

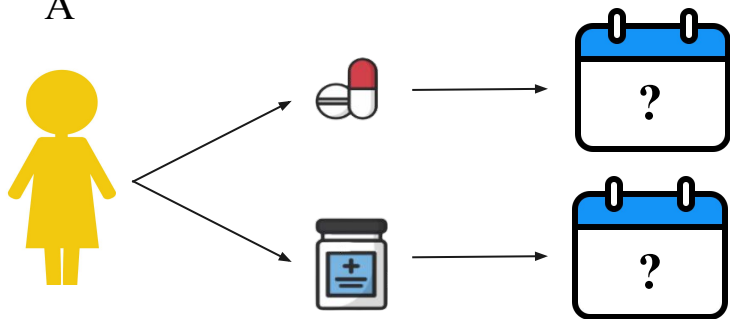
# **Heterogeneous Treatment Effect in Time-to-Event Outcomes: Harnessing Censored Data with Recursively Imputed Trees**

Tomer Meir, Uri Shalit, and Malka Gorfine

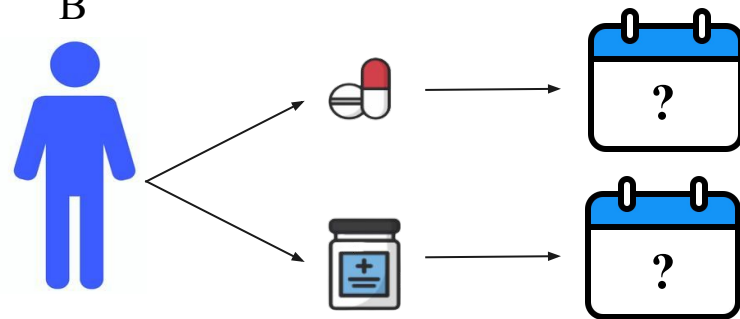
# Motivation

Which cancer treatment would lead to longer survival for each patient?

Patient  
A

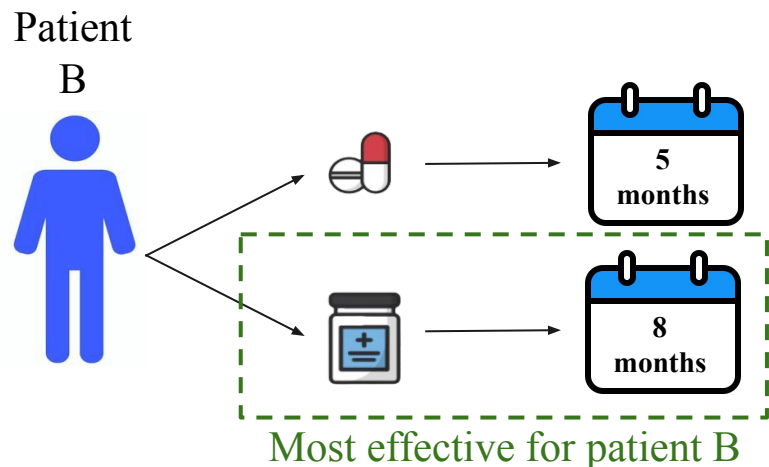
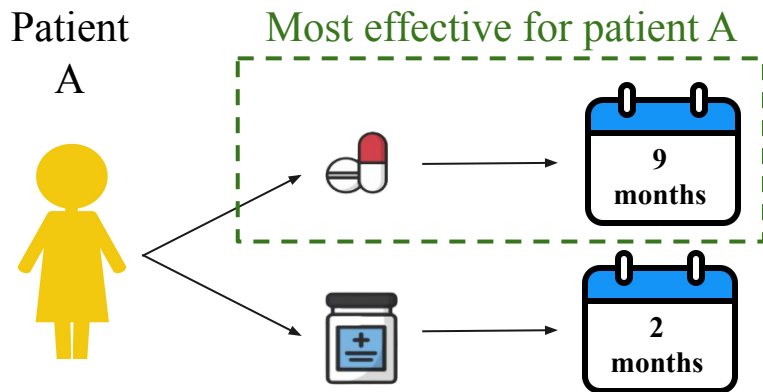


Patient  
B



# Motivation

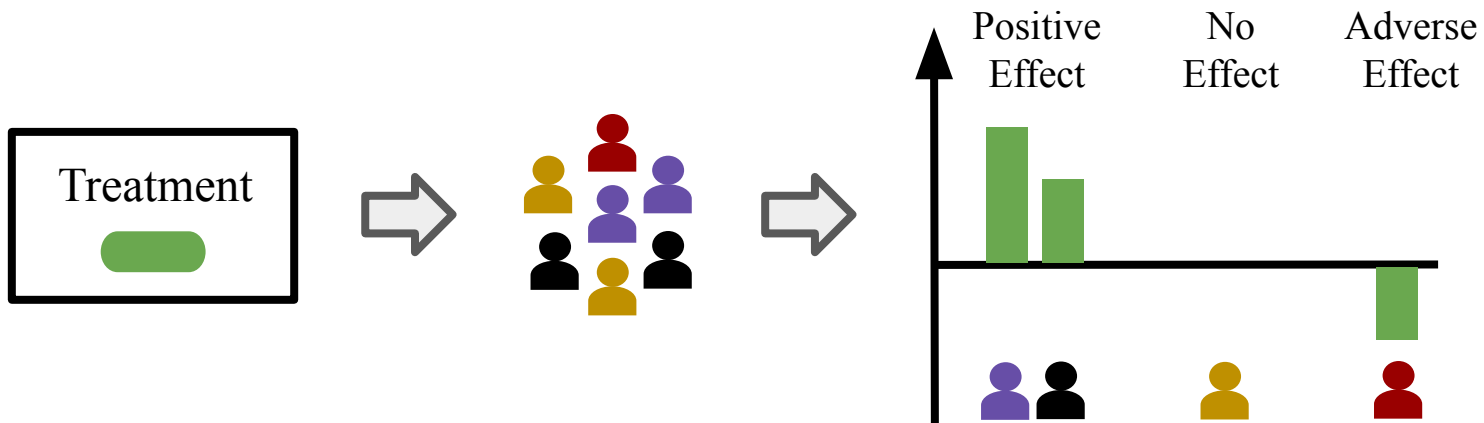
Which cancer treatment would lead to longer survival for each patient?



Knowing this is a key step toward the selection of the **most effective treatment** for each individual.

# Heterogeneous Treatment Effects

*Heterogeneous Treatment Effects (HTE)* describe the way treatments impact different subgroups.

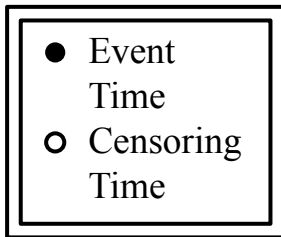
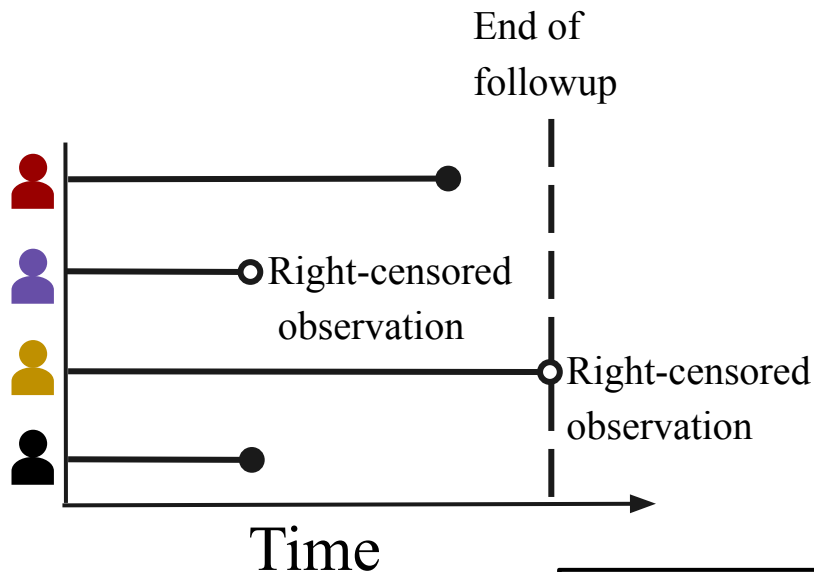


# Time to Event Analysis

*Time-to-event (TTE)* analysis is used  
 when the **outcome is time until a  
 specific event**.

Right-censored observations:

- Exact event-time is unknown.
- Event-free until a certain time.



# Estimand

Our goal is to estimate the HTE in TTE data,  
 defined by

$$\tau(x) = E \left\{ g(\tilde{T}_i^0) - g(\tilde{T}_i^1) | X_i = x \right\}$$

for a known function  $g(\cdot)$ .

## Notation

$N$  iid observations

$X_i \in \mathbb{R}^p$  - vector of  $p$  covariates of  
 observation  $i$

$\tilde{T}_i \in \mathbb{R}_+$  - event time

$W_i \in \{0, 1\}$  - treatment assignment

$\{\tilde{T}_i^0, \tilde{T}_i^1\}$  - potential outcomes under

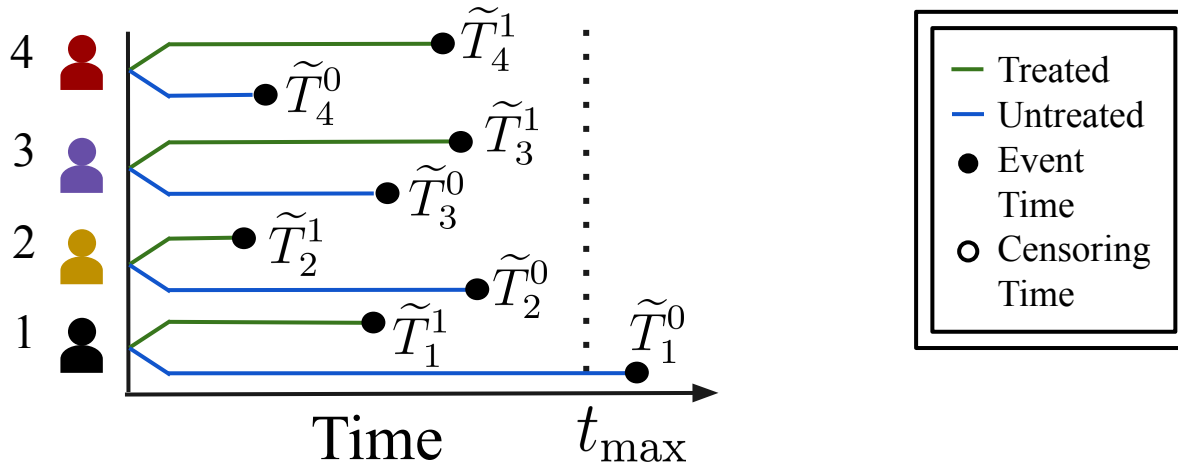
$W_i = 0$  and  $W_i = 1$

Examples of the HTE for specific functions  $g(\cdot)$

$g(\cdot)$	$\tau(\cdot)$
$g(\tilde{T}_i) = \min(\tilde{T}_i, h)$	difference in restricted mean survival time (RMST)
$g(\tilde{T}_i) = I(\tilde{T}_i \geq h)$	differences in survival functions

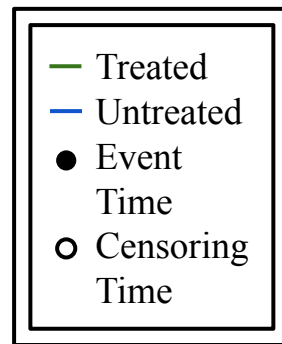
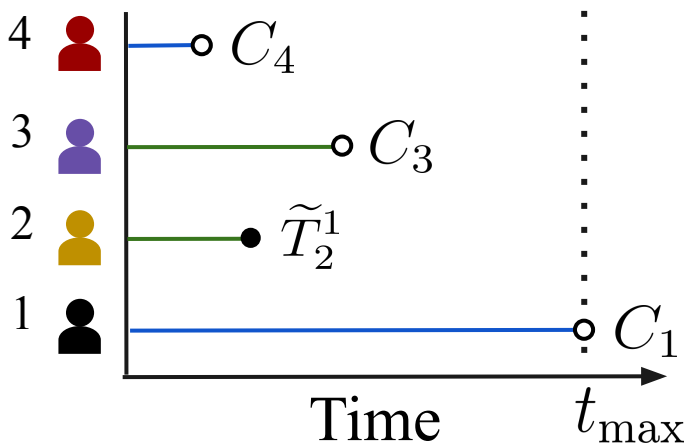
# Ideal Data

Each sample would include **both potential outcomes** - under treatment and under no treatment.



# Observed Data

Each sample receives a **single treatment**, with **right-censored observations** providing only a lower bound on survival under that arm.

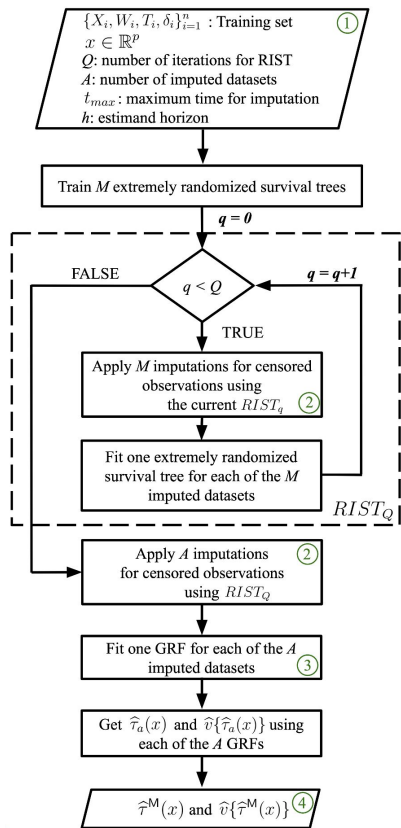


# Existing Methods

Existing methods, such as causal survival forests (CSF) with inverse probability weighting and a doubly robust estimator [1], **struggle under heavy censoring and cannot handle instrumental variables (IV).**

# MISTR - An Overview

MISTR is a non-parametric multiple imputation based algorithm for estimating HTE in TTE data.

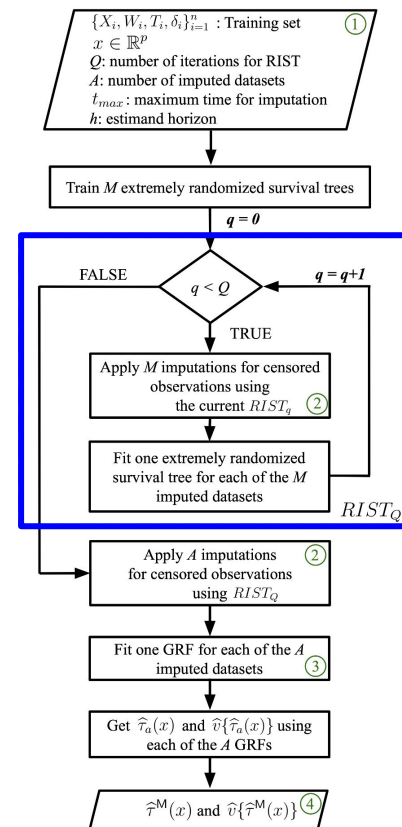


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MISTR is a non-parametric multiple imputation based algorithm for estimating HTE in TTE data.

MISTR utilizes:

- **Recursively Imputed Survival Trees (RIST) [2]**



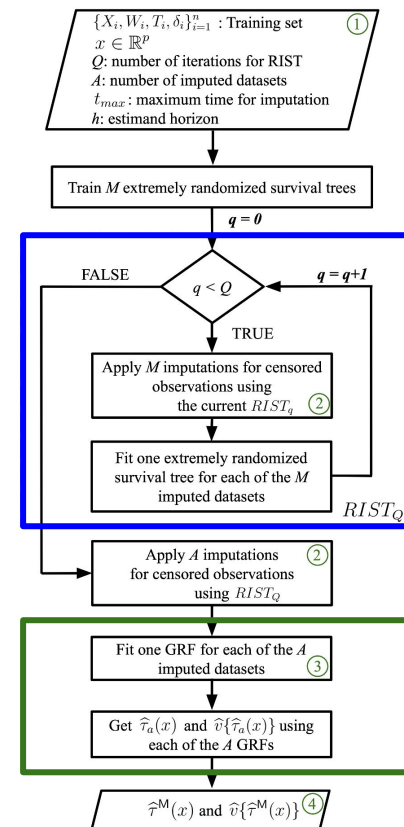
# MISTR - An Overview

MISTR is a non-parametric multiple imputation based algorithm for estimating HTE in TTE data.

MISTR utilizes

- Recursively Imputed Survival Trees (RIST) [2]
- Generalized Random Forest (GRF) [3]

to overcome existing limitations.

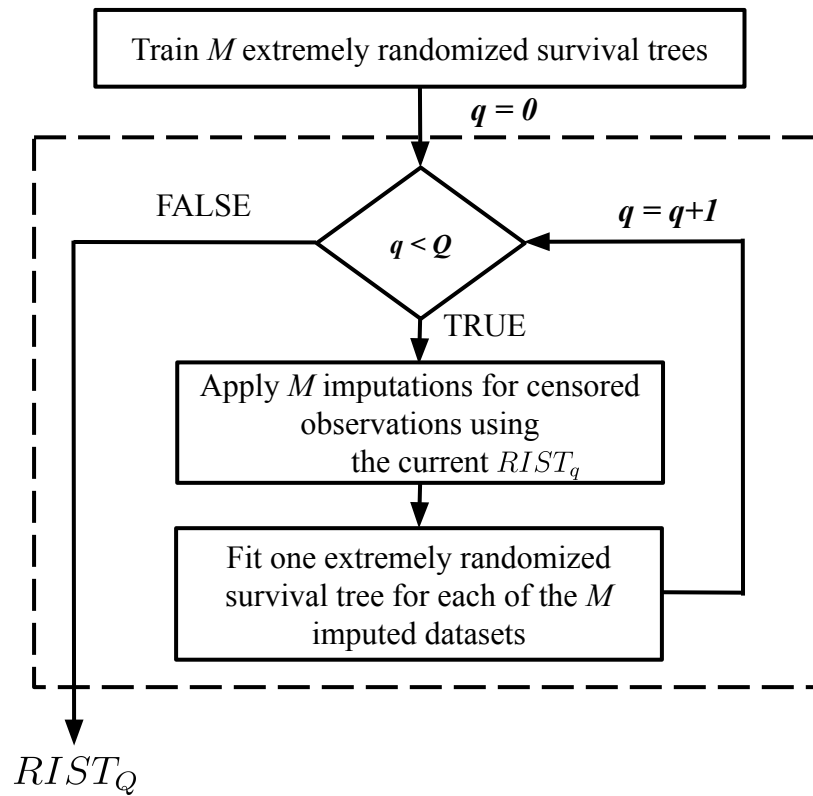


[2] Zhu, R. and Kosorok, M. R. *Recursively Imputed Survival Trees*. Journal of the American Statistical Association, 107(497):331–340, March 2012.

[3] Athey, S., Tibshirani, J., and Wager, S. *Generalized random forests*. The Annals of Statistics, 47(2), April 2019.

# Q-RIST

Q-RIST [2] is a forest of  $M$  extremely randomized survival trees, trained over  $Q$  recursive steps.



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# Q-RIST

Q-RIST [2] is a forest of  $M$  extremely randomized survival trees, trained over  $Q$  recursive steps.

It is used for **estimating the conditional survival function while extracting more information from censored data.**

$$\Pr(\tilde{T}_i > t \mid X_i, W_i, C_i, \tilde{T}_i > C_i) = \frac{\Pr(\tilde{T}_i > t \mid X_i, W_i)}{\Pr(\tilde{T}_i > C_i \mid X_i, W_i, C_i)}$$

$X_i \in \mathbb{R}^p$  - vector of  $p$  covariates of observation  $i$

$\tilde{T}_i \in \mathbb{R}_+$  - event time

$C_i \in \mathbb{R}_+$  - censoring time

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# GRF

GRF [3] can be used for **estimating HTE** in non-censored data.

$$\tau(x) = E \left\{ g(\tilde{T}_i^0) - g(\tilde{T}_i^1) | X_i = x \right\}$$

**Heterogeneity is captured through similarity weights**, defined as the fraction of trees in which a given training and test sample share the same leaf.

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[2] Zhu, R. and Kosorok, M. R. ***Recursively Imputed Survival Trees***. Journal of the American Statistical Association, 107(497):331–340, March 2012.

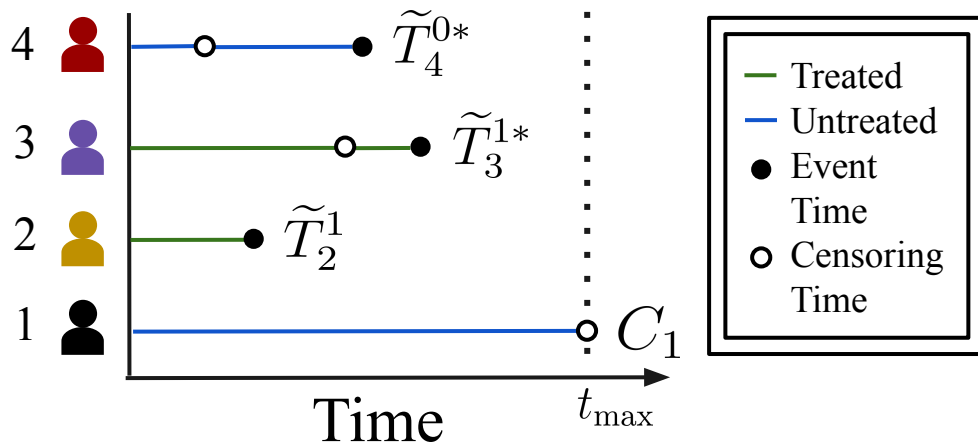
[3] Athey, S., Tibshirani, J., and Wager, S. ***Generalized random forests***. The Annals of Statistics, 47(2), April 2019.

# MISTR

Initially, RIST is employed to impute event times for censored observations.

The imputed and non-censored observations are merged to form complete uncensored datasets.

## Observed Data with a Single Imputation



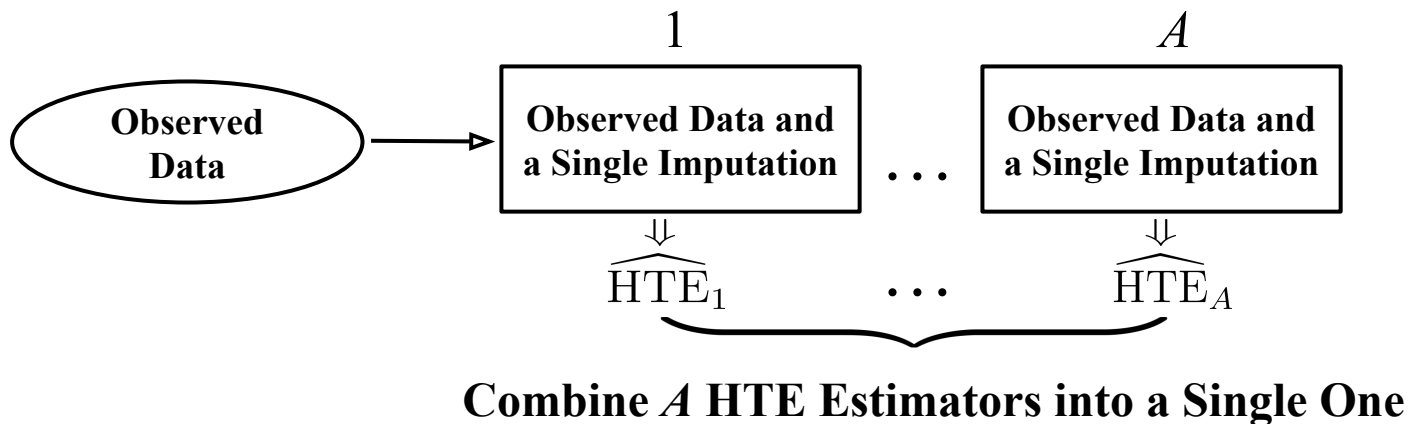
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# MISTR

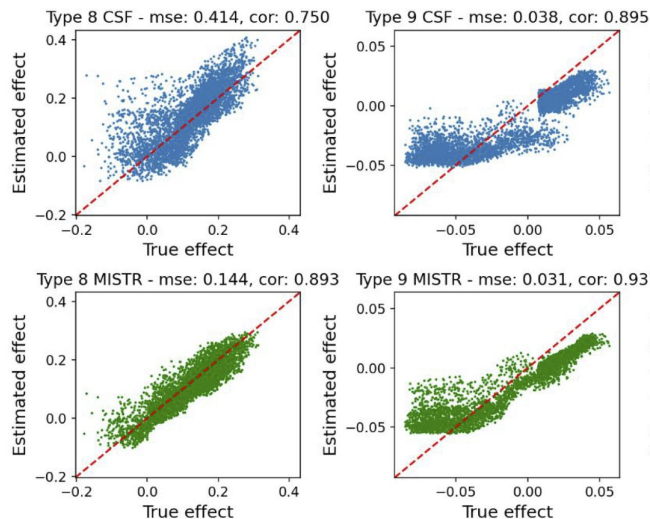
GRF is then used to produce  $A$  HTE estimates, each based on different imputed event times.

Our proposed estimate is the average of these  $A$  HTE estimates.

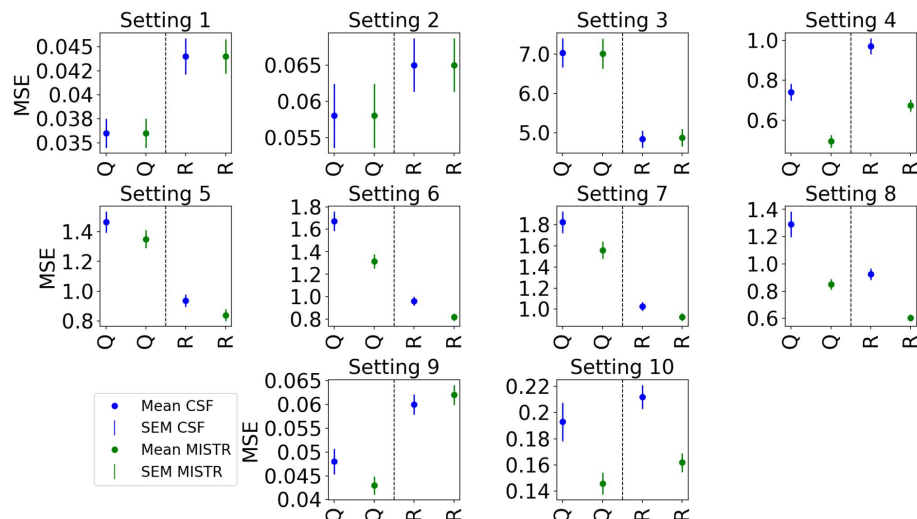


# Simulations

We show that MISTR outperforms prior methods under heavy censoring.



True vs Estimated effect Calculated over one random test sample of 5000 observations.



Mean squared error (MSE) on randomly sampled test set (R) and on covariates quantiles test set (Q).

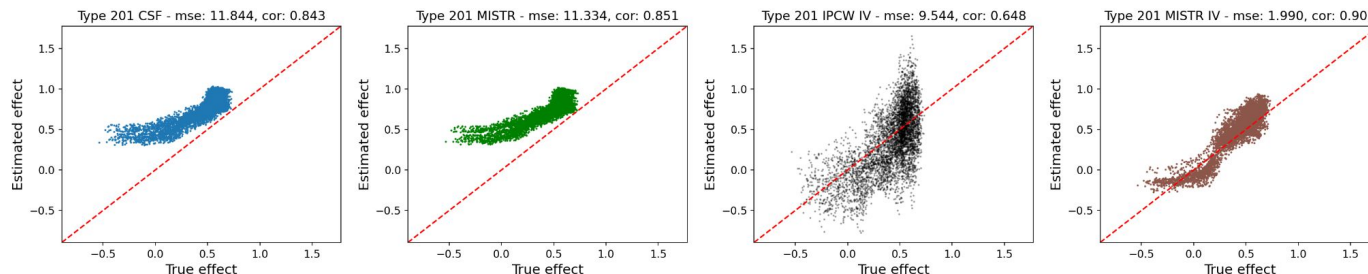
# Simulations

MISTR can be extended to the setting of instrumental variable (MISTR-IV).

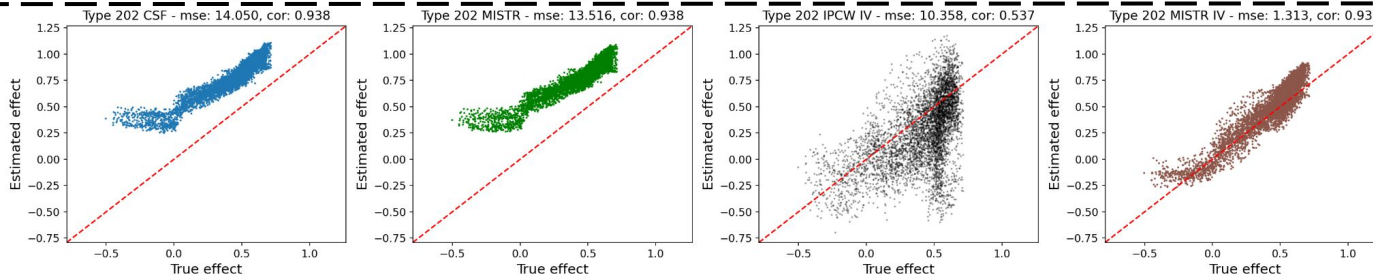
## Non-IV methods

## IV methods

Type 201



Type 202



True vs Estimated effect Calculated over one random test sample of 5000 observations.

# Use-Case: HIV Clinical Trial

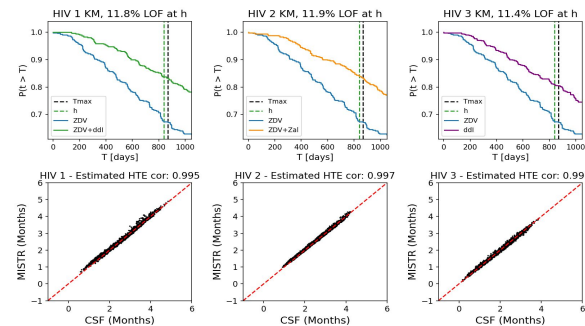
**Which treatment results in longer survival with no AIDS progression?**

$N = 2,139$  HIV-infected patients

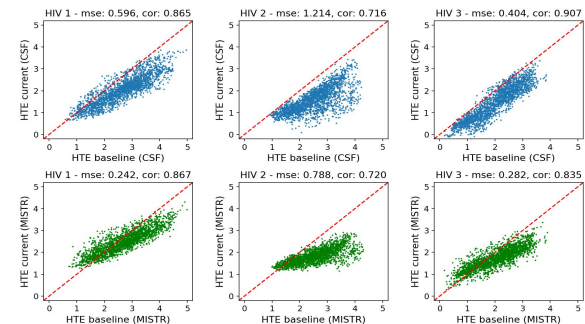
$p = 12$  covariates

4 treatment groups

We analyze the original data and the data with added censoring.



Original  
Data



Added  
Censoring

# Use-Case: HIV Clinical Trial

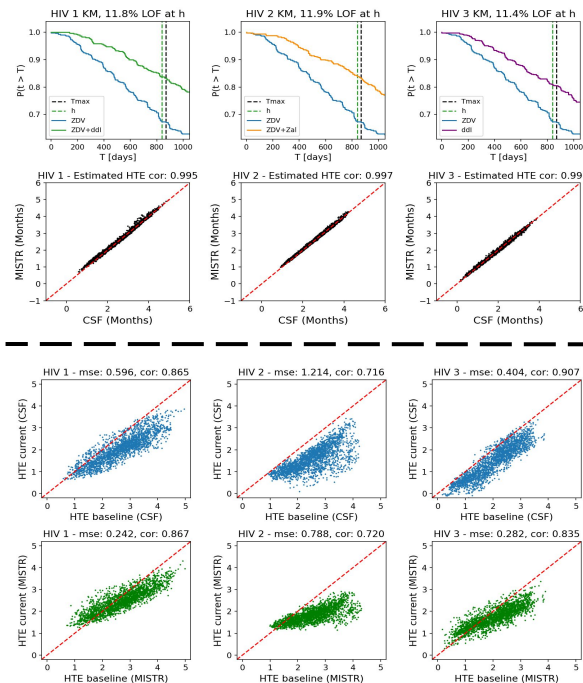
**Which treatment results in longer survival with no AIDS progression?**

$N = 2,139$  HIV-infected patients

$p = 12$  covariates

4 treatment groups

- Similar results when censoring rate is low.
- MISTR outperforms CSF at high censoring rates.



Original  
Data

Added  
Censoring

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This work presents **MISTR** - a novel non-parametric approach for estimating HTE and its variance in TTE data:

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- **MISTR outperforms other existing approaches**, especially in heavy censoring rates.
- **MISTR can incorporate an instrumental variable.**

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This work presents **MISTR** - a novel non-parametric approach for estimating HTE and its variance in TTE data:

- **MISTR expands the range of cases** that can be effectively addressed.
- **MISTR outperforms other existing approaches**, especially in heavy censoring rates.
- **MISTR can incorporate an instrumental variable.**

Additional details, simulations, and a real-world use case with IV are included in the paper.

# Thank You!

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**Further details are available at:**

**Full paper:** <https://arxiv.org/abs/2502.01575>

**Project Github:** <https://github.com/tomer1812/mistr>