



**Collins Aerospace** 



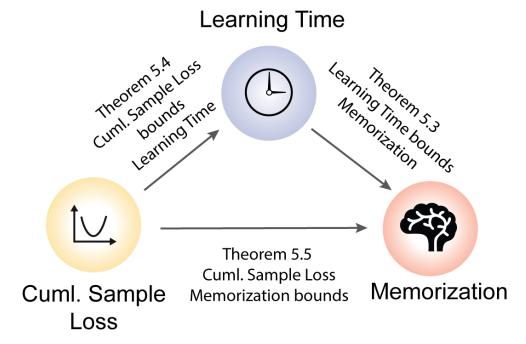
# Towards Memorization Estimation: Fast, Formal and Free

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#### Overview



## The Challenge: Understanding Memorization in Deep Learning Our Contribution: Cumulative Sample Loss (CSL)

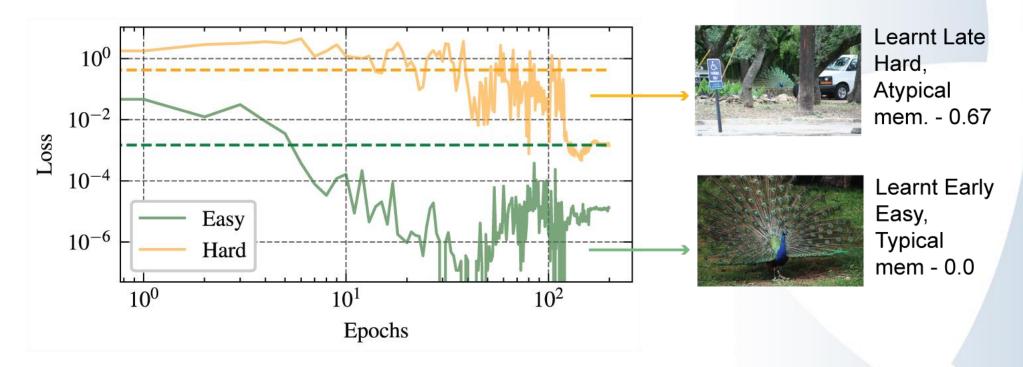
- We introduce a new, efficient proxy for memorization: Cumulative Sample Loss (CSL). CSL is simply the accumulated loss of a sample over the entire training process.
- We establish a theoretical framework that connects CSL to both learning time and stability-based memorization.

#### **Key Benefits & Applications**

- Fast & Free: CSL is 10,000x faster than stability-based methods and can be obtained with zero extra overhead during training.
- Practical Applications: State-of-the-art performance for identifying mislabeled examples and detecting duplicates in datasets.



#### Intuition and CSL



Visualizing peacock class learning in ImageNet. Average loss is shown for easy and hard-to-learn peacocks. The dashed line represents average loss, while solid lines show actual loss. Easy images are less memorized, while hard images are memorized more.



#### Define Learning Time and CSL

Cumulative Sample Loss: 
$$CSL(\vec{z}_i) = \sum_{t=0}^{\infty} l(\vec{w}_t, \vec{z}_i)$$

Sample Learning Condition: 
$$E[||\nabla_{x_i} l(\vec{w}_R)||_2^2] \leq \tau$$

Learning Time: 
$$T_{Z_i} = \min_T \{T: \mathbb{E}[||\nabla_{x_i} l(\overrightarrow{w}_R)||_2^2] \le \tau\}$$



#### Theoretical Results

#### Assumptions:

Bounded loss (or cross entropy), Lipschitznes, bounded gradient variance and unbiased grad. est.

Lemma 5.1: Input gradient norm is bound by weight gradient norm.

Theorem 5.2: Convergence in input gradient norm for SGD converges is root of iterations.

Theorem 5.3: Learning Time bounds Memorization.

$$E_{z_i}[mem(\vec{z}_i)] \le \kappa_T E_{z_i}[T_{z_i}] + \frac{\beta}{L}$$

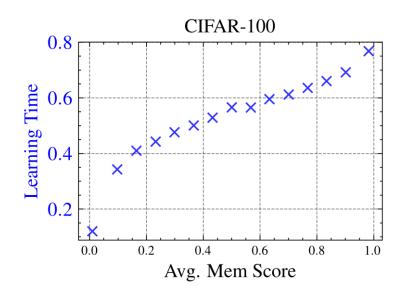
Theorem 5.4: Cumulative loss bounds learning time. 
$$\kappa_T \ \mathbf{E}_{z_i} \big[ T_{z_i} \big] \leq \frac{\mathbf{E}_{z_i} [\mathit{CSL}(\vec{z}_i)] - \xi}{L}$$

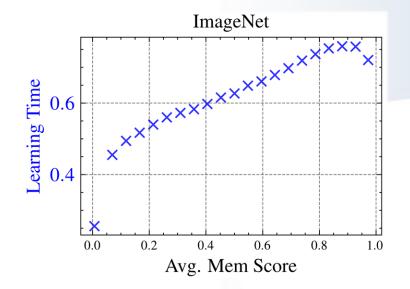
Theorem 5.5 Cumulative Sample Loss bounds Memorization.

$$E_{z_i}[mem(\vec{z}_i)] \le \frac{E_{z_i}[CSL(\vec{z}_i)] + \beta - \xi}{L}$$



#### Validating Theoretical Results





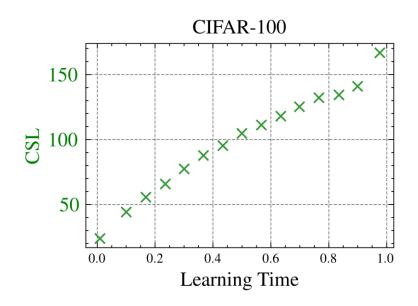
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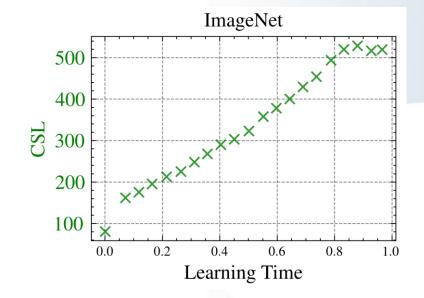
$$E_{z_i}[mem(\vec{z}_i)] \le \kappa_T E_{z_i}[T_{z_i}] + \frac{\beta}{L}$$

The expectation is that it has a linear relation between learning time and memorization score. This is validated by results on CIFAR100 and ImageNet



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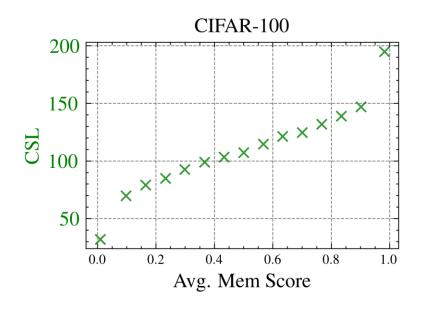


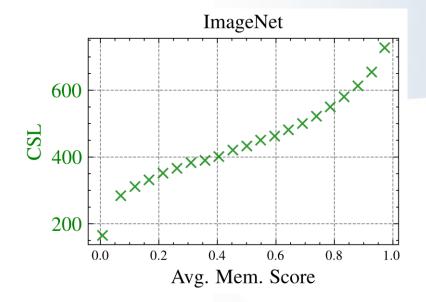
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#### Validating Theoretical Results





Theorem 5.5 Cumulative Sample Loss bounds Memorization.

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### Similarity with Memorization

Dataset	Arch.	Subset	Method	CS	PC
	Inception	Top 5k	Final Sample Loss	0.33	0.06
			Curv	0.87	0.16
			Loss Sensitivity	0.97	0.39
			Forget Freq.	0.96	0.29
CIEAR 100			CSL (Ours)	0.93	0.4
CIFAR-100		All	Final Sample Loss	0.24	0.17
			Curv	0.69	0.49
			Loss Sensitivity	0.81	0.76
			Forget Freq.	0.76	0.59
			CSL (Ours)	0.87	0.79
	ResNet50	Top 50k	Final Sample Loss	0.78	0.12
			Curv	0.84	0.05
			Loss Sensitivity	0.79	0.04
			Forget Freq.	0.68	0.15
L NL			CSL (Ours)	0.94	0.21
ImageNet		All	Final Sample Loss	0.64	0.5
			Curv	0.62	0.33
			Loss Sensitivity	0.49	0.17
			Forget Freq.	0.49	0.04
			CSL (Ours)	0.79	0.64

CSL correlation and similarity with memorization compared to other methods across CIFAR-100 and ImageNet datasets. CS denotes cosine similarity and PC denotes Pearson correlation.



### Identifying Mislabeled

Dataset	Method	1% Noise	2% Noise	5% Noise	10% Noise
CIFAR-10	Thr. Learning Time	0.4951 ± 0.0248	$0.4954 \pm 0.0044$	$0.4911 \pm 0.0071$	0.4948 ± 0.0057
	In Conf.	0.8781 ± 0.0177	$0.8072 \pm 0.0130$	$0.7254 \pm 0.0214$	0.6528 ± 0.0042
	CL	0.8651 ± 0.0127	$0.8905 \pm 0.0115$	$0.8874 \pm 0.0019$	0.8551 ± 0.0030
	SSFT	0.9626 ± 0.0018	$0.9551 \pm 0.0020$	$0.9498 \pm 0.0042$	$0.9360 \pm 0.0020$
	Curv.	0.9715 ± 0.0045	0.9776 ± 0.0033	$0.9800 \pm 0.0003$	$0.9819 \pm 0.0006$
	CSL (Ours)	0.9845 ± 0.0026	0.9864 ± 0.0004	0.9870 ± 0.0003	0.9869 ± 0.0005
CIFAR-100	Thr. Learning Time	0.5256 ± 0.0012	$0.5227 \pm 0.0100$	$0.5161 \pm 0.0051$	$0.5203 \pm 0.0029$
	In Conf.	0.7258 ± 0.0102	$0.7236 \pm 0.0047$	$0.7069 \pm 0.0069$	0.6884 ± 0.0053
	CL	0.8723 ± 0.0208	$0.8838 \pm 0.0006$	$0.8733 \pm 0.0010$	0.8536 ± 0.0006
	SSFT	0.8915 ± 0.0045	$0.8893 \pm 0.0013$	$0.8784 \pm 0.0030$	0.8664 ± 0.0024
	Curv.	0.9856 ± 0.0009	$0.9865 \pm 0.0011$	$0.9876 \pm 0.0021$	$0.9892 \pm 0.0012$
	CSL (Ours)	0.9891 ± 0.0003	0.9895 ± 0.0002	0.9895 ± 0.0001	0.9897 ± 0.0001

Evaluating the performance of mislabeled detection of the proposed framework against existing methods on CIFAR-10 and CIFAR-100 datasets under various levels of label noise.



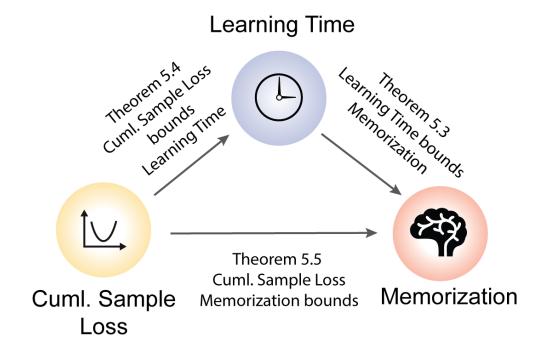
#### Identifying Duplicate Samples

CSL (Ours)	0.9821 ± 0.0006	0.9886 ± 0.0008	
Curv.	0.9536 ± 0.0030	0.9639 ± 0.0030	
SSFT	$0.8490 \pm 0.0034$	0.7938 ± 0.0045	
CL	$0.5533 \pm 0.0031$	0.5873 ± 0.0090	
In Conf.	0.9237 ± 0.0114	$0.8623 \pm 0.0131$	
Thr. LT	0.7029 ± 0.0058	0.7419 ± 0.0059	
Method	CIFAR-10	CIFAR-100	

Result of duplicate detection using the proposed methods and other baselines on CIFAR-10 and CIFAR-100 datasets.



#### Summary



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## Thank you!

