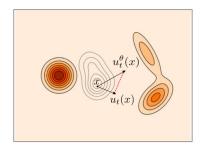
Lipman et al. Flow Matching Guide and Code. arXiv 2024

TimeFlow Loss





(b) Path design.



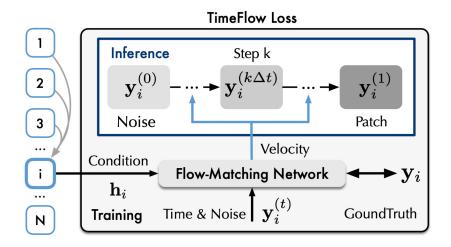
(c) Training.

Why FM?

Error in Prediction

Objective	ETTm1	ETTm2	ETTh1	ETTh2	ECL	Weather	Avg.
TimeFlow Diffusion MSE	0.336	0.258	<u>0.411</u>	0.333	0.169	0.234	0.290
Diffusion	0.362	0.265	0.444	0.360	0.202	0.252	0.314
MSE	0.360	0.264	0.404	0.341	<u>0.175</u>	0.231	0.296

Conditional Flow-Matching



Velocity Prediction:

Parameterized by a small MLP

$$\mathcal{L}_{ ext{TimeFlow}} = \sum_{i=1}^{N} \left\| ext{FM-Net}\left(\mathbf{y}_{i}^{(t)}, t, \mathbf{h}_{i}\right) - \left(\mathbf{y}_{i} - \mathbf{y}_{i}^{(0)}\right) \right\|^{2}$$
Predicted v at t Real v

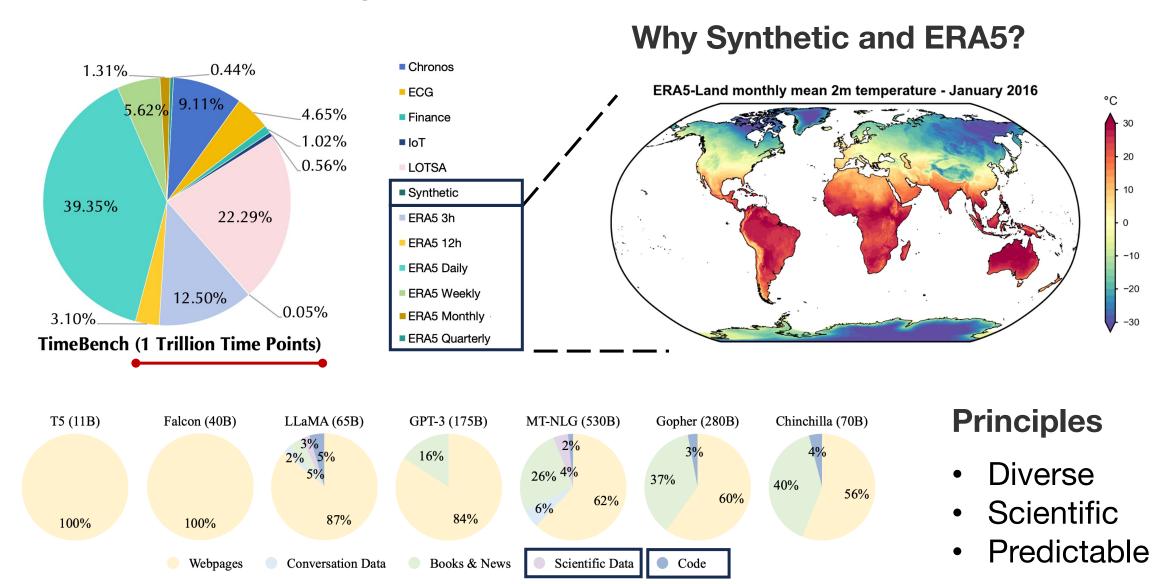
Push-Forward:

$$\mathbf{y}_{i}^{(t+\Delta t)} = \mathbf{y}_{i}^{(t)} + \text{FM-Net } \left(\mathbf{y}_{i}^{(t)}, t, \mathbf{h}_{i}\right) \Delta t$$

Predicted noise at t

Uniform-step trajectory

TimeBench: Data Collection



°C

20

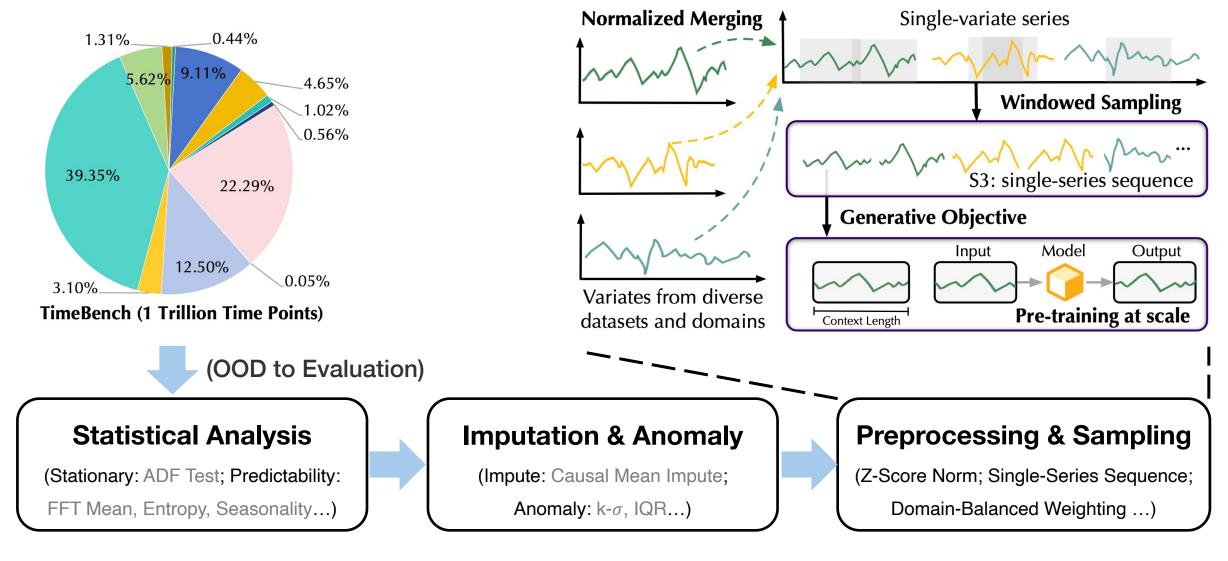
10

-10

-20

Zhao et al. A Survey of Large Language Models. arXiv 2023.

TimeBench: Data Curation



Liu et al. Timer: Generative Pre-trained Transformers Are Large Time Series Models. ICML 2024.

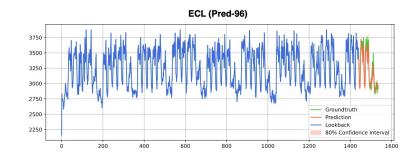
Zero-Shot Forecasting

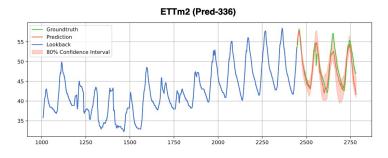
Long-Term Forecasting (Time-Series-Library)

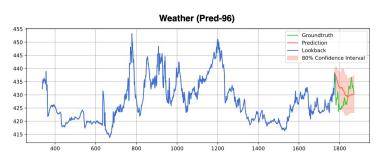
7.57% MSE \downarrow **4.71% MAE** \downarrow (to Prev. SoTA)

Models		ial _{Small} urs)	Sundi (Ou			ial _{Large} urs)	Time-1	MoE _{Base} 24b)		MoE _{Large} 24b)	Time-1 (20	MoE _{Ultra} 24b)	Time (2024		Moirai (2024		Moirai _{Larg} (2024)		nos _{Base} 024)		10s _{Large} 124)	Time (202	
Metric	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE N	MAE	MSE M	IAE	MSE MAI	EMSE	MAE	MSE	MAE	MSE	MAE
ETTm1	0.354	0.388	0.336	0.377	0.331	0.369	0.394	0.415	0.376	0.405	0.356	0.391	0.373 0	.392	0.406 0	.385	0.422 0.39	0.645	0.500	0.555	0.465	0.433	0.418
ETTm2	0.265	0.324	0.258	0.320	0.254	0.315	0.317	0.365	0.316	0.361	0.288	0.344	0.273 0).336	0.311 0	.337	0.329 0.34	3 0.310	0.350	0.295	0.338	0.328	0.346
ETTh1	0.390	0.418	0.411	0.434	0.395	0.420	0.400	0.424	0.394	0.419	0.412	0.426	0.404	.417	0.417 0	.419	0.480 0.439	0.591	0.468	0.588	0.466	0.473	0.443
ETTh2	0.340	0.387	0.333	0.387	0.334	0.387	0.366	0.404	0.405	0.415	0.371	0.399	0.347 0	.388	0.362 <u>0</u>	.382	0.367 0.37 ′	0.405	0.410	0.455	0.427	0.392	0.406
ECL	0.169	0.265	0.169	0.265	0.166	0.262	-	-	-	-	[-	-	0.174 0	.278	0.187 0	.274	0.186 0.27	0 0.214	0.278	0.204	0.273	1-	-
Weather	0.233	0.271	0.234	0.270	0.238	0.275	0.265	0.297	0.270	0.300	0.256	0.288	0.256 0	.294	0.287 0	.281	0.264 0.27	3 0.292	0.315	0.279	0.306	-	-
1st Count	7	2	8	5	16	16	0	1	0	0	2	1	1	3	0	2	0 <u>6</u>	0	0	0	0	0	0

Showcases







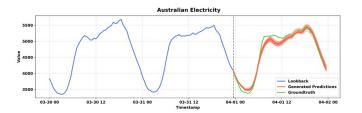
Zero-Shot Forecasting

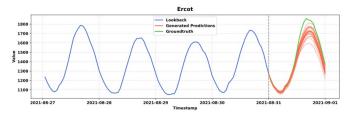
GIFT-Eval (Salesforce) w/o Prior in Evaluation Metrics

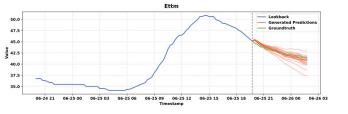
We introduce the General Time Series ForecasTing Model Evaluation, GIFT-Eval, a pioneering benchmark aimed at promoting evaluation across diverse datasets. GIFT-Eval encompasses 24 datasets over 144,000 time series and 177 million data points, spanning seven domains, 10 frequencies, multivariate inputs, and prediction lengths ranging from short to long-term forecasts.

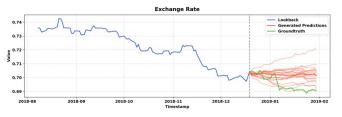
🏅 Ove				
_	model	MASE	▲ CRPS	_
	TiRex	0.650	0.421	
	Toto Open Base 1.0	0.673	0.437	
	sundial base 128m	0.673	0.472	
	TTM-R2-Finetuned (code)	0.679	0.492	
	timesfm 2 0 500m (code)	0.680	0.465	
	TabPFN-TS	0.692	0.46	
	YingLong_300m	0.716	0.463	
	<pre>chronos bolt base (code)</pre>	0.725	0.485	
	YingLong_110m	0.726	0.471	
	YingLong_50m	0.738	0.479	
	chronos_bolt_small (code)	0.738	0.487	

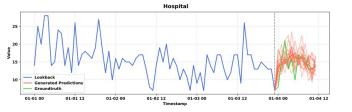
Generated Results







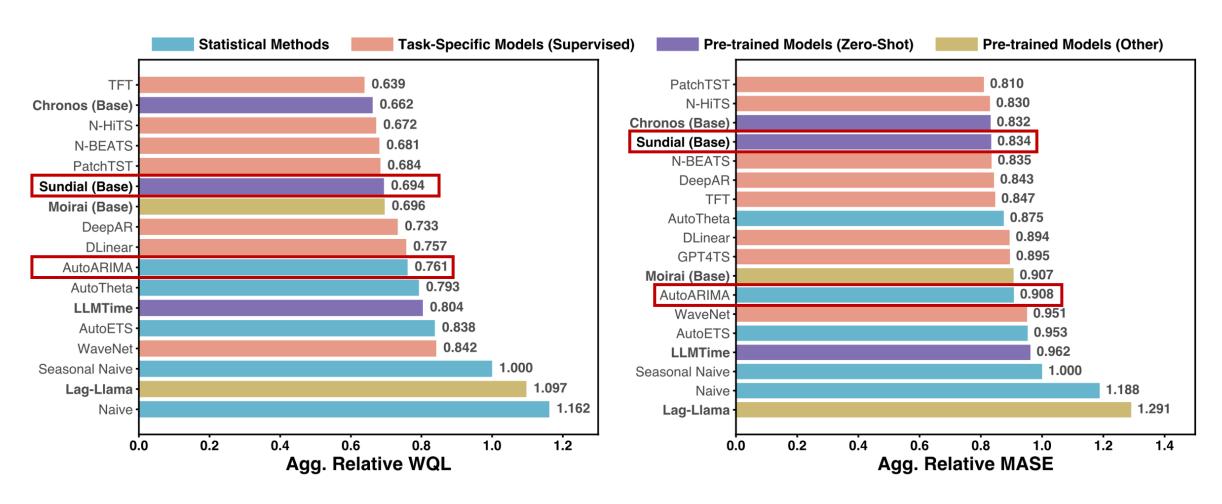




Zero-Shot Forecasting

FEV (AutoGluon)

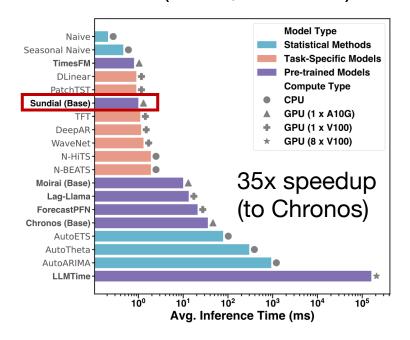
Surpass Auto-ARIMA(p,d,q) Fitting and Tuning on In-Distribution Data



Ansari et al. Chronos: Learning the Language of Time Series. TMLR 2024.

Inference Speed

On GPU (A100, ms-level)



On CPU (Apple M1, sec-level)

Lookback Length	Prediction Length	# Generated Samples	Inference Time	Accelerate By
672	16	1	249ms	-
2880	16	1	510ms	FlashAttention
2880	720	1	510ms	Multi-Patch Prediction
2880	1440	1	789ms	KV Cache
2880	720	20	949ms	Shared Condition

Ref: Auto-ARIMA

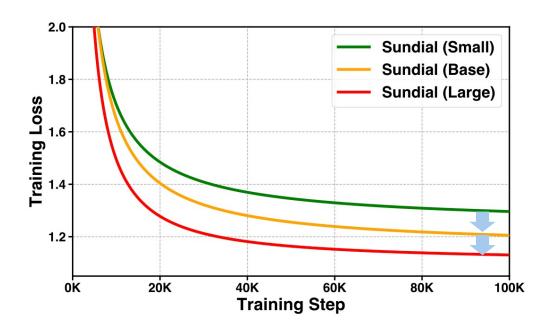
Dataset Size	Manual Tuning	Automated Search
Small (<1000 pts)	10 min – 2 hours	30 sec – 5 min
Medium (100~100K pts)	1 hour – 10 hours	5 min – 1 hour

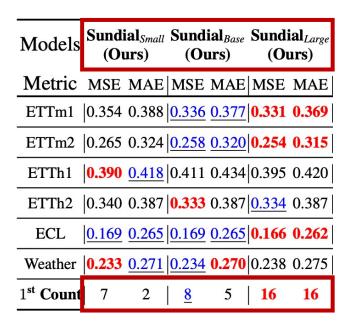
Ansari et al. Chronos: Learning the Language of Time Series. TMLR 2024.

Scaling

Scalable Training

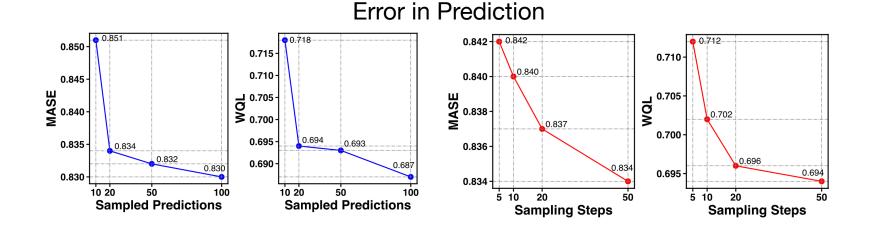
- ✓ Large Model ↑
- ✓ More Data ↑





Test-Time Calibration

- ✓ Use More Samples ↑
- √ Fine-Grained FM Steps ↑



Resource: Out-of-the-Box Models

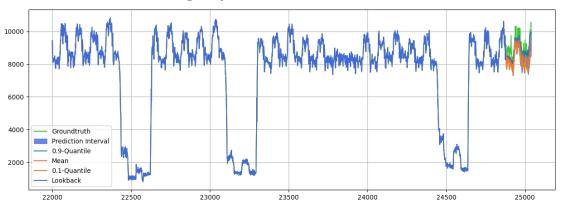
Time Series Foundation Models 2



Pretrained models and datasets for out-of-the-box time series forecasting 🖉



Point forecasting, quantile, and interval estimation



More details: https://github.com/thuml/Sundial



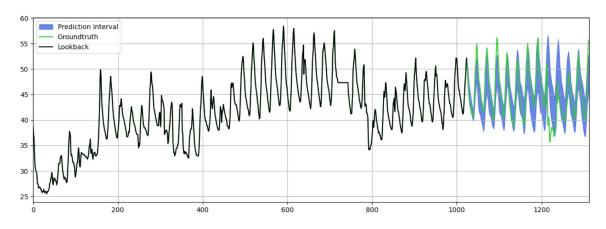
All you need is a HuggingFace account

Note that Sundial can generate multiple probable predictions

batch_size, lookback_length = 1, 2880

forecast_length = 96
num samples = 20

segs = torch.randn(batch size, lookback length)

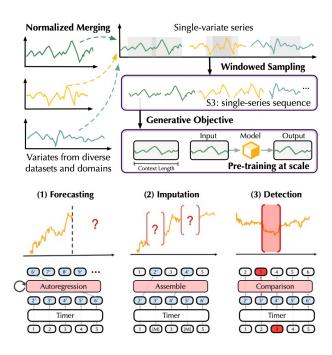


Code for fine-tuning will be available soon!

A Path Towards Time Series Foundation Model

Timer-v1 (ICML 2024)

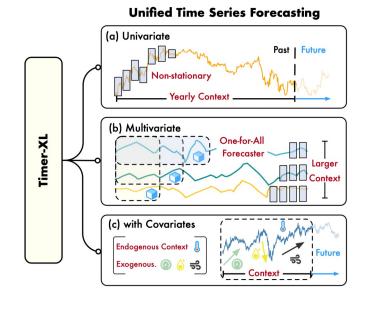
- Unified Pre-training
- Multiple Tasks



Multitask, Few-Shot

Timer-XL (ICLR 2025)

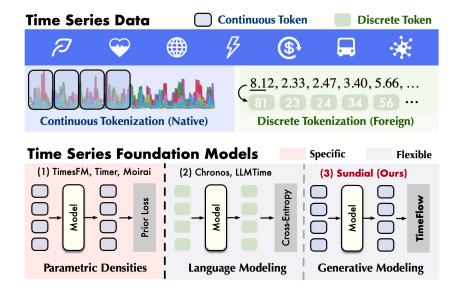
- Long-Context Extension
- Unified Time Series Attention
- Enhanced Position Embedding



2D Context, Zero-Shot

Sundial (ICML 2025)

- Trillion-Scale Pre-training
- Native Time Series Transformer
- Flow-Matching for Generation



Probabilistic, Flexible

1440/67M/28B 2880/84M/260B 2880/444M/1032B

Limitations and Future Directions

From A Model Developer's Perspective

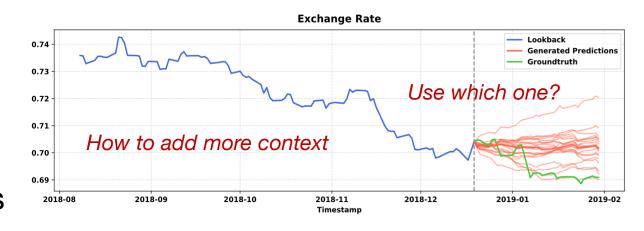
- Data: Effective Data Expansion/Synthetics
- Architecture: Unified Context Modeling
- Generalization: OOD and Ambiguity

From A Practitioner's Perspective

- Finetuning: Univariate -> Multivariate
- Domain-Specific: Physics-driven
- Generative + RL: Rewarded Predictions

Pre-Trained Domains





Thank you!





Code



Yong Liu



Guo Qin



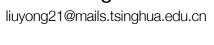
Zhiyuan Shi

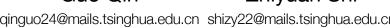


Zhi Chen



Caiyin Yang chenzhi21@mails.tsinghua.edu.cn ycy23@mails.tsinghua.edu.cn









Xiangdong Huang huangxdong@tsinghua.edu.cn



Jianmin Wang jimwang@tsinghua.edu.cn



Mingsheng Long mingsheng@tsinghua.edu.cn





