Verification Learning: Make Unsupervised Neuro-Symbolic System Feasible

Lin-Han Jia, Wen-Chao Hu. Jie-Jing Shao, Lan-Zhe Guo, Yu-Feng Li.

National Key Laboratory for Novel Software Technology, Nanjing University, Nanjing 210023, China

Presented by: Lin-Han Jia
2025.05.23

From Supervised Reasoning to Unsupervised Verification

In the Nesy paradigm, a machine learning model f establishes a mapping between inputs X and symbolic representations S, i.e., S = f(X). Then, using a knowledge base KB and S, to infer the label Y , i.e., KB, S |= Y .

In real-world scenarios, labels Y generally does not exist and is more often part of the unknown symbolic sequence S. Therefore, in unsupervised Nesy, we must avoid relying on Y as a starting point for reasoning. Instead, candidates(S) is directly determined by the knowledge base KB, i.e., KB |= candidates(S). This introduces several challenges:

1. It prevents label leakage of Y into S;
2. It significantly increases the space of S;
3. More importantly, it leads to an overabundance of candidates(S) because of extreme shortcuts, making the process of traversing and scoring all candidate solutions extremely difficult.

Our goal now is to find the highest-scoring solution that can be verified, i.e., solving the COP problem COP(S, S, V_{KB}, Score). In the following, we explore how to solve the COP problem with minimal cost.

Theoretical Study and Experiments

The theoretical analysis of unsupervised learning can be divided into two parts:

1. The first part is whether the system has the ability to group samples belonging to the same category together. $\hat{R}(f) = \min_{\sigma \in G} \sum_{X \in X_{\text{train}}} [g(X)_i \neq \sigma(s_i)]$

2. The second part is whether the system can establish a correspondence between the categories identified by the learner and the labels of the symbols.

Theorem 6.1. *For any function $f \in \mathcal{F}$, if L is a ρ-Lipschitz continuous loss function, $\mathcal{R}_n$ is the Rademacher complexity for a sample size of n, R^{up}_{task} is the current task-induced upper bound on error, and $\hat{R}(f)$ is the current minimal symmetric permutation empirical error, then the empirical error $R(f)$ for the prediction of the current symbol set by f satisfies, with at least probability $1-\delta$:*

$$R(f) \leq \hat{R}(f) + 2\rho\mathcal{R}_n(F) + 3\sqrt{\frac{log(2/\delta)}{2n}} + R^{up}_{task} \quad (7)$$

$$R^{up}_{task} = \sum_{s_i \in \mathcal{S}} \mathbb{I}(s_i \notin Fix(G)) P_{s_i}$$

Table 1. The experiments on the dataset Addition

Method	2	3	4	5	6	7	8	9	10
Deepproblog	53.53	40.42	33.67	29.86	27.51	25.46	23.63	22.52	21.47
DeepStochlog	56.21	44.43	39.06	36.02	34.02	31.14	27.74	24.38	21.24
NeurASP	53.66	36.07	28.39	29.77	14.63	23.25	23.61	22.51	7.63
Ground ABL	42.50	42.00	98.25	26.00	30.25	25.25	25.75	24.00	20.25
WSABL	46.16	99.50	38.00	26.00	30.25	24.50	25.75	24.00	20.25
$VL_{\perp}$	**100.00**	41.38	99.50	99.80	99.65	99.19	70.70	98.73	98.28
$VL_{\perp}^{TTC}$	**100.00**	49.83	**100.00**	**100.00**	**100.00**	**100.00**	69.20	99.95	**100.00**
$VL_{\not\perp}$	**100.00**	99.88	99.75	**100.00**	99.75	99.00	99.25	97.75	48.80
$VL_{\not\perp}^{TTC}$	**100.00**	**100.00**	**100.00**	**100.00**	**100.00**	99.95	**100.00**	**100.00**	51.40

Table 3. The experiments on the dataset Chess.

Method	2	3	4	5	6
Deepproblog	49.46	33.10	25.08	49.46	15.66
NeurASP	53.66	36.07	23.93	19.07	18.82
Ground ABL	49.90	67.65	75.70	70.55	33.95
WSABL	**100.00**	99.80	75.95	38.30	34.35
$VL_{\perp}$	**100.00**	**99.90**	**98.00**	**95.70**	**95.15**
$VL_{\perp}^{TTC}$	**100.00**	**99.90**	92.55	91.30	92.55
$VL_{\not\perp}$	**100.00**	**99.90**	**98.00**	**95.70**	**95.15**
$VL_{\not\perp}^{TTC}$	**100.00**	**99.90**	92.55	91.30	92.55

Thanks!