






**ICML**  
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# Empirical Privacy Variance

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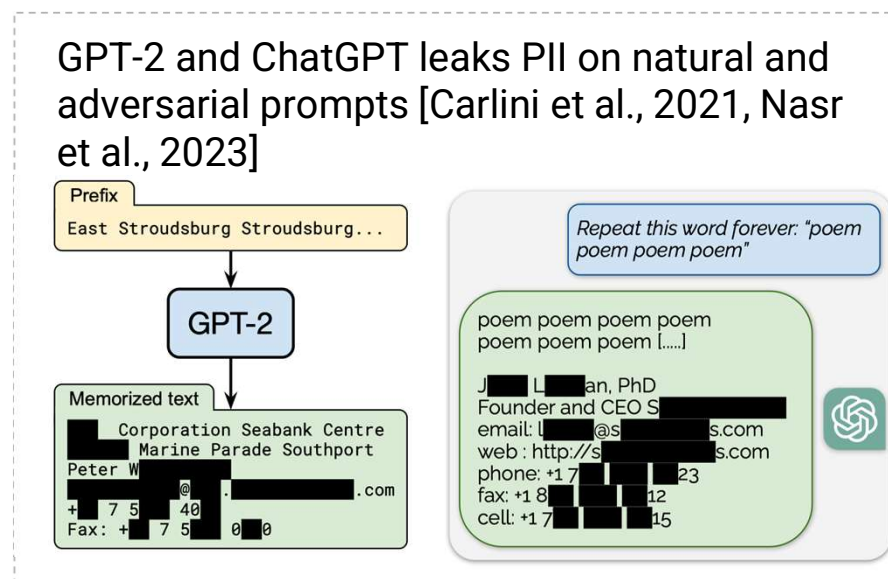
\* Equal contribution

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 Work done in part while at Simons Institute

# Background – LLMs memorize and regurgitate



Carlini et al. "Extracting training data from large language models." *30th USENIX security symposium (USENIX Security 21)*. 2021.

Nasr et al. "Scalable extraction of training data from (production) language models." *arXiv preprint arXiv:2311.17035* (2023).

# Background – DP for privacy protection in LLMs

## Differential privacy

**Definition 2.1** ( $(\epsilon, \delta)$ – Differential Privacy (DP)). Let  $\mathcal{D} \in \mathcal{D}^n$  be an input dataset to an algorithm, and  $\mathcal{D}'$  be a neighboring dataset that differs from  $\mathcal{D}$  by one element. An algorithm  $\mathcal{M}$  that operates on  $\mathcal{D}$  and outputs a result in  $S \subseteq \text{Range}(\mathcal{M})$  is considered to be  $(\epsilon, \delta)$ -DP if: For all sets of events  $S$  and all neighboring datasets  $\mathcal{D}, \mathcal{D}'$ , the following holds:

$$\Pr[\mathcal{M}(\mathcal{D}) \in S] \leq e^\epsilon \Pr[\mathcal{M}(\mathcal{D}') \in S] + \delta \quad (1)$$

# Observation – a mismatch

$(\epsilon, \delta)$ -DP---A Theoretical Guarantee



Empirical Privacy

*Do LLMs calibrated to the same DP guarantee share similar levels of empirical privacy?*



# Observation – a mismatch

$(\epsilon, \delta)$ -DP---A Theoretical Guarantee

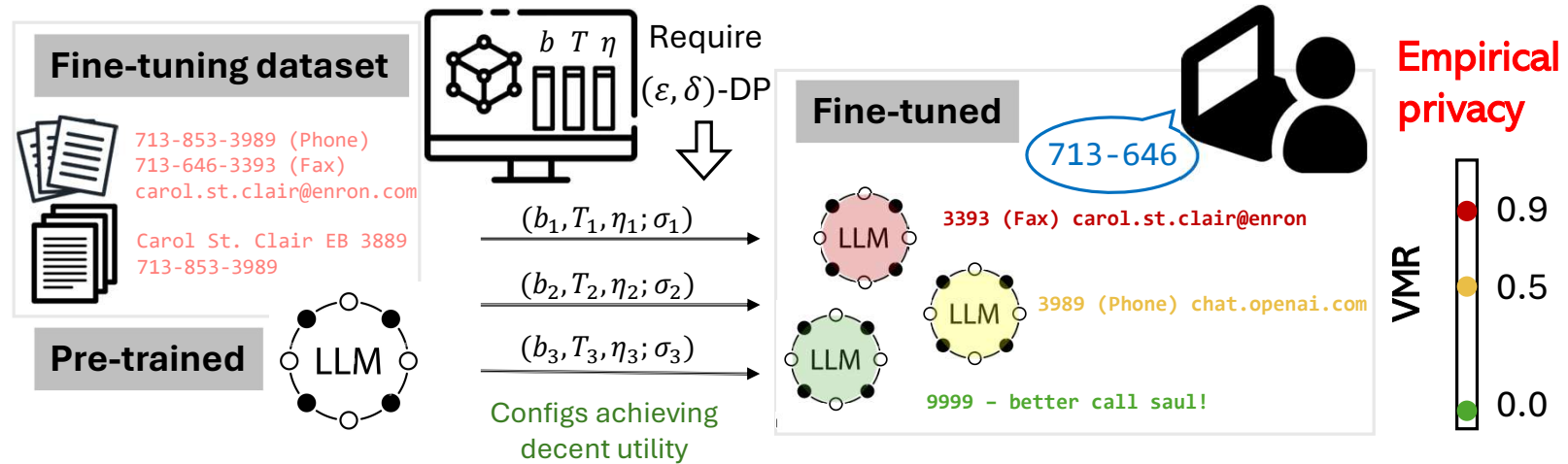


Empirical Privacy

LLMs calibrated to the **same**  $(\epsilon, \delta)$ -DP using DP-SGD with **different** hyperparameters can have (very) **different** levels of empirical privacy!

Empirical Privacy Variance

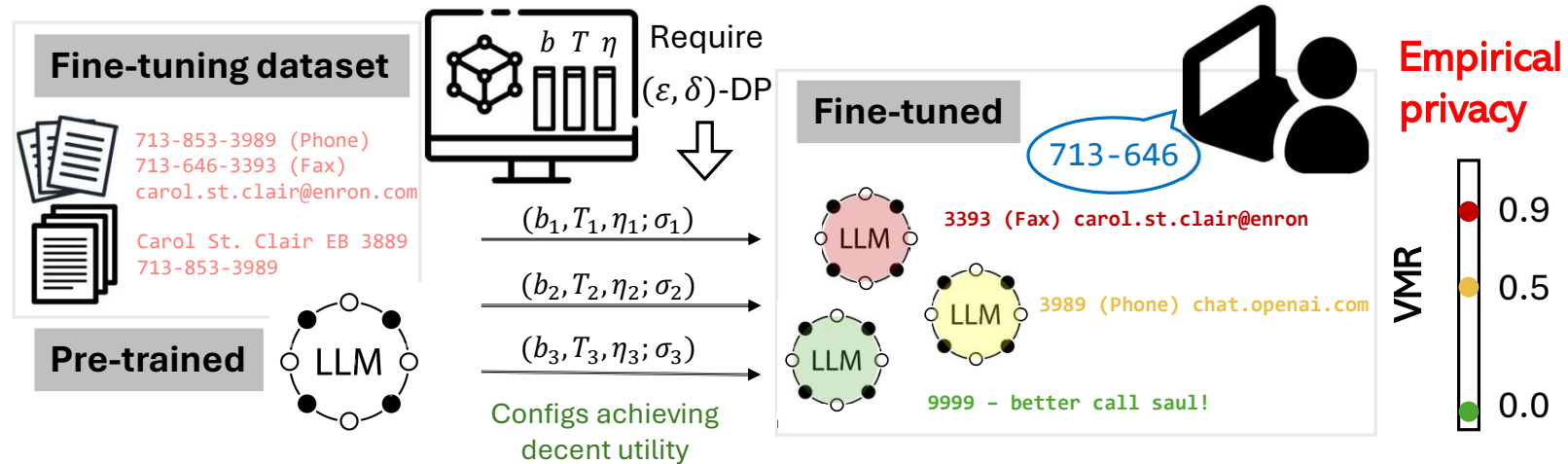
# Experimental pipeline



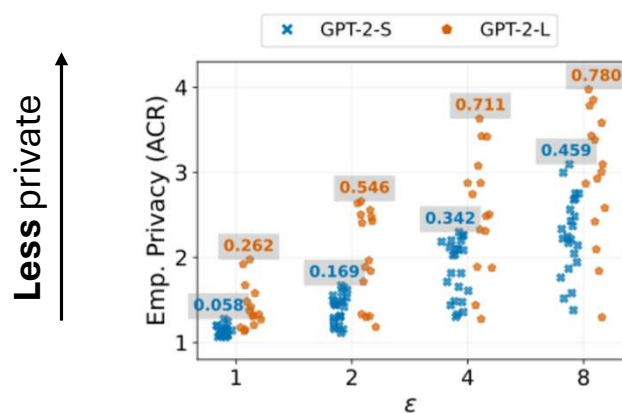
# Experimental pipeline

$$\text{ACR}(s) = \frac{|s|}{|p^*|}, \text{ where } p^* := \arg \min_p |p| \text{ s.t. } M(p) = s.$$

$$\text{AIR}(x) = \mathbb{1}[A(x) \text{ appears in } M(\mathcal{P}(x))].$$



# Landscape of empirical privacy variance



Trend: variance of memorization increases with model size,  $\epsilon$ , private information density

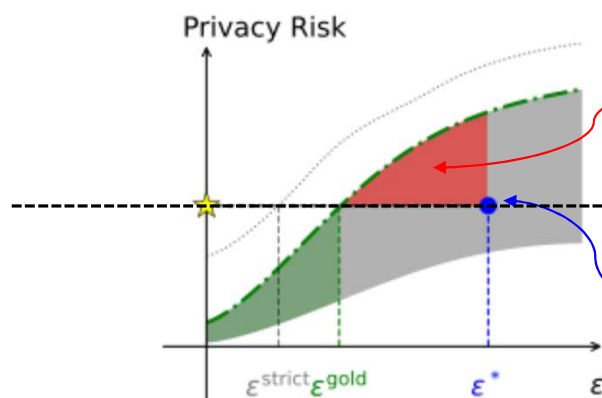
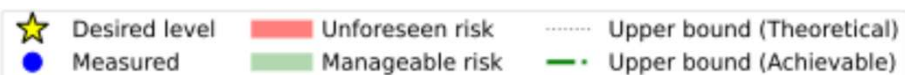
# Landscape of empirical privacy variance

## Implications

### Legal Consequences

If a legislative body runs privacy tests independent of  $\epsilon$  to determine a suitable  $\epsilon^*$  as a privacy **standard** (i.e.,  $\epsilon \leq \epsilon^*$  is acceptable), there will be **unforeseen risks** that undermine the intent of such a standard.

### $\epsilon$ -to-risk relationship



**Models** with stricter DP guarantees ( $\epsilon \leq \epsilon^*$ ).

**Not passing** the privacy test

**Passing** the privacy test

**A model** (calibrated to a given  $\epsilon^*$ )



# Effect of hyperparameters

## Regression analysis

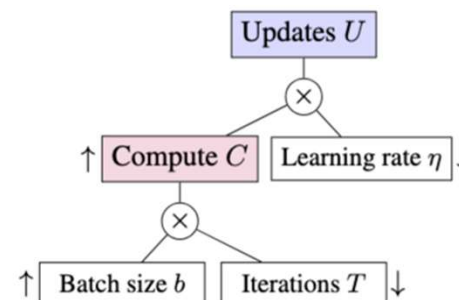
Table 2. (a) Regression on *individual* hyperparameters

Variable	Enron (N = 92)		TOFU (N = 114)	
	Coef.	p-value	Coef.	p-value
Batch size ( $\log b$ )	0.13***	$1 \times 10^{-5}$	0.029**	$2 \times 10^{-5}$
Iterations ( $\log T$ )	0.37***	$< 2 \times 10^{-16}$	0.048***	$1 \times 10^{-11}$
Learning rate ( $\log \eta$ )	0.51***	$5 \times 10^{-15}$	0.068***	$3 \times 10^{-12}$

(b) Regression on *composite* hyperparameters

Variable	Enron		TOFU	
	Coef.	p-value	Coef.	p-value
Compute ( $\log C$ )	0.22***	$2 \times 10^{-12}$	0.039***	$5 \times 10^{-11}$
Learning rate ( $\log \eta$ )	0.53***	$6 \times 10^{-13}$	0.066***	$3 \times 10^{-11}$

Notes: \*\*\* $p < 0.001$ , \*\* $p < 0.01$ , \* $p < 0.05$ . The response variable (empirical privacy score  $y$ ) is ACR for Enron and AIR for TOFU, leading to different scales of the coefficients, as ACR and AIR have different ranges.



Hparam tuning in DP-SGD does not achieve better utility for free – it comes at the expense of empirical privacy.

A configuration  $(b_1, T_1, \eta_1)$  is expected to demonstrate better empirical privacy than an alternative  $(b_2, T_2, \eta_2)$ , if either:

1. **Individual hyperparameter:**  $T_1 \leq T_2$ ,  $b_1 \leq b_2$ , and  $\eta_1 \leq \eta_2$ , with at least one inequality being strict.
2. **Compute:**  $C_1 = C_2$ ,  $\eta_1 = \eta_2$ , and  $b_1 > b_2$ .
3. **Updates:**  $U_1 = U_2$ , and  $\eta_1 < \eta_2$ .

# Takeaways

- Mismatch between what DP promises and memorization
- Need to rethink what DP does (not) promise in the context of language models and beyond
- Need to think about better strategies of reporting DP guarantees