Towards Better-than-2 Approximation for Constrained Correlation Clustering

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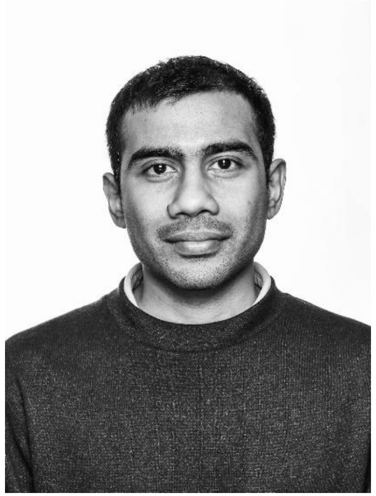

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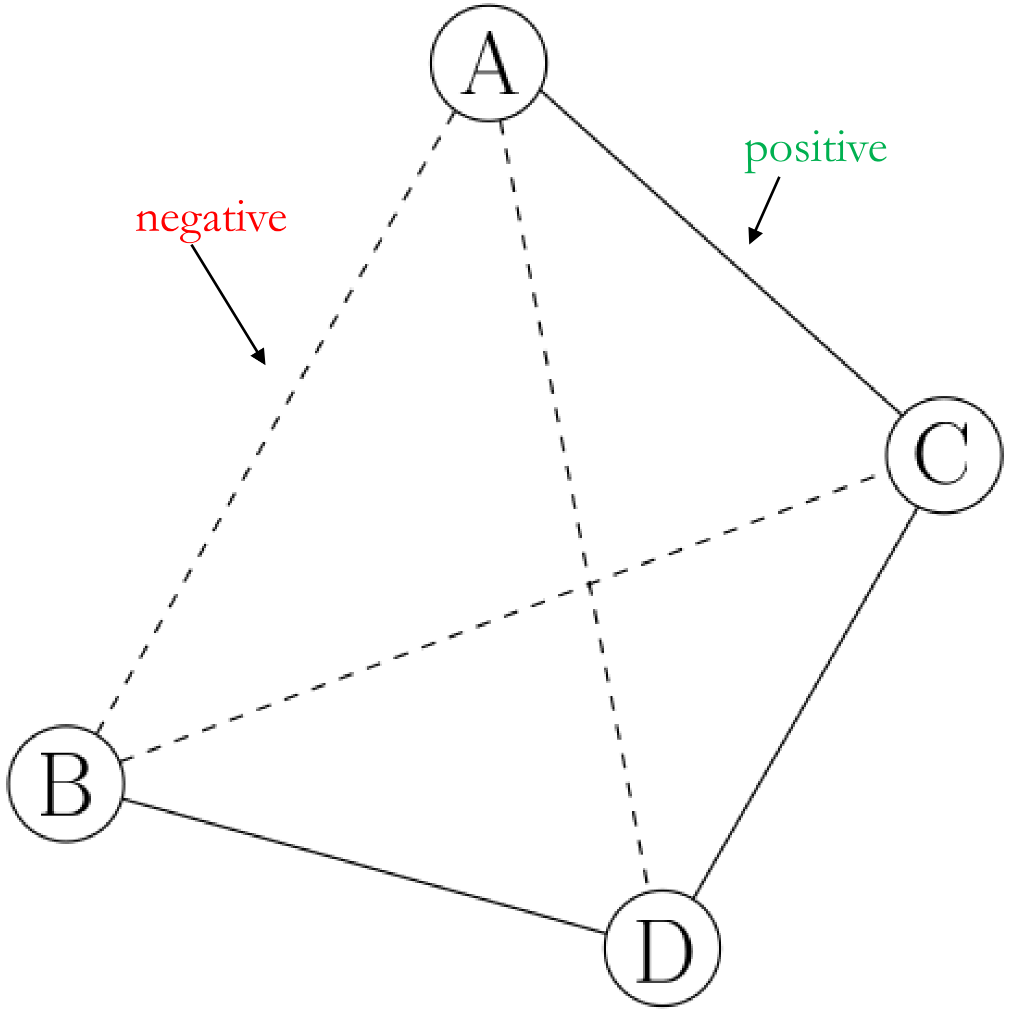

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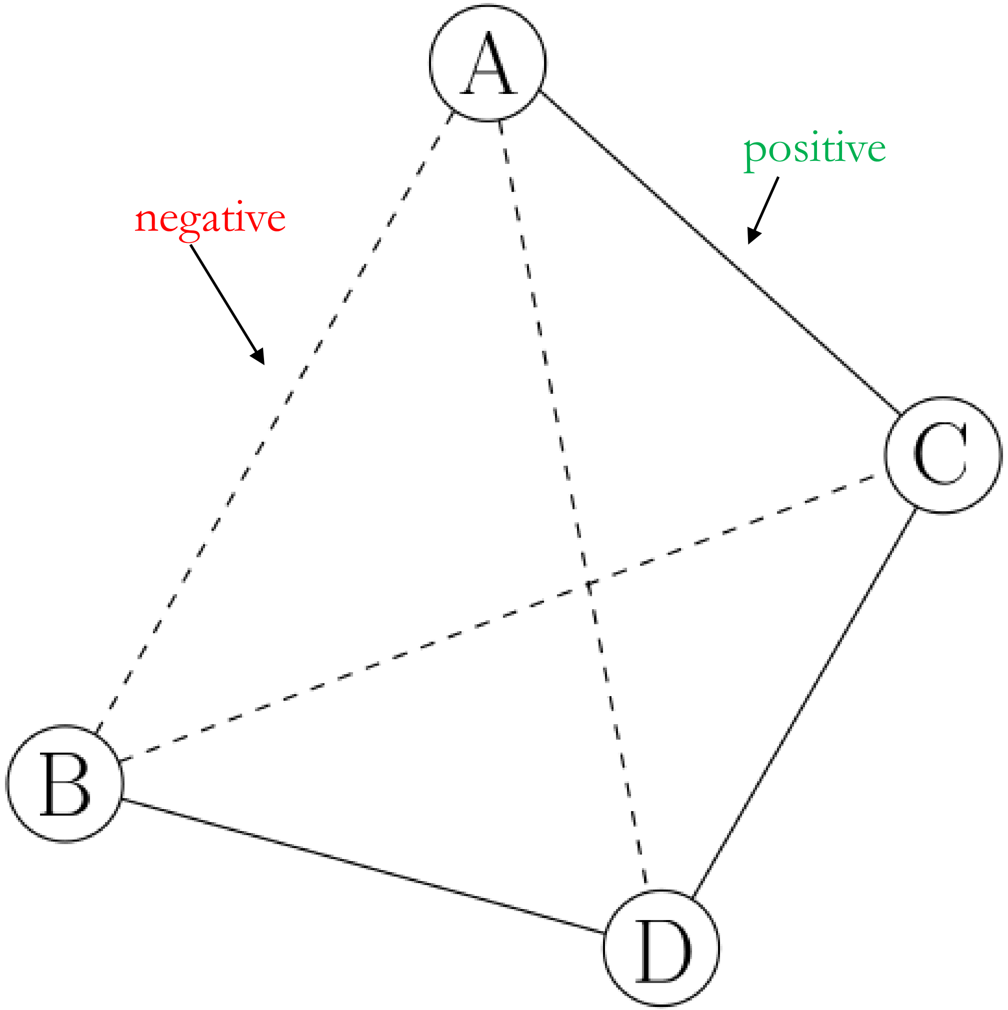

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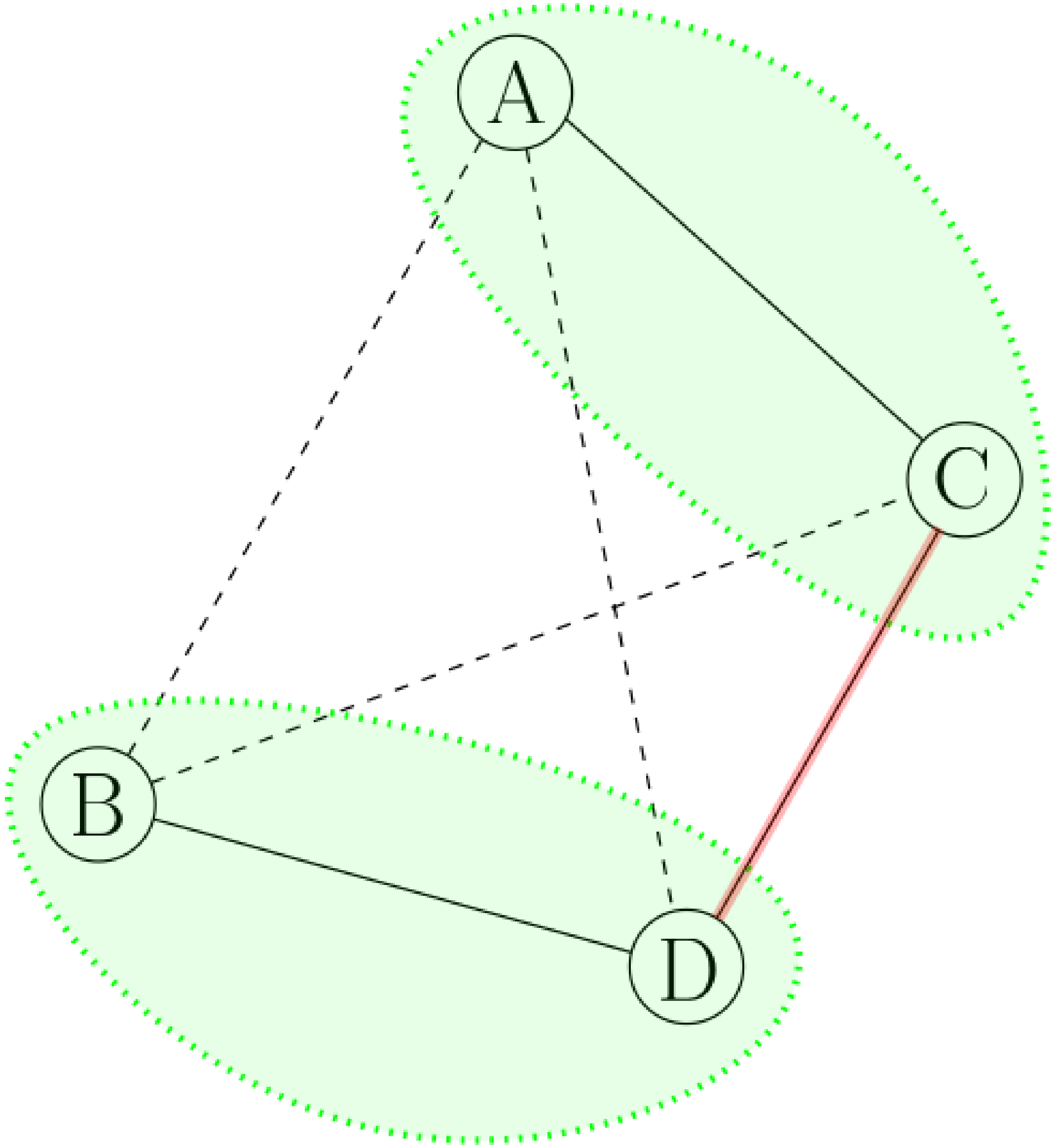
Example: Clustering violates only the preference of CD ⇒ Cost = 1



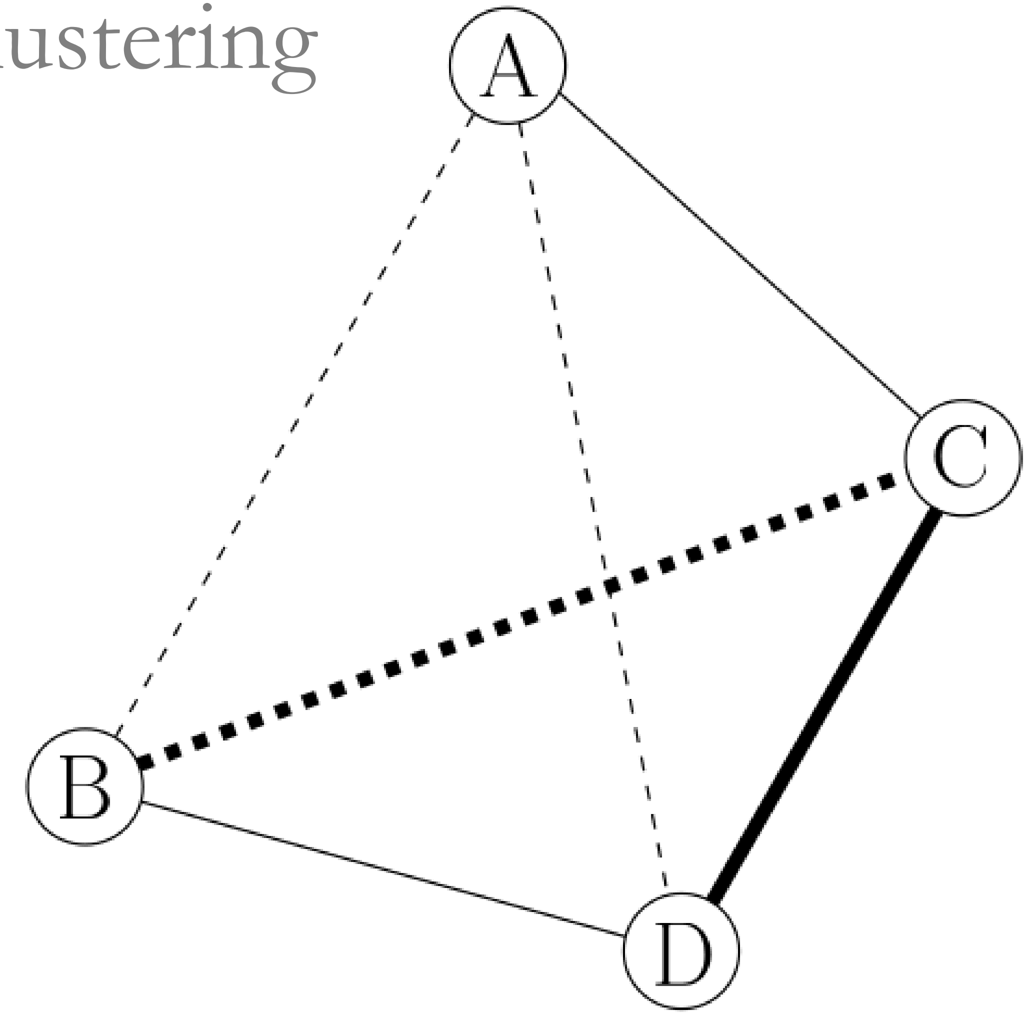
Constrained Correlation Clustering

Input:

- Complete graph with preferences on each edge (positive or negative)
- Some preferences are **hard constraints**

Output:

- A **partition** of nodes violating the least number of pairwise preferences and **satisfies** all **hard constraints**



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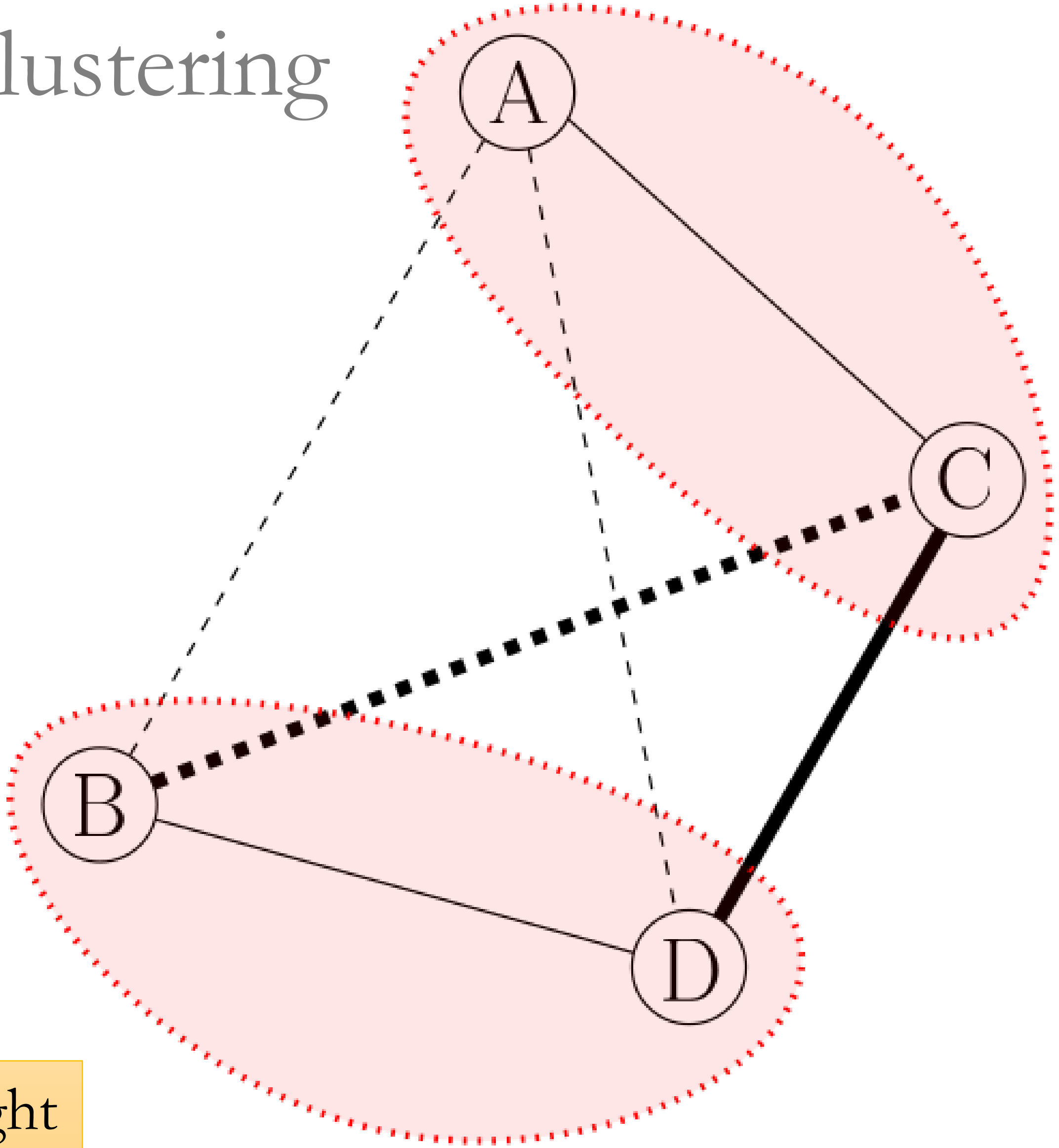

Best clustering for **unconstrained** instance might **not** be **feasible**

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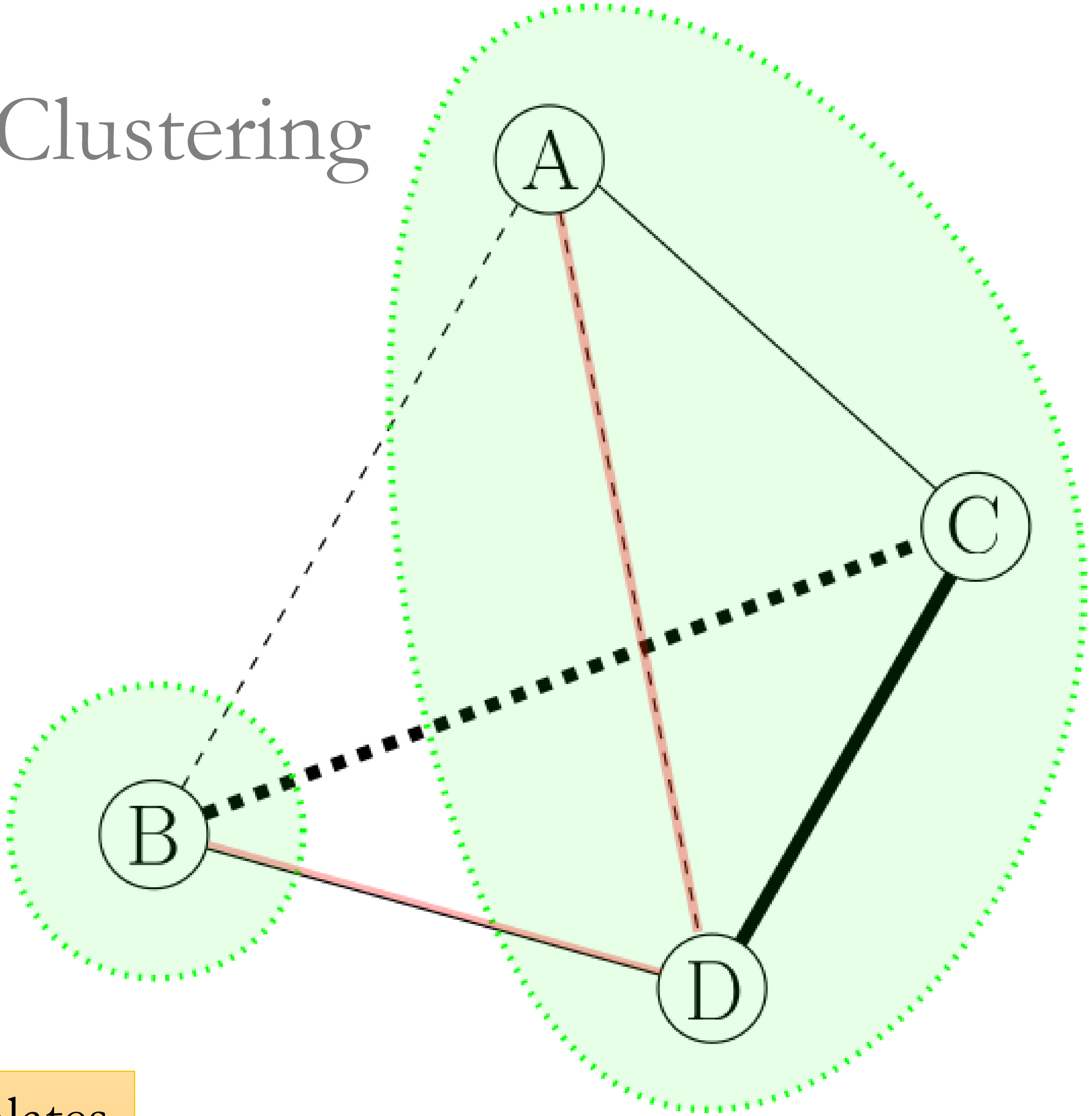

Example: Feasible constrained clustering violates preferences of AD, BD ⇒ Cost = 2

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NP-hard

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Constrained Correlation Clustering

- 3-approximation (LP) [van Zuylen & Williamson, SODA '07]
- 16-approximation (Faster combinatorial algorithm) [Fischer et al., STACS '25]

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Novel way of **combining** large **LP** and **Local Search** techniques

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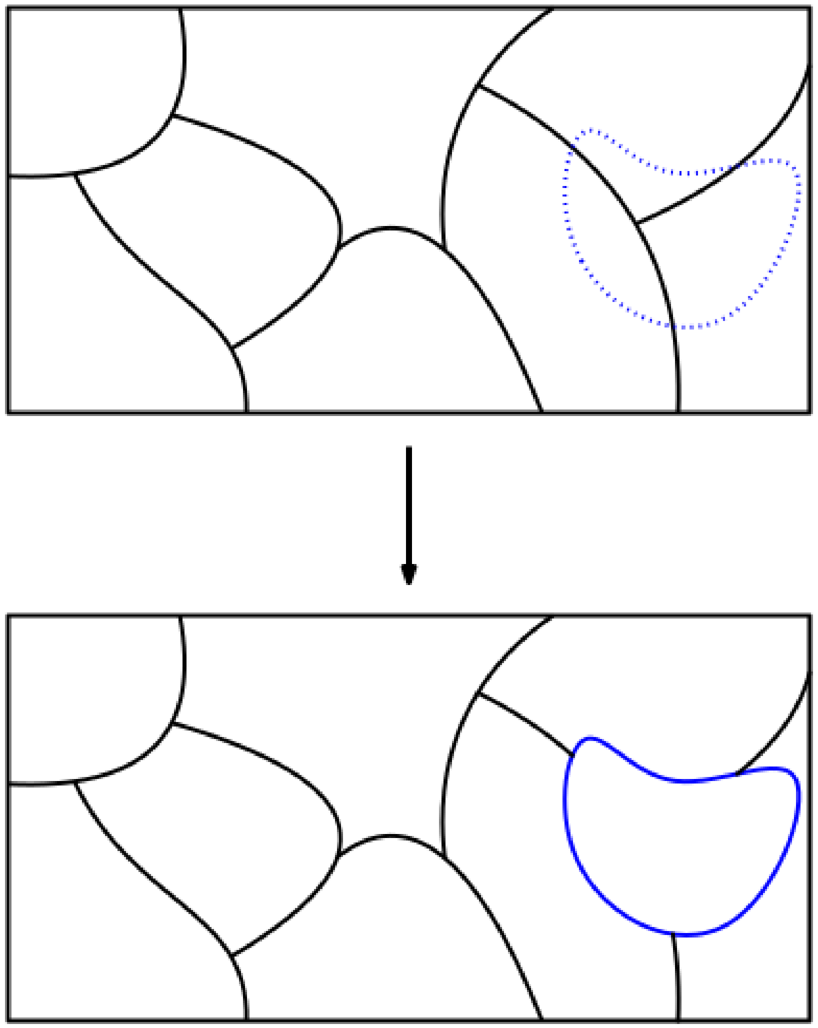
Better-than-2 approximation for Constrained Correlation Clustering, <u>conditional on solution to a large Constrained LP</u>

Main Idea

Local Search move: **swap** a **cluster** in the clustering

Local Search



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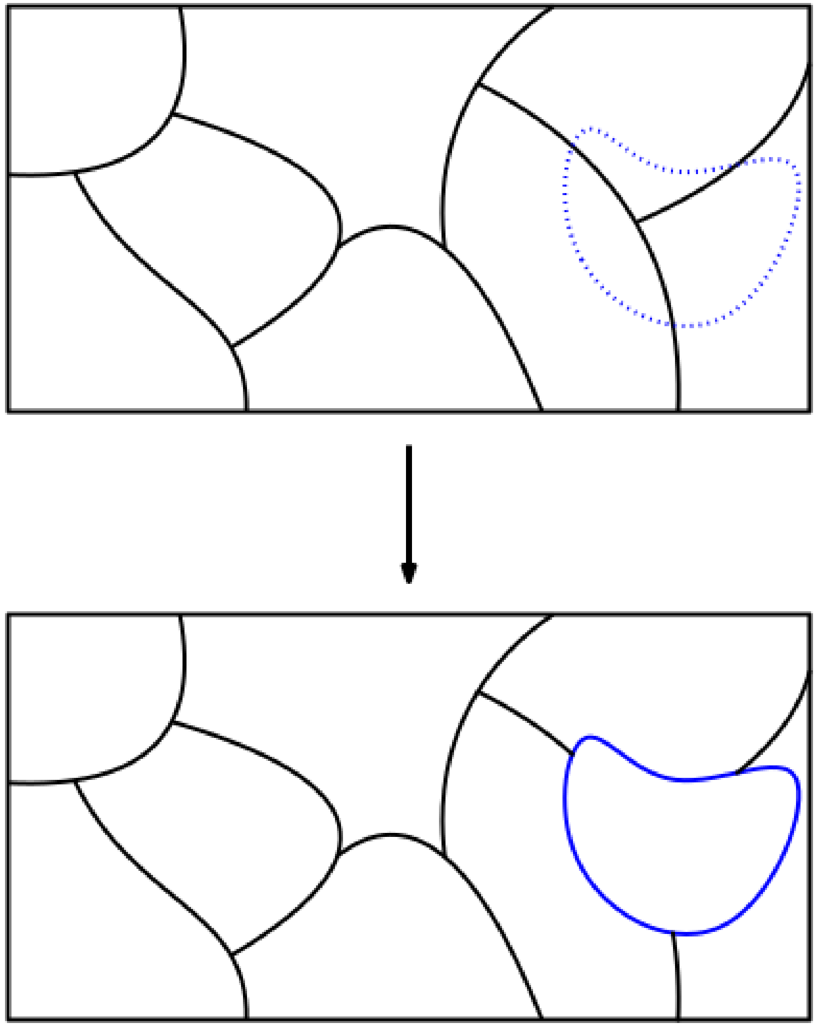

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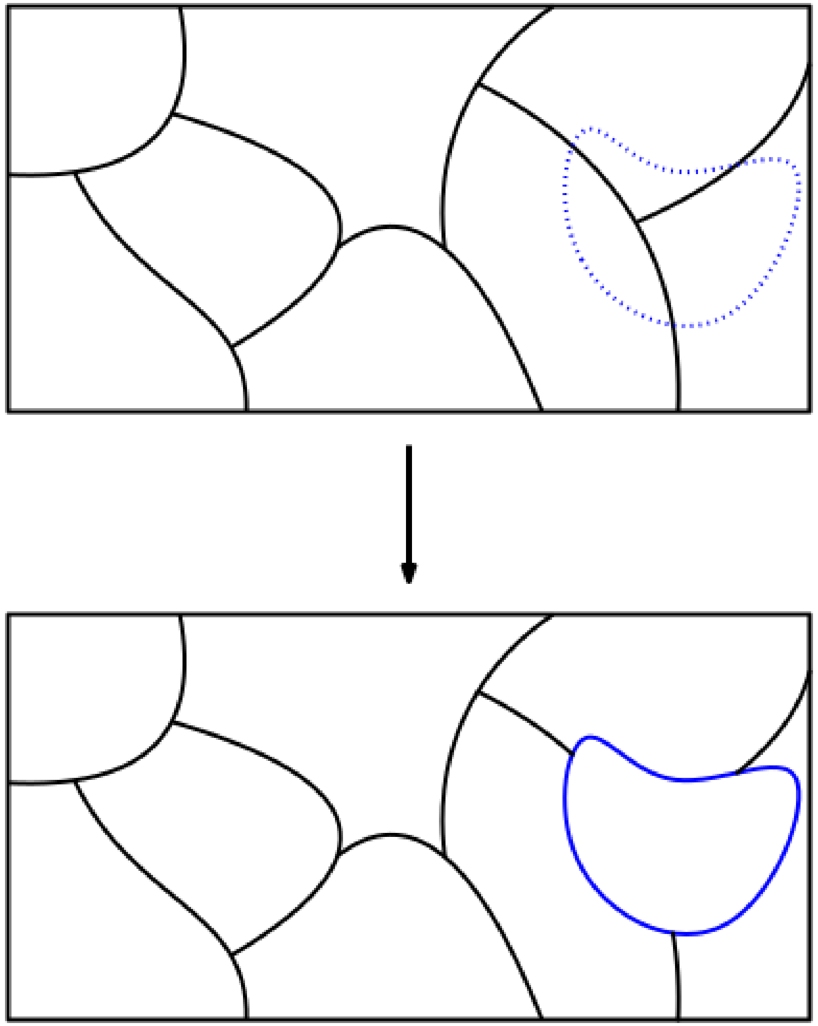

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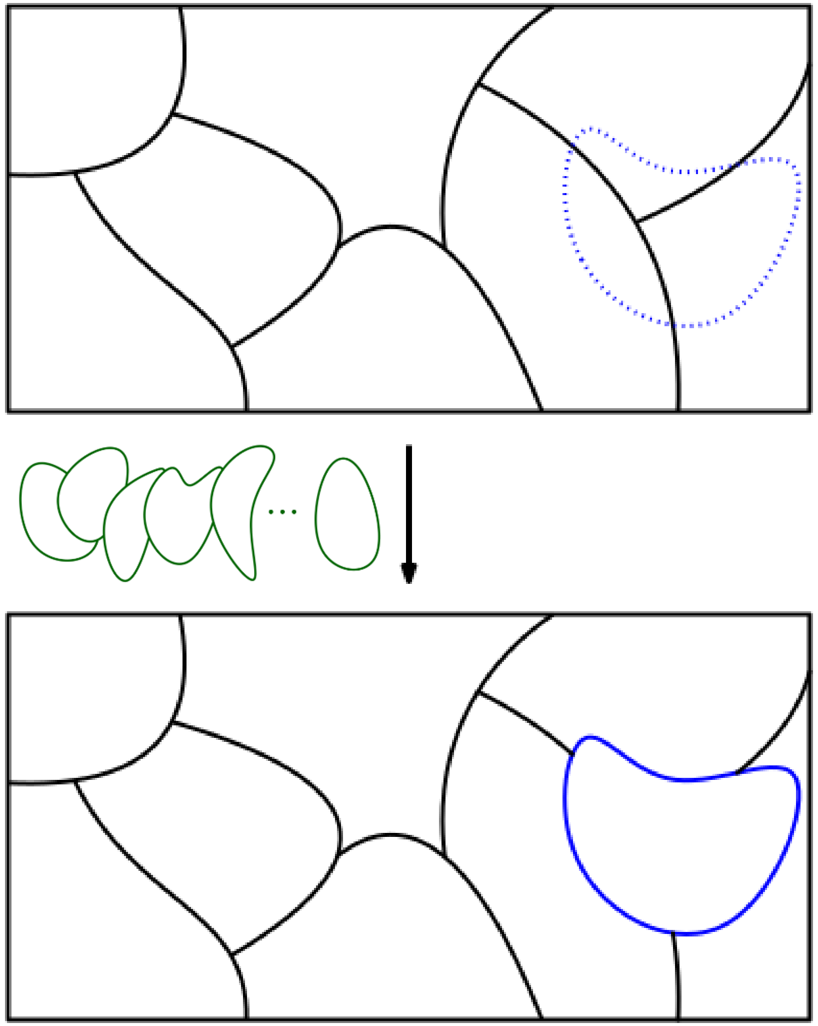

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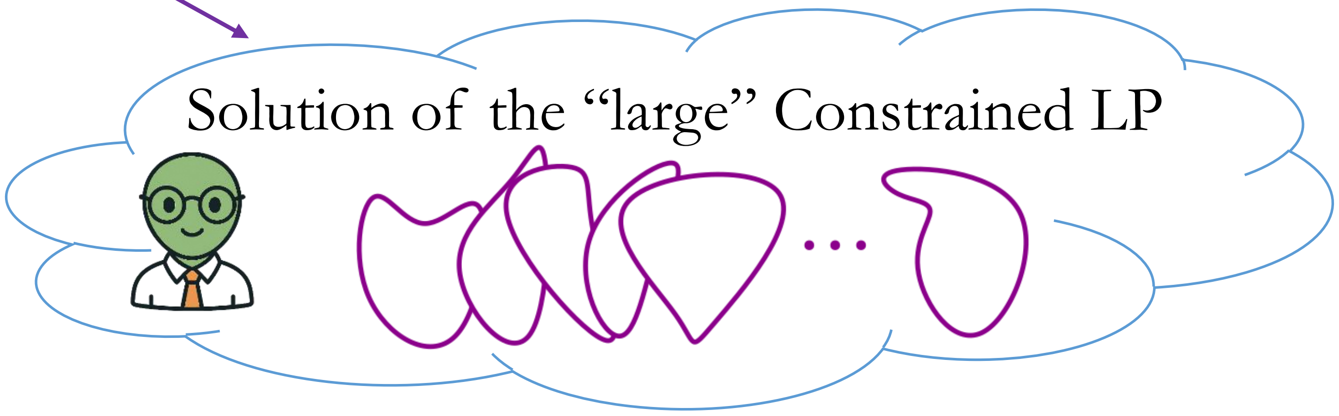

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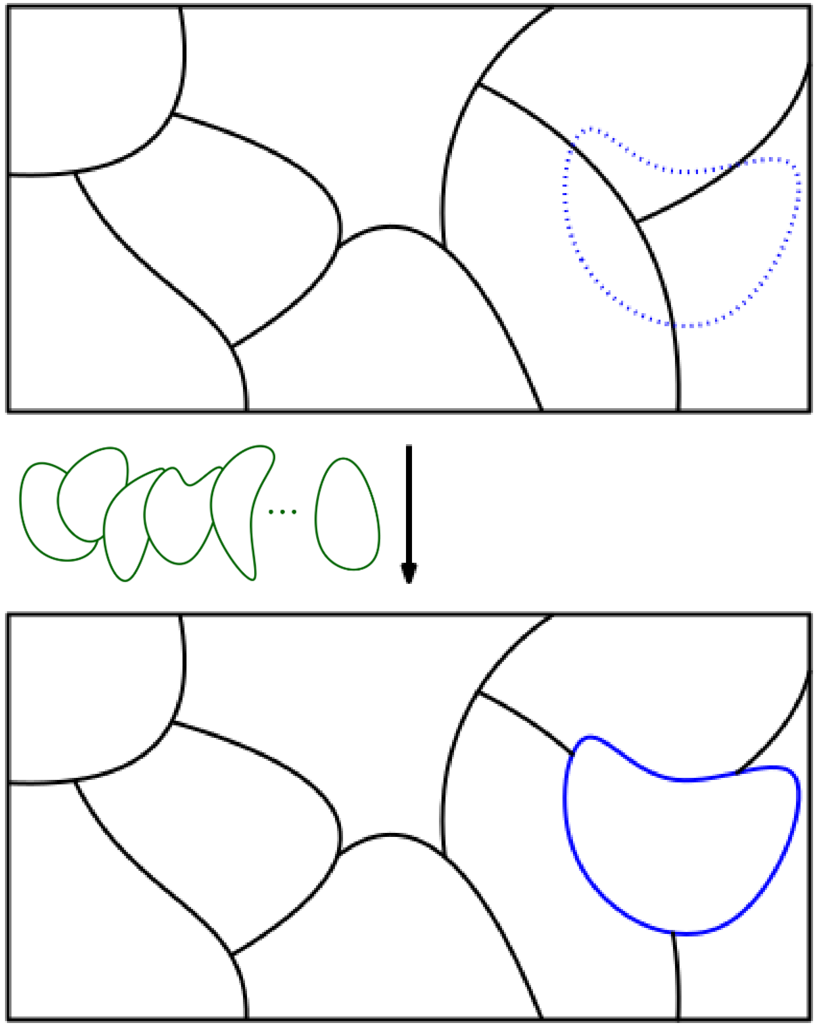

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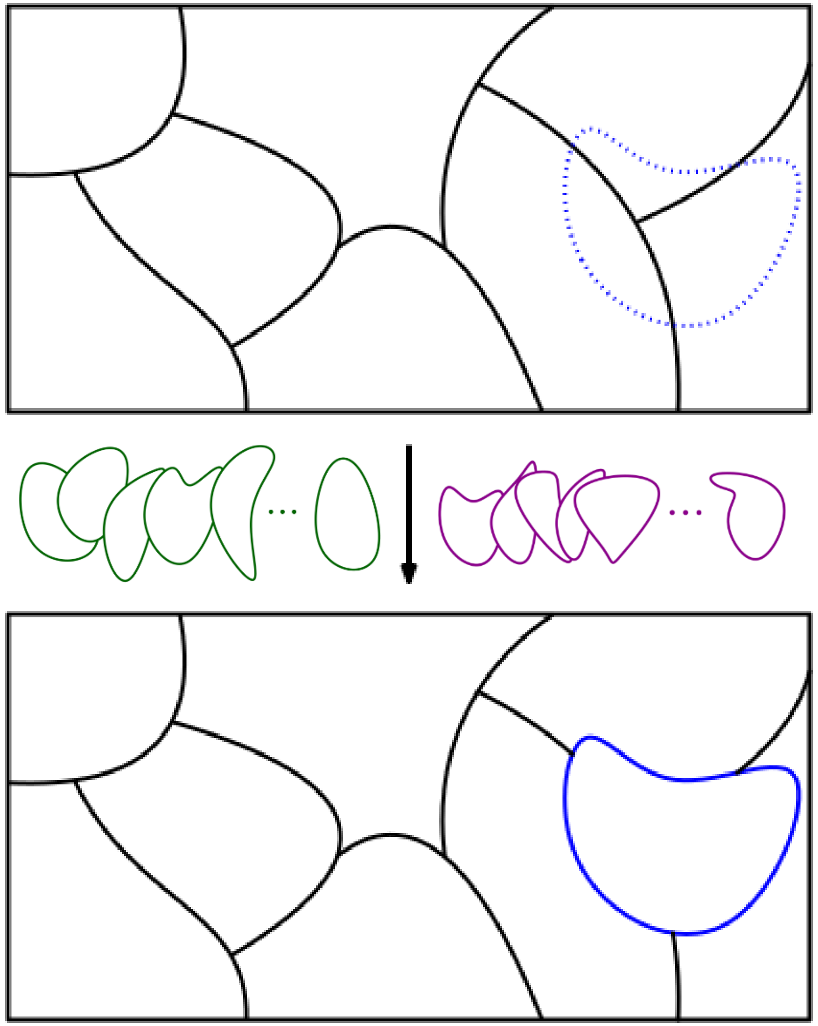

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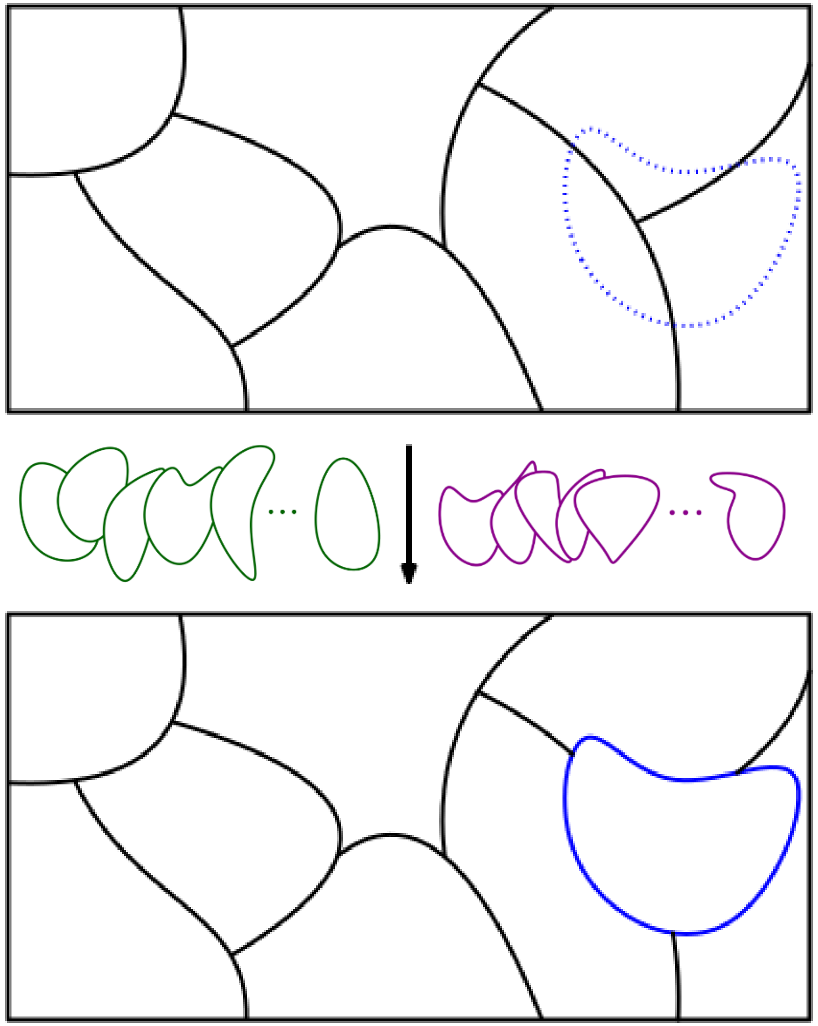

Result: Better-than-2 approximation for both variants!

Open Directions

- Efficient solution to the "large" Constrained LP
- Broader applications of combining LP and Local Search techniques
- Inapproximability of Constrained Correlation Clustering

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Thank You!