Optimal Sensor Scheduling and Selection for Continuous-Discrete Kalman Filtering with Auxiliary Dynamics

Mohamad Al Ahdab, John Leth, Zheng-Hua Tan

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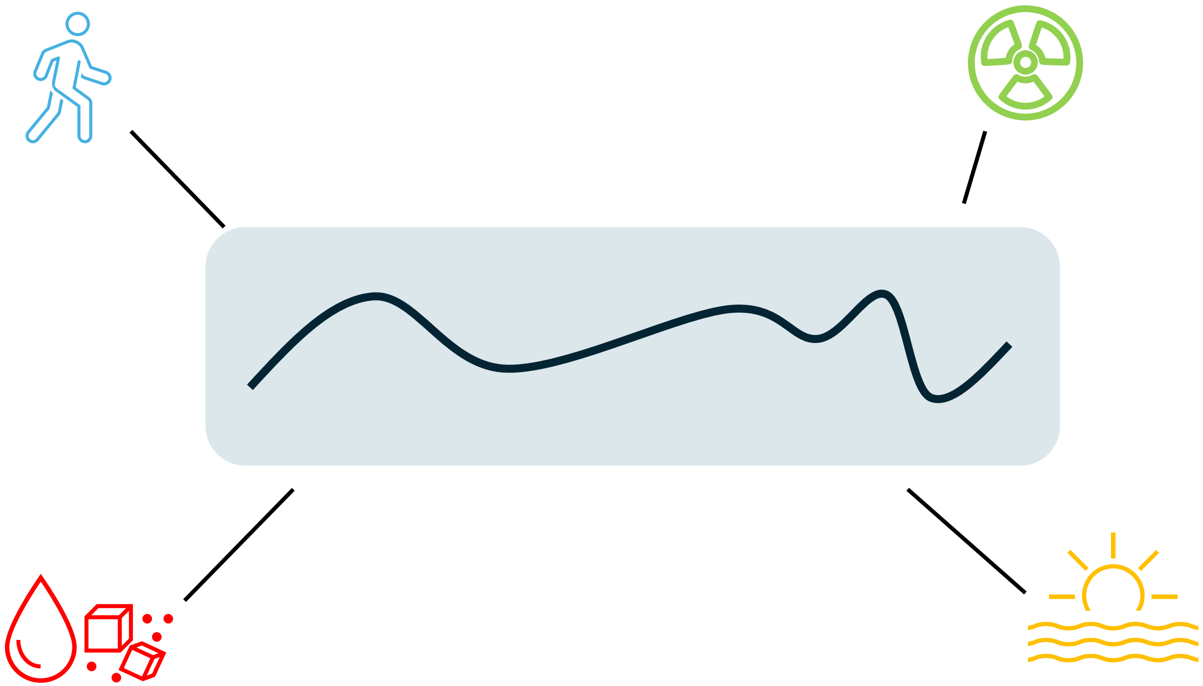
Introduction

- Many physical signals of interest evolves *continuously* in time.



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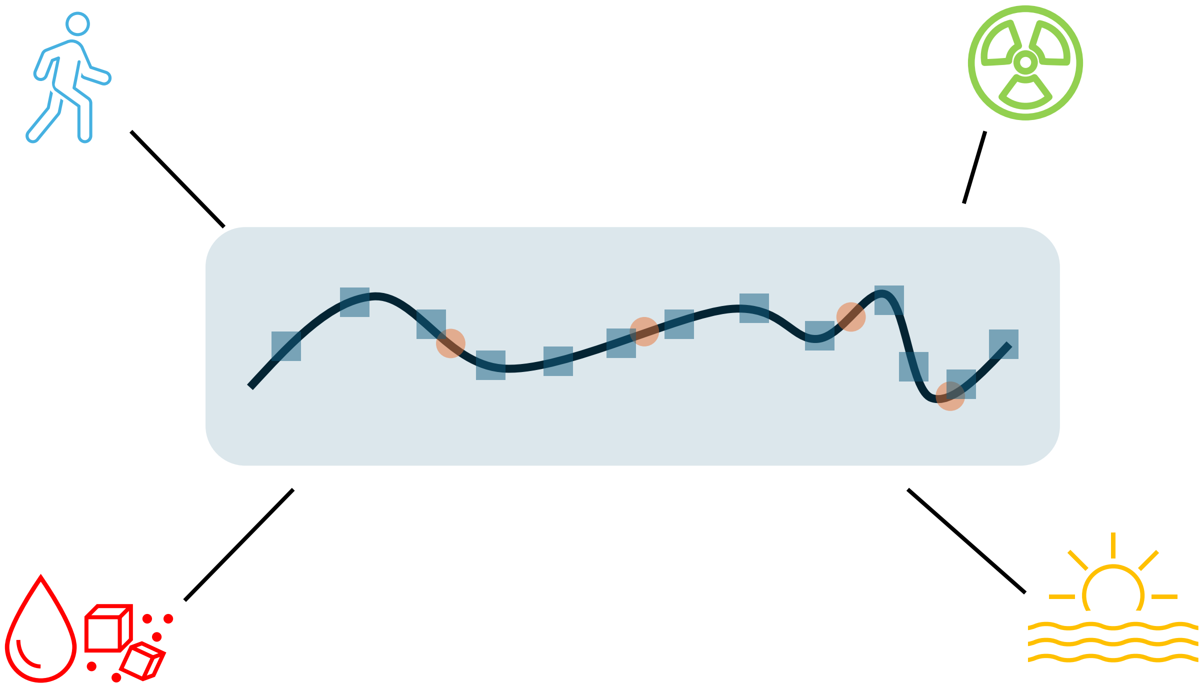

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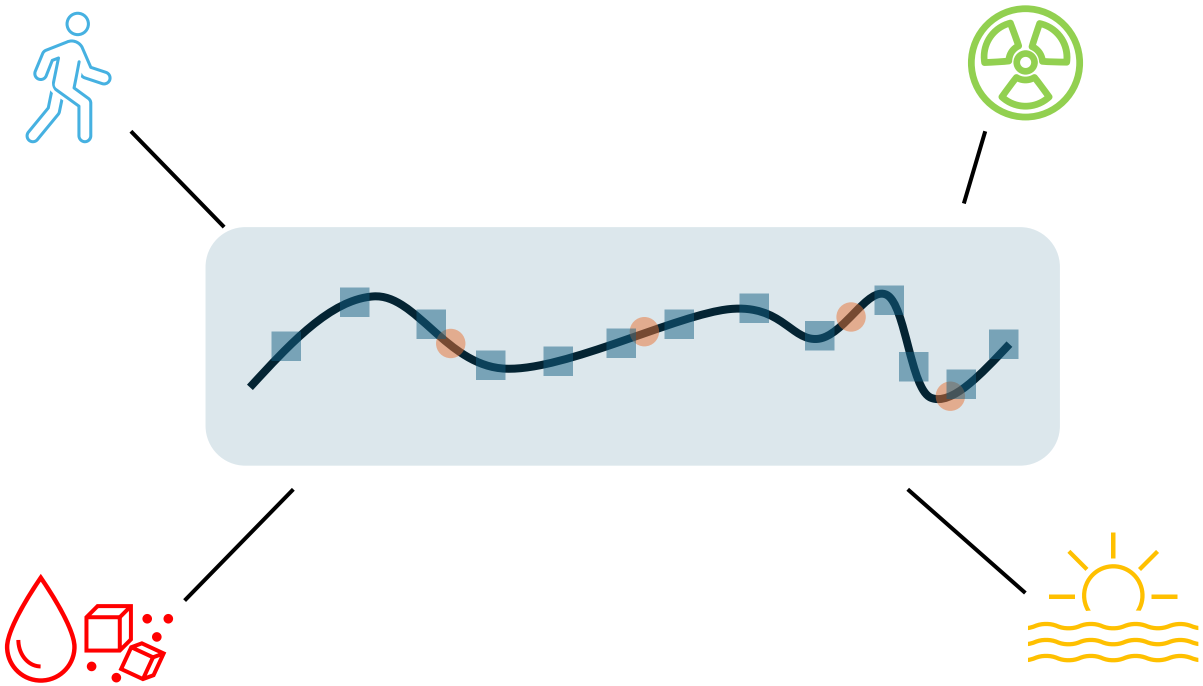

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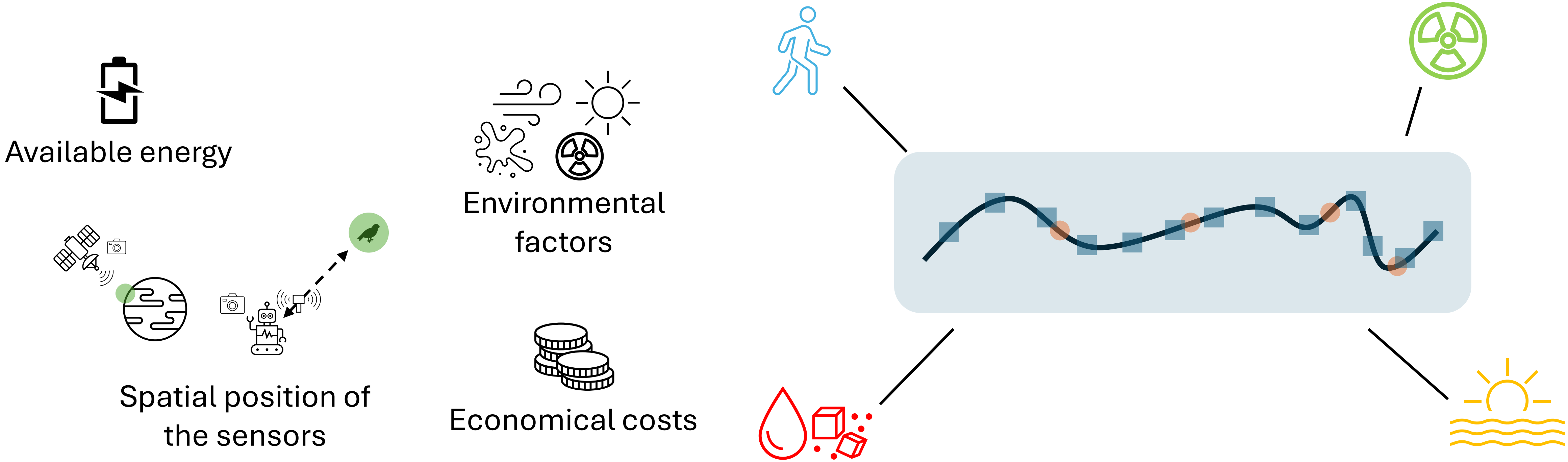

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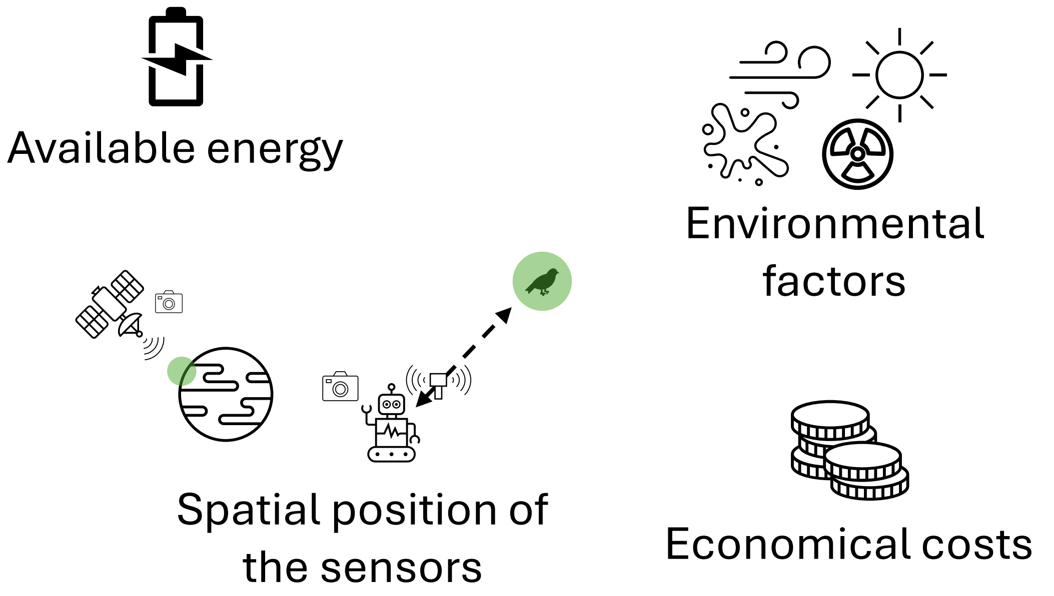
Introduction

- We will refer to the variables interacting with the sensors as *Auxiliary* variables (ξ).



Introduction

The main question we attempt to answer in the paper is:

How can we schedule and select sensors with discrete measurements for continuously evolving signals while considering auxiliary variables?

Setup

We consider a State-Space Model (SSM) for the signal of interest:

$$dx = A(\xi, t)x\,dt + \sigma(\xi, t)\,dW, \qquad x_0 \sim \mathcal{N}(\mu_0, \Sigma_0),$$

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with measurements from sensor s at time t_i:

$$y^s(t_i) = C_s(\xi(t_i), t_i)x(t_i) + v^s(\xi(t_i), t_i), \qquad v^s(\xi(t_i), t_i) \sim \mathcal{N}\big(0, R_s(\xi(t_i), t_i)\big)$$

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Regression with Gaussian Processes for many covariance functions is *equivalent* to Kalman filtering and then smoothing [*].

[*] See for example: Hartikainen, Jouni, and Simo Särkkä. "Kalman filtering and smoothing solutions to temporal Gaussian process regression models." *2010 IEEE international workshop on machine learning for signal processing*. IEEE, 2010.

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Setup

For the auxiliary variables:

$$\frac{d\xi_p}{dt} = f_p(\xi, u, t) + \sum_{s=1}^{S} g_s(\xi, u, t) \sum_{i=1}^{N_s(t)} \delta_{t_i^s},$$

$$\frac{d\xi_u}{dt} = f_u(\xi_u, u, t),$$

$$\xi = \begin{bmatrix} \xi_u \\ \xi_p \end{bmatrix}$$

Setup

For the auxiliary variables:

Perturbed by measurements

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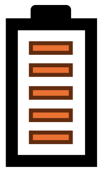

ξ_p is stored energy

Setup

For the auxiliary variables:

Number of measurements from sensor s

Perturbed by measurements

$$\frac{d\xi_p}{dt} = f_p(\xi, u, t) + \sum_{s=1}^{S} g_s(\xi, u, t) \sum_{i=1}^{N_s(t)} \delta_{t_i^s},$$

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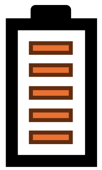
Dirac delta (measurement event)



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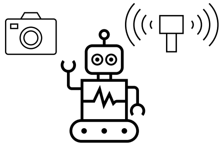

ξ_u is spatial position

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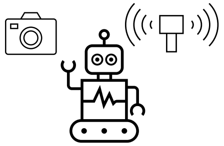
u: represents inputs/actions to the auxiliary dynamics.



u is acceleration

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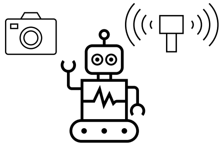

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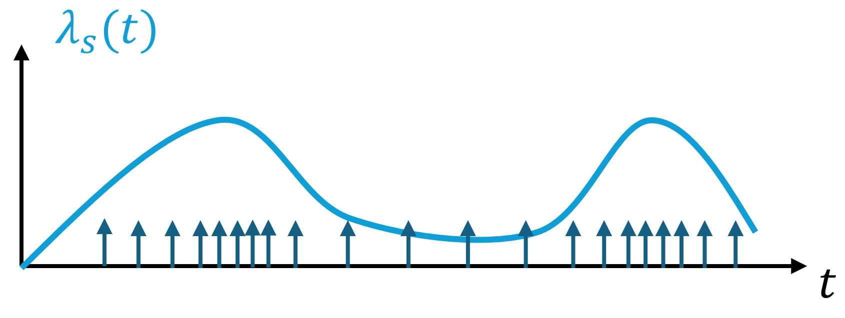
Summary of the Methods

- We model the measurements events to be according to a Poisson process with variable rates ($\lambda_s(t)$ for sensor s).
- This allows us to obtain *continuously differentiable* upper bounds in the rates on the mean covariance matrix for the Kalman filter along the mean auxiliary state [*].



[*] assuming that:

$$\frac{d\xi_p}{dt} = f_p(\xi, u, t) + \sum_{s=1}^{S} g_s(\xi, u, t) \sum_{i=1}^{N_s(t)} \delta_{t_i^s}$$

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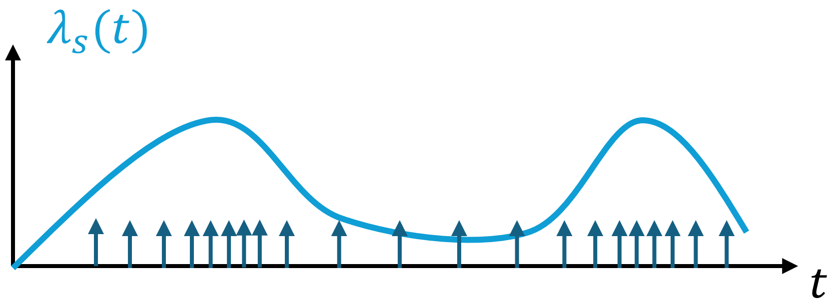

We also provide approximate solutions for other assumptions in the paper

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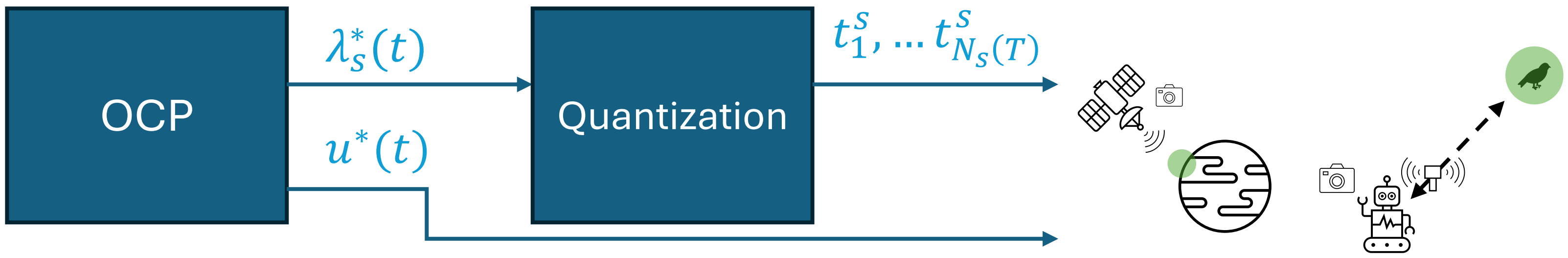
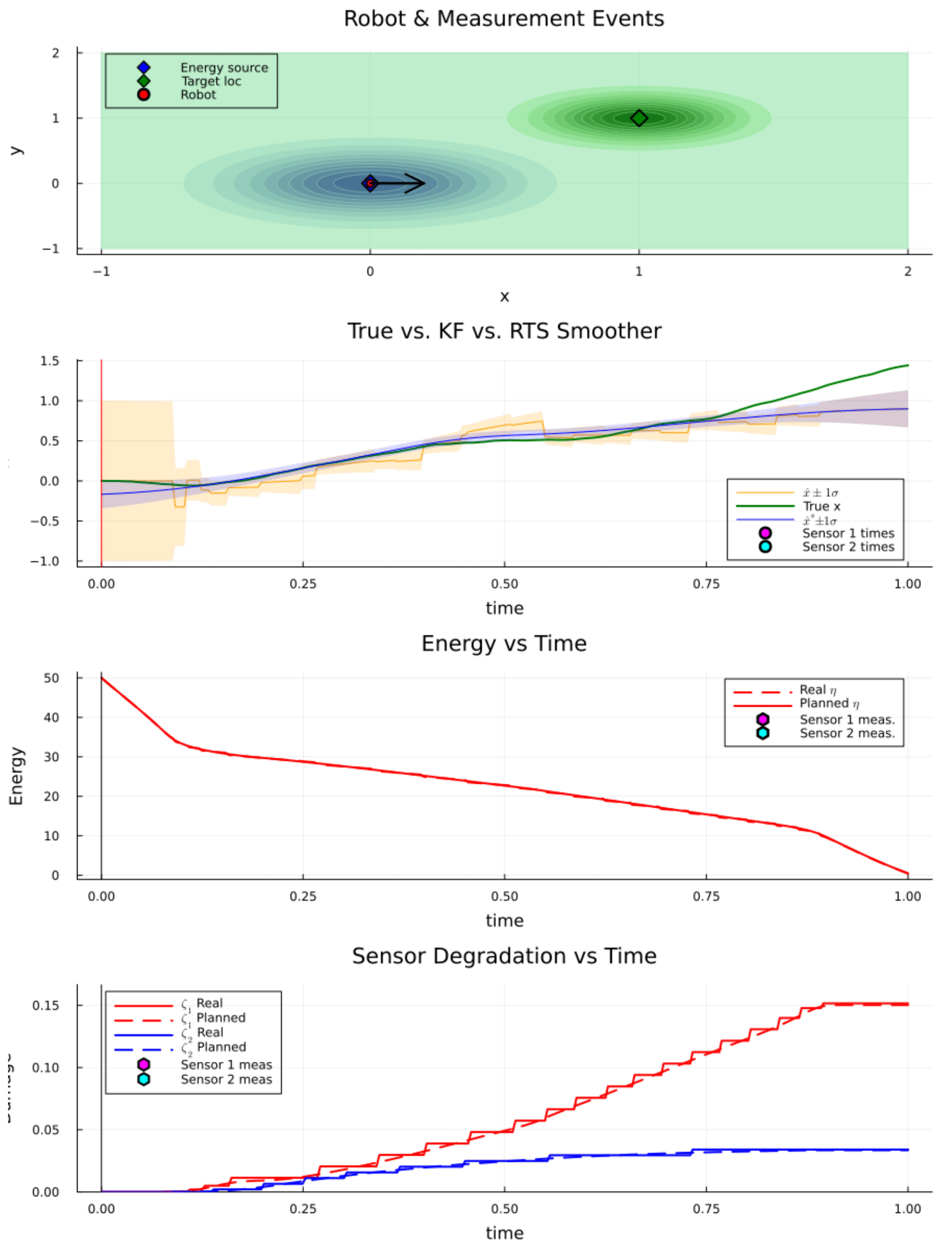
- Using the bounds, we formulate an optimal control problem (OCP) to optimize over the rates for each sensor ($\lambda_s(t)$) and the inputs/actions ($u(t)$) for a horizon of T.
- Using optimal quantization, we derive a deterministic rule to select the measurement times for each sensor based on the corresponding optimized rate ($\lambda_s^*(t)$) to approximate the mean behavior.



Experiments



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Experiments

Covariance Trace

Method	Mean	Std	Maximum
Optimized	1.90221	0.3406	2.94496
M-Optimized	1.92461	0.364315	3.00148
Greedy	2.60768	0.487677	3.21866
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Energy η

Method	Mean	Std	Maximum
Optimized	21.5435	10.1709	50.0
M-Optimized	22.3106	10.1279	50.0
Greedy	-1.67343	14.0586	50.0
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Degradation $(\zeta_1 + \zeta_2)$

Method	Mean	Std	Maximum
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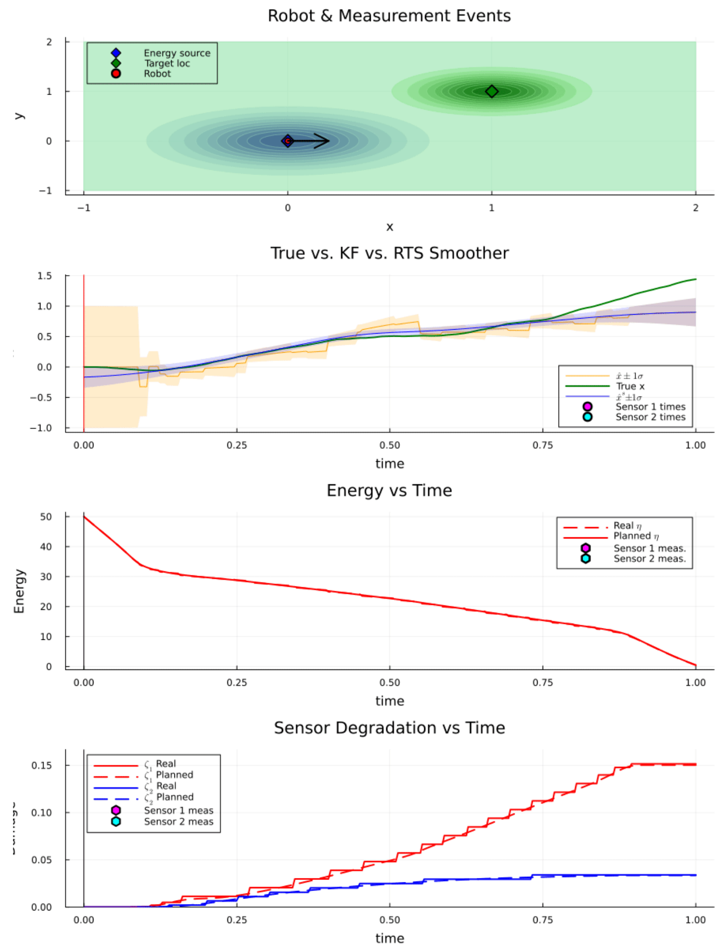

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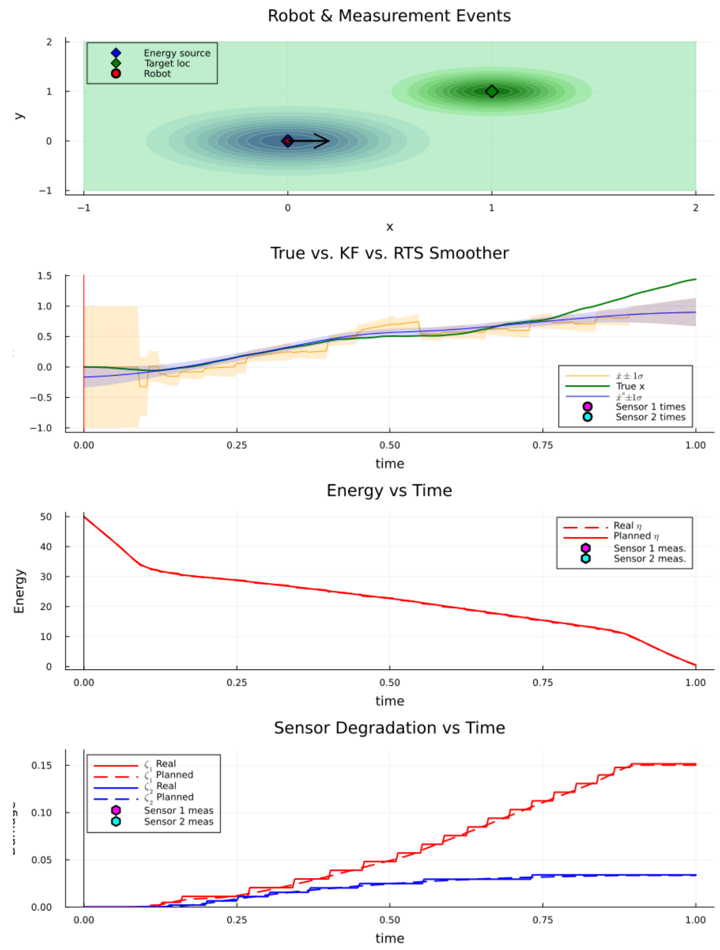

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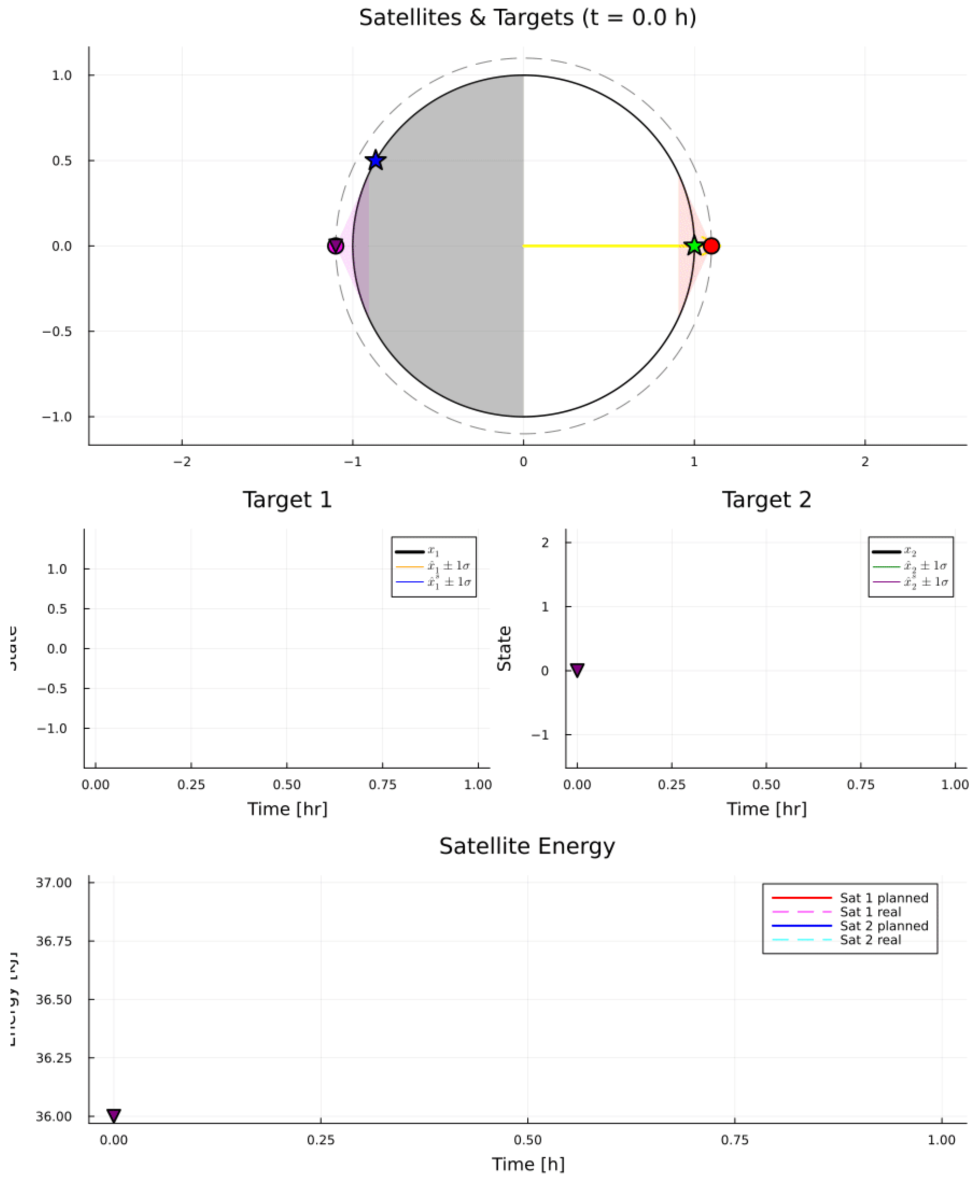

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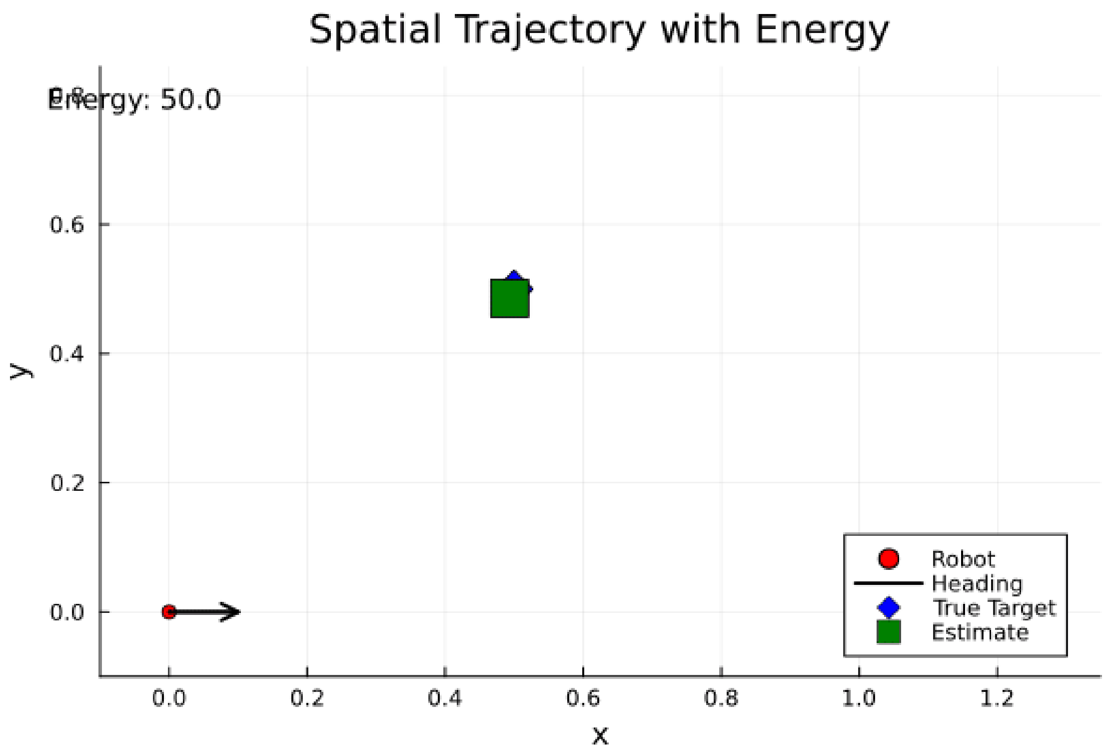

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