Optimal Auction Design in the Joint Advertising

RENMIN UNIVERSITY OF CHINA

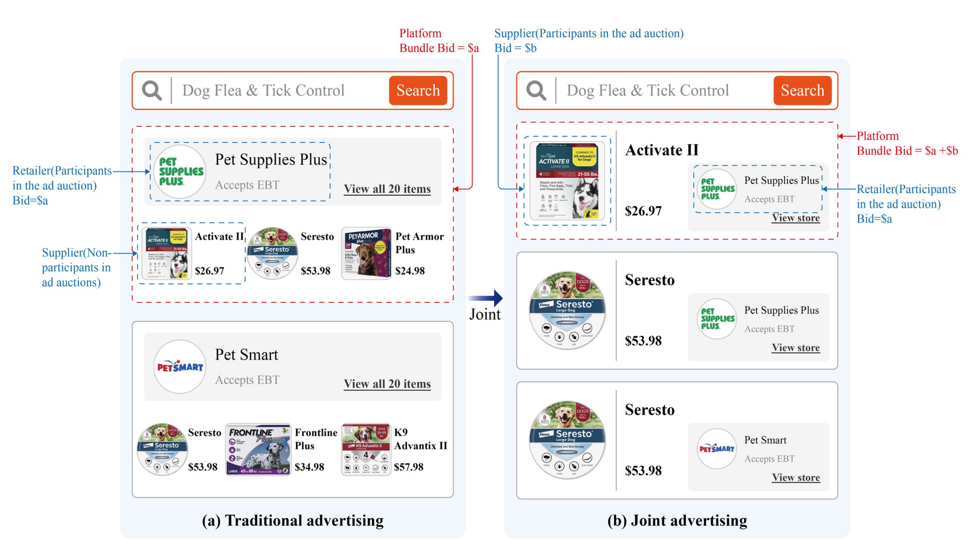
Yang Li[1*], Yuchao Ma[1], Qi Qi[1†],

[1]Gaoling School of Artificial Intelligence, Renmin University of China

ICML International Conference On Machine Learning

Background

- In recent years, a novel auction scenario known as "joint advertisement" emerge on online advertising platforms such as Facebook.



- Existing mechanisms for joint advertising fail to realize the optimality, as they tend to focus on individual advertisers and overlook bundle structures.

Contribution

- First, we identify the optimal mechanism for single-slot joint advertisement.
- Additionally, we propose a novel neural network architecture and introduce a new incentive compatibility constraint method for multi-slot joint advertisements.
- Our approach not only improves platform revenue but also ensures approximate incentive compatibility and individual rationality .
- Extensive experiments demonstrate that our method achieves state-of-the-art performance.

Settings

- Retailer advertisers set R and Supplier advertisers set S: $R \cap S = \emptyset$.
- n joint ad : $E = (e_1, e_2, \dots, e_n), E \in R \times S$. Bipartite Graph $G = (R, S, E)$
- The click-through rates for m ad slots : $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \dots, \lambda_m)$.
- b_i: the bid of advertiser i, $i \in R \cup S$.
- $v_i \sim F_i$: the private value of advertiser i, $i \in R \cup S$.
- **b** & **v** the bid profile and private value profile of all advertisers.
- Joint ad auction mechanism $\mathcal{M} = (\mathbf{x}, \mathbf{p})$:
  - Allocation Rule: $x^e(\boldsymbol{v}) = x_r^e(\boldsymbol{v}) = x_s^e(\boldsymbol{v}), \forall e = (r, s) \in E$.
  - Payment Rule: $p^e(\boldsymbol{v}) = p_r^e(\boldsymbol{v}) + p_s^e(\boldsymbol{v}), \ \forall e = (r, s) \in E$.
  - $x_i(\boldsymbol{v}) = \sum_{e \in E_i} x^e(\boldsymbol{v}), p_i(\boldsymbol{v}) = \sum_{e \in E_i} p_i^e(\boldsymbol{v}), \forall i \in R \cup S$.
  - $\sum_{e \in E} x^e(\boldsymbol{v}) \le \mathbf{1}_m$.
  - The utility of advertiser i: $u_i(v_i, v_i^*) = \mathbb{E}_{v_{-i} \sim V_{-i}}[v_i \cdot x_i(v_i^*, v_{-i})\boldsymbol{\lambda}^T - p_i(v_i^*, v_{-i})]$.
  - DSIC : $u_i(v_i, v_i) \ge u_i(v_i, v_i^*)$, $\forall v_i, v_i^* \in V_i$
  - IR: $u_i(v_i, v_i) \ge 0, \forall v_i \in V_i$.
  - Revenue: $U_0 = \mathbb{E}_{v \sim V}[\sum_{e \in E} p^e(\boldsymbol{v})]$.
- Optimal Joint Auction:

$$\max_{\mathcal{M} = (\mathbf{x}, \mathbf{p})} U_0$$
$$s.t.\ u_i(v_i, v_i) \ge u_i(v_i, v_i^*),\ \forall v_i, v_i^* \in V_i,$$
$$u_i(v_i, v_i) \ge 0, \forall v_i \in V_i$$

Optimal Joint Auction Design with Single Slot

Theorem: For the single-slot joint advertisement with regular bidders, A deterministic joint auction mechanism $\mathcal{M}$ is optimal if and only if for all $i \in R \cup S$,

(i) Step Function: $x_i^{\mathcal{M}}(v_i, v_{-i}) = \begin{cases} 1 & v_i > \hat{v}_i(v_{-i}) \\ 0 & otherwise \end{cases}$.

(ii) Critical Value: $p_i^{\mathcal{M}}(v_i, v_{-i}) = \begin{cases} \hat{v}_i(v_{-i}) & v_i > \hat{v}_i(v_{-i}) \\ 0 & otherwise \end{cases}$.

where the critical value $\hat{v}_i(v_{-i})$ is defined as follows:

- For $r \in R$, the critical value $\hat{v}_r(v_{-r})$ is:

$$\hat{v}_r(v_{-r}) = \inf\{b_r | c^{e_r^M}(b_r, v_{s_r^M}) \ge v_0 \wedge c^{e_r^M}(b_r, v_{s_r^M}) \ge c^{\hat{e}}(v_{\hat{r}}, v_{\hat{s}}), \forall \hat{e} \in E_{-r}\}$$

- For $s \in S$, the critical value $\hat{v}_s(v_{-s})$ is defined similarly:

$$\hat{v}_s(v_{-s}) = \inf\{b_s | c^{e_s^M}(v_{r_s^M}, b_s) \ge v_0 \wedge c^{e_s^M}(v_{r_s^M}, b_s) \ge c^{\hat{e}}(v_{\hat{r}}, v_{\hat{s}}), \forall \hat{e} \in E_{-s}\}$$

Optimal Joint Auction Design in General Case

RegretNet-Like Method(JRegNet):

$$\min_{w \in \mathbb{R}^{d_w}} -\mathbb{E}_{v \sim V}[\textstyle\sum_{i \in R \cup S} p_i(\boldsymbol{v}; w)]$$
$$s.t.\ rgt_i(w) = 0, \ \forall i \in R \cup S$$

$$rgt_i(w) = \mathbb{E}_{v \sim V}[\max_{v_i' \in \mathcal{V}_i}[u_i(v_i; (v_i', \boldsymbol{v}_{-i}); w) - u_i(v_i; (v_i, \boldsymbol{v}_{-i}); w)]]$$

$\sum_{i \in R \cup S} rgt_i(w) \le \sum_{e \in E} rgt^e(w)$ →

Our Method:

$$\min_{w \in \mathbb{R}^{d_w}} -\mathbb{E}_{v \sim V}[\textstyle\sum_{e \in E} p^e(\boldsymbol{v}; w)]$$
$$s.t. rgt^e(w) = 0, \ \forall e \in E$$

$$rgt^e(w) = \mathbb{E}_{v \sim V}[\max_{v_r' \in \mathcal{V}_r}[u_r^e(v_r; (v_r', \boldsymbol{v}_{-r}); w) - u_r^e(v_r; (v_r, \boldsymbol{v}_{-r}); w)] + \max_{v_s' \in \mathcal{V}_s}[u_s^e(v_s; (v_s', \boldsymbol{v}_{-s}); w) - u_s^e(v_s; (v_s, \boldsymbol{v}_{-s}); w)]]$$

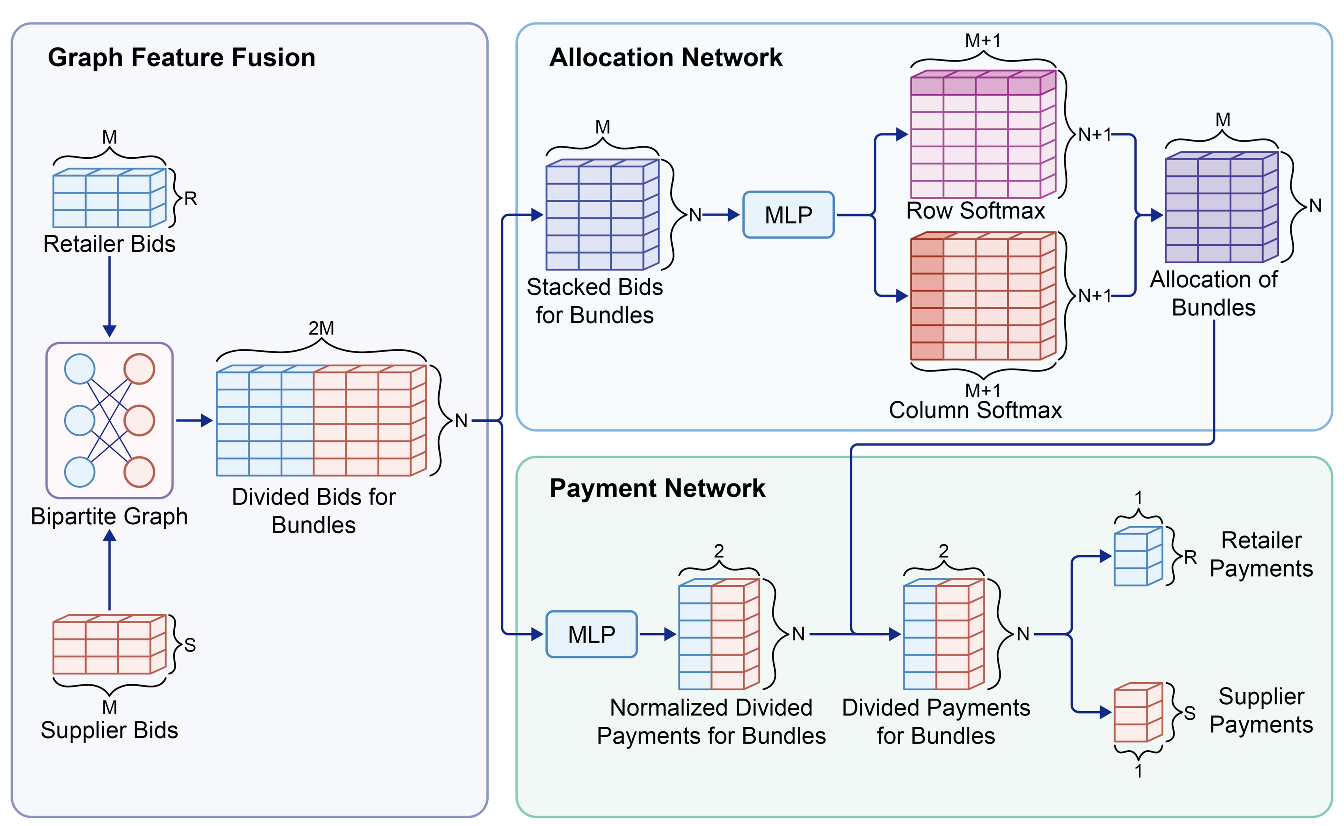


Figure 1. A diagram of BundleNet, including the Graph Feature Fusion, Allocation Network and Payment Network, where N ads and M slots are input.

Algorithm 1 BundleNet Training

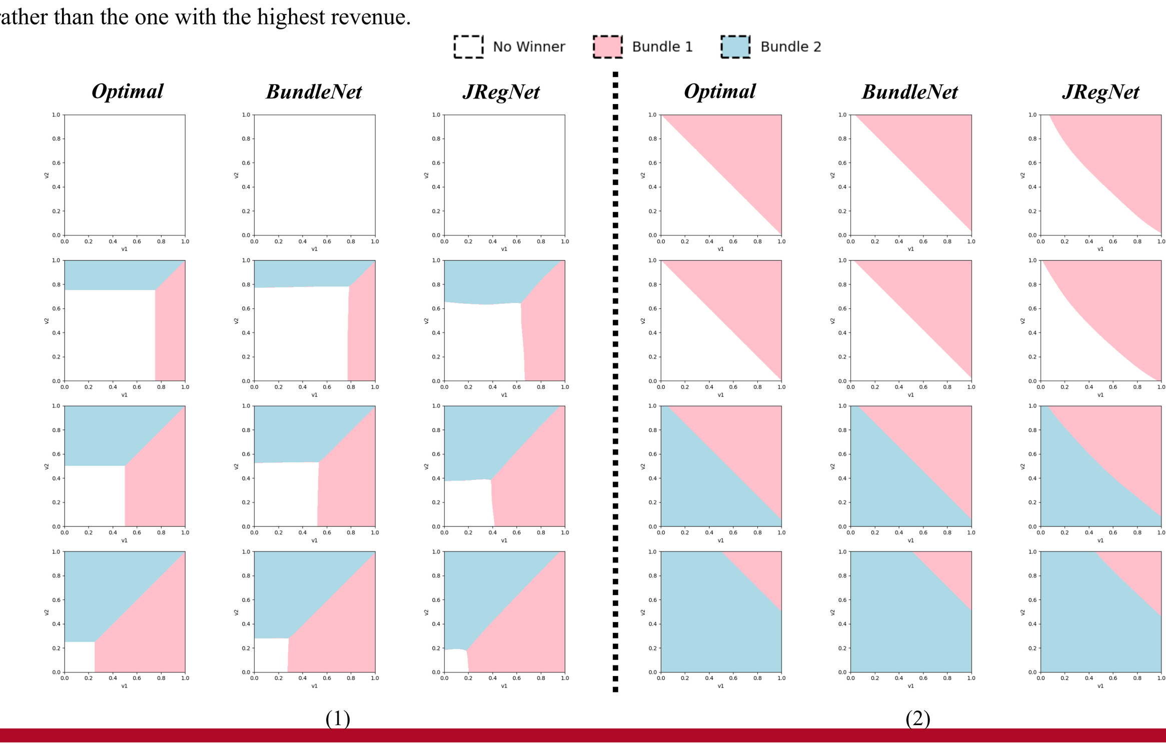
Input: Minibatches $\mathcal{B}_1, \dots, \mathcal{B}_T$ of size C
Parameters: $\forall t, \rho_t > 0, \gamma > 0, \eta > 0, \Gamma \in \mathbb{N}, K \in \mathbb{N}$
Initialize: $w^0 \in \mathbb{R}^d, \lambda^0 \in \mathbb{R}^m$
for $t = 0$ **to** T **do**
  Receive $\mathcal{B}_t = \{G^{(1)}, \dots, G^{(B)}\}$
  Initialize $v_r^{\prime(\ell)} \in V_r, v_s^{\prime(\ell)} \in V_s, \forall \ell \in [B], r \in R, s \in S$
  for $r = 0$ **to** Γ **do**
    for $\ell = 1$ **to** B **do**
      $\forall \ell \in [C], i \in R \cup S$:
        $v_i^{\prime(\ell)} \leftarrow v_i^{\prime(\ell)} + \gamma \nabla_{v_i'}[u_i^w(v_i^{(\ell)}; (v_i', v_{-i}^{(\ell)}))]\big|_{v_i' = v_i^{\prime(\ell)}}$
    end for
  end for
  Compute Lagrangian gradient and update w^t
  $w^{t+1} \leftarrow w^t - \eta \nabla_w \mathcal{L}_{\rho_t}(w^t, \mu^t)$
  if t is a multiple of H **then**
    $\mu_e^{t+1} \leftarrow \mu_e^t + \rho_t \widehat{rgt}^e(w^{t+1}), \forall e \in E$
  else
    $\boldsymbol{\lambda}^{t+1} \leftarrow \boldsymbol{\lambda}^t$
  end if
end for

Experiment

Single slot:

Alg.	Setting							
	U_2	U_3	U_4	U_5	E_2	E_3	E_4	E_5
Ours								
BundleNet	**0.5286**	**0.6681**	**0.7805**	**0.8802**	**0.4248**	**0.5460**	**0.6354**	**0.7215**
IC Violation	0.0006	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001
IC Baselines								
RVCG	0.3811	0.6003	0.7455	0.8607	0.2820	0.4649	0.5927	0.7041
Optimal	0.5247	0.6705	0.7826	0.8819	0.4249	0.5479	0.6470	0.7376
Baseline with IC Violation								
JRegNet	0.5622	0.7287	0.7791	0.7882	0.4727	0.5892	0.6306	0.6943
IC Violation	0.0005	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001

Table 1: The experimental results compare BundleNet, JRegNet, Revised VCG, and the Optimal Mechanism as the number of bundles increases under different settings in the single-slot scenario with CTR $\boldsymbol{\lambda} = (1)$. The notations U_2, U_3, U_4, U_5 represent cases where the number of bundles is 2, 3, 4 and 5, respectively, under the uniform distribution $U(0,1)$. Similarly, E_2, E_3, E_4, E_5 correspond to scenarios with 2, 3, 4 and 5 bundles under the truncated exponential distribution $E(2)$. In this table, we use bold to indicate the method among BundleNet, JRegNet, and RVCG that is closest to the optimal mechanism, rather than the one with the highest revenue.



Multi-slots:

Alg.	Setting							
	$U_{5\times5}$	$U_{6\times5}$	$U_{7\times5}$	$U_{8\times5}$	$U_{9\times5}$	$U_{10\times5}$	$U_{11\times5}$	$U_{12\times5}$
Ours								
BundleNet	**1.4982**	**1.7162**	**1.9233**	**2.0890**	**2.2210**	**2.4047**	**2.5649**	**2.6495**
IC Violation	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001
IC Baselines								
RVCG	0.8420	1.2854	1.6142	1.8831	2.0991	2.2800	2.4564	2.5868
Baseline with IC Violation								
JRegNet	1.4972	1.6849	1.8244	1.9350	1.9804	1.9622	1.9763	1.9973
IC Violation	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001

Table 2: The experimental results of BundleNet, JRegNet, RVCG as the number of bundles increases under different settings in the multi-slots scenario. Similar to those of Table 1, the notation $U_{5\times5}, \dots, U_{12\times5}$ represent the settings where the number of bundles varies from 5 to 12, while letting the CTRs of these 5 slots as (1, 0.8, 0.6, 0.4, 0.2).

Alg.	Setting							
	$LN_{5\times5}$	$LN_{6\times5}$	$LN_{7\times5}$	$LN_{8\times5}$	$LN_{9\times5}$	$LN_{10\times5}$	$LN_{11\times5}$	$LN_{12\times5}$
Ours								
BundleNet	**2.6560**	**3.0093**	**3.4346**	**3.7031**	**3.9952**	**4.2366**	**4.4618**	**4.6038**
IC Violation	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001
IC Baselines								
RVCG	1.4365	2.1764	2.7130	3.1579	3.5104	3.8332	4.1019	4.3449
Baseline with IC Violation								
JRegNet	2.6020	2.9237	3.1845	3.4139	3.441	3.2535	3.2747	3.3848
IC Violation	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001

Table 3: The experimental results of BundleNet, JRegNet, RVCG as the number of bundles increases under different settings in the multi-slots scenario. Similar to those of Table 2, the notation $LN_{5\times5}, \dots, LN_{12\times5}$ represent the settings where the number of bundles varies from 5 to 12, under the truncated lognormal distribution $LN(0.1,1.44)$ over the interval $(0, 1)$,while letting the CTRs of these 5 slots as (1, 0.8, 0.6, 0.4, 0.2).