

Renyi Neural Processes



Xuesong Wang

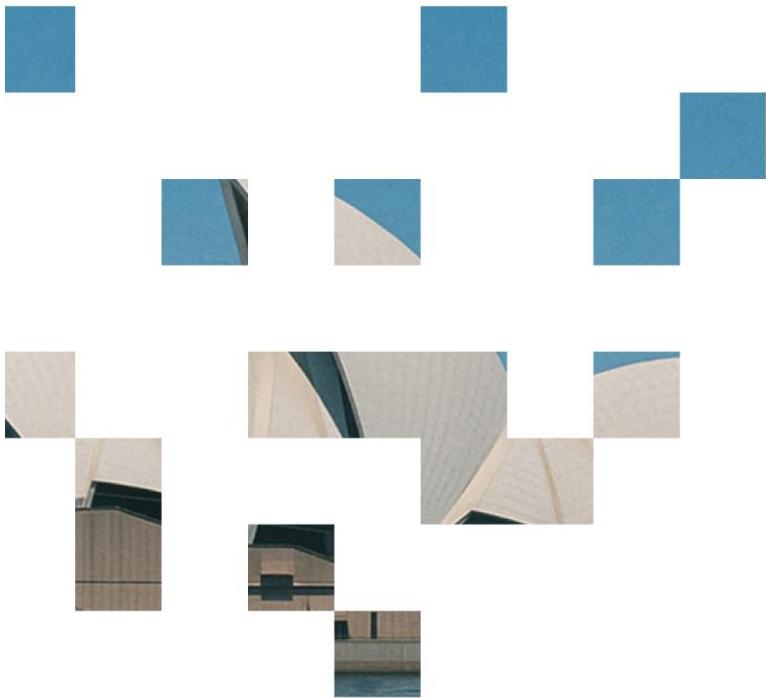


He Zhao

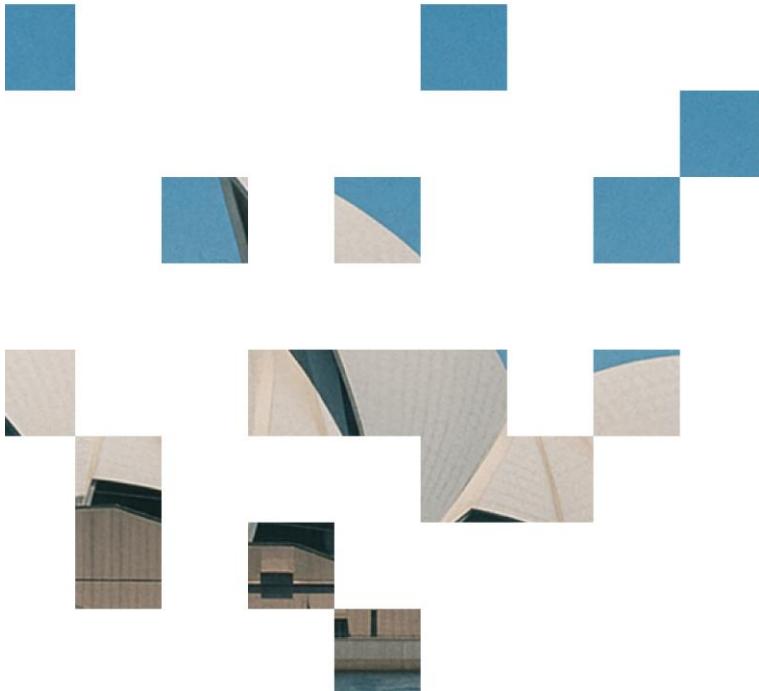


Edwin V. Bonilla

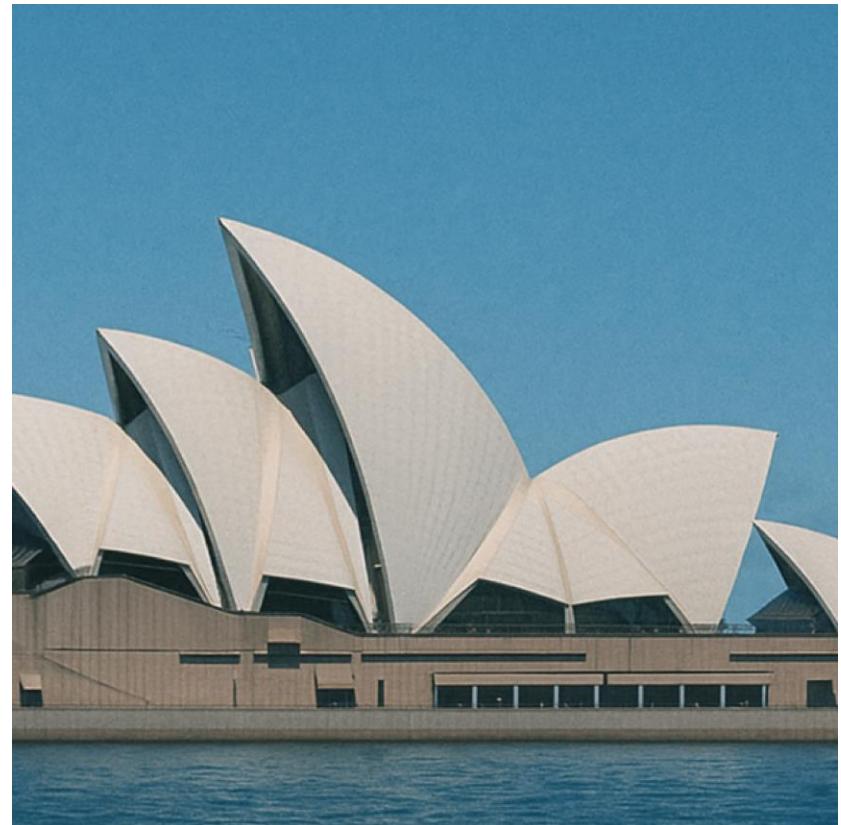
Where is this ?



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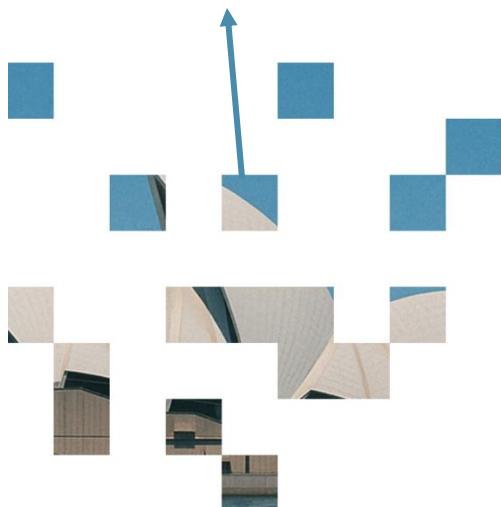
Sydney Opera House !





Neural Processes: a predictive model based on contexts

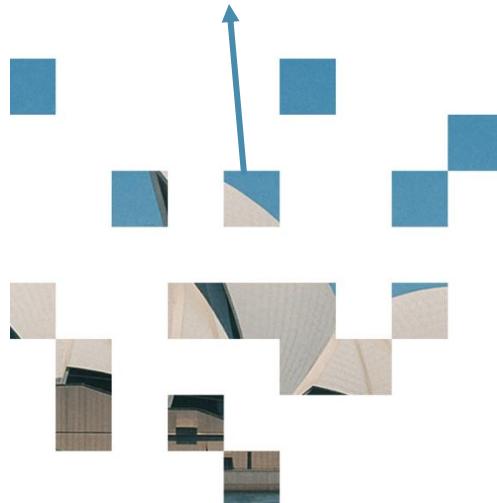
Context set
 $(x_C, y_C) \in C$



Neural Processes: a predictive model based on contexts

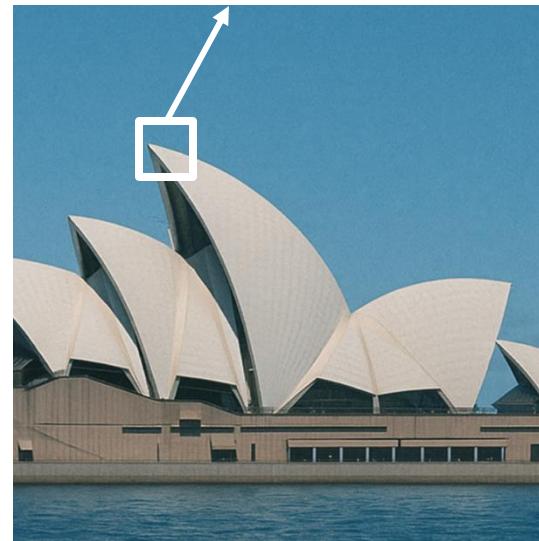
Context set

$$(x_C, y_C) \in C$$



Target set

$$(x_T, y_T) \in T$$



$$p(y_T | x_T, C) = ?$$



Why is this important ?

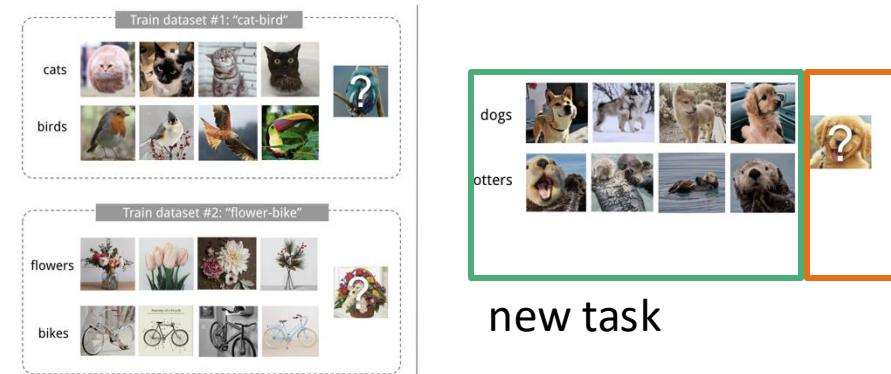
fast adaptation by conditioning on a new context set



Why is this important ?

fast adaptation by conditioning on a new context set

- Meta learning & multitask learning

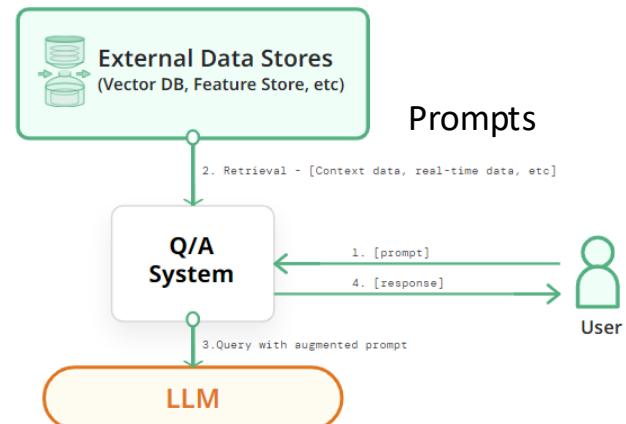




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- Meta learning & multitask learning
- Retrieval-augmented generation (RAG) for LLMs

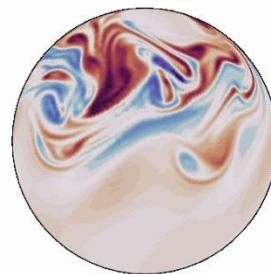




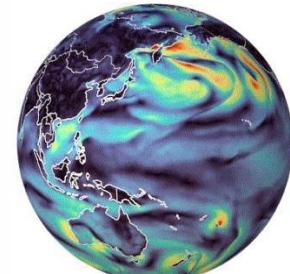
Why is this important ?

fast adaptation by conditioning on a new context set

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- **Sim2real in scientific computing**
- ...



simulation



physical

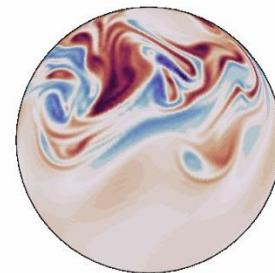


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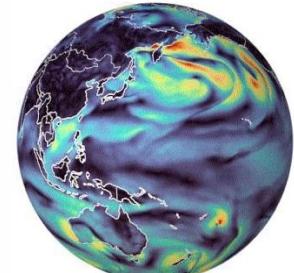
fast adaptation by conditioning on a new context set

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NPs empowers an interpretable and robust decision-making process.

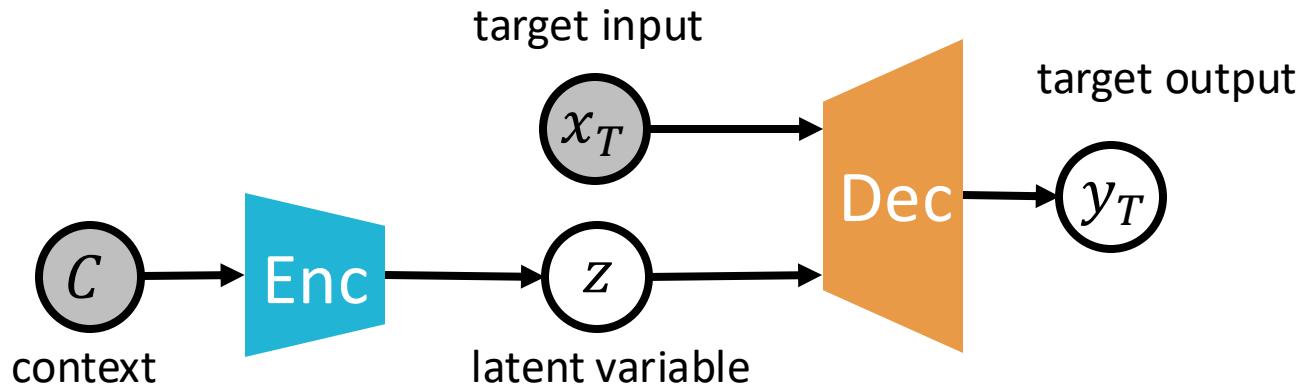


simulation

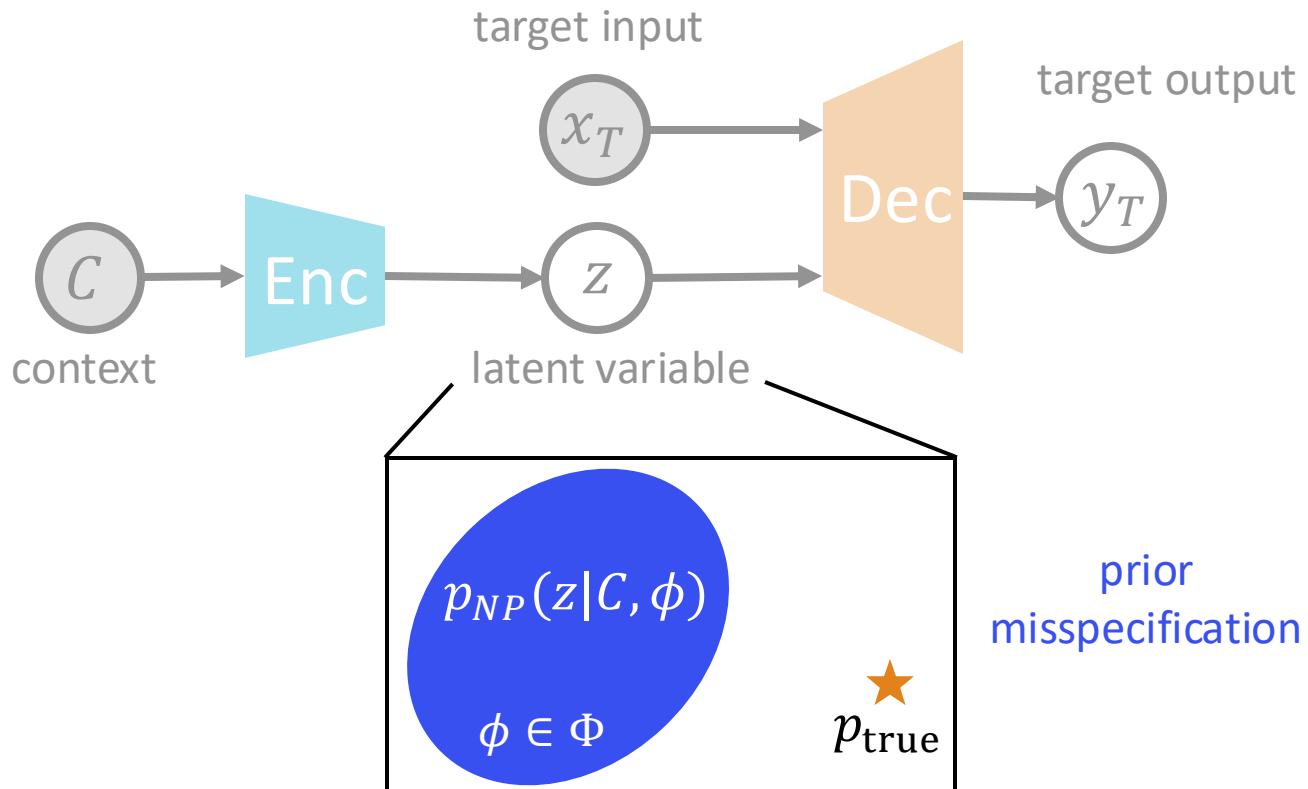


physical

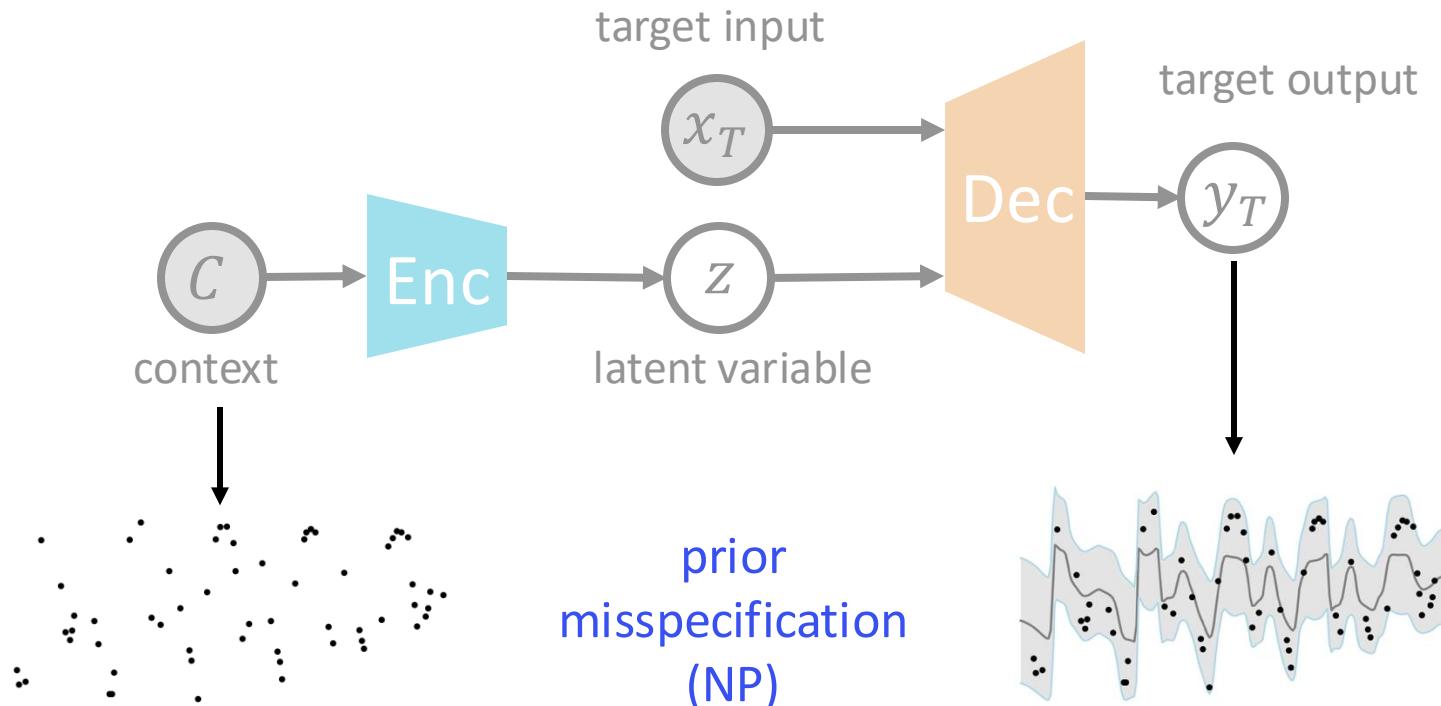
Challenge with encoding the context information & Prior misspecification



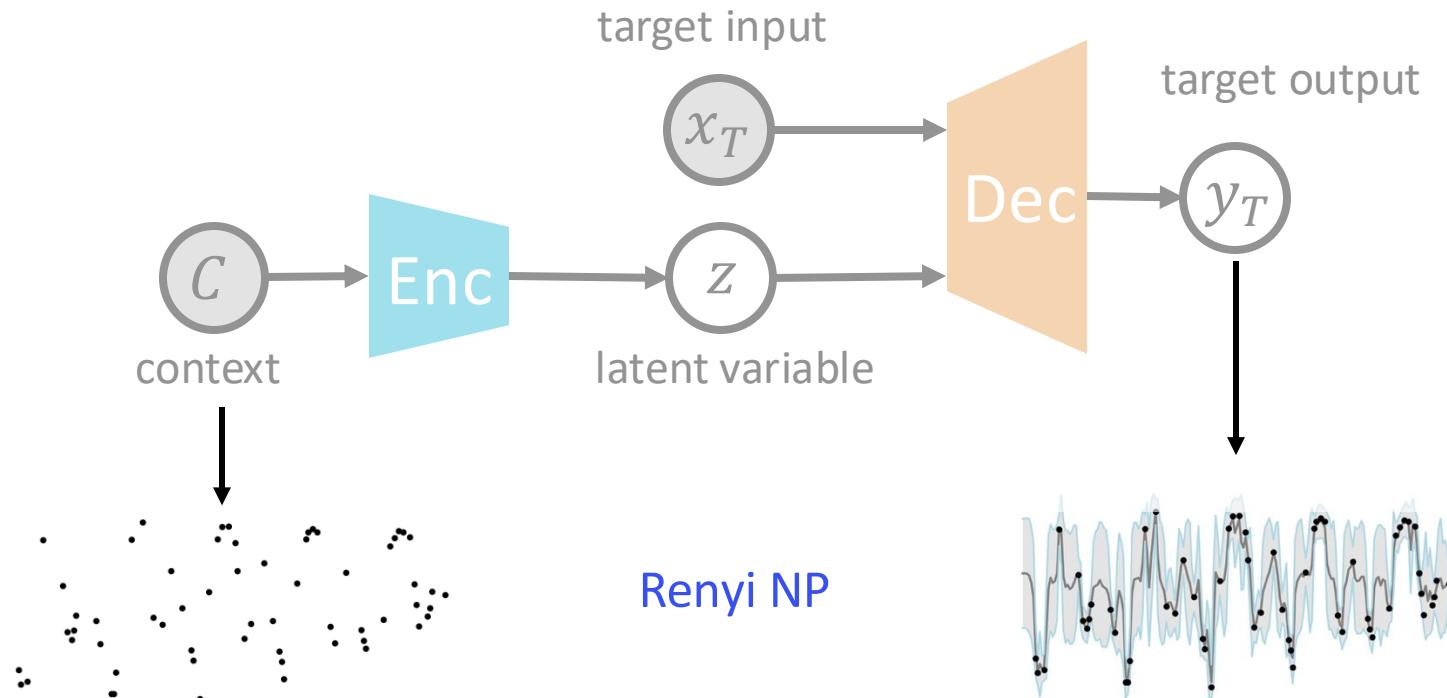
Challenge with encoding the context information & Prior misspecification



Challenge with encoding the context information & Prior misspecification



Challenge with encoding the context information & Prior misspecification





Cause for prior misspecification

$$-\mathcal{L}_{VI}(\theta, \phi) = \mathbb{E}_{q_\phi(\mathbf{z}|C, T)} \log p_\theta(Y_T | X_T, \mathbf{z}) - D_{KL}(q_\phi(\mathbf{z}|C, T) || p(\mathbf{z}|C))$$



Cause for prior misspecification

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Likelihood Posterior Prior

The equation shows the variational lower bound (ELBO) for a variational autoencoder (VAE). The first term is the likelihood, which is the expected log probability of the observed data Y_T given the latent variable \mathbf{z} and parameters θ . The second term is the Kullback-Leibler divergence (D_{KL}) between the approximate posterior distribution $q_\phi(\mathbf{z}|C, T)$ and the true prior distribution $p(\mathbf{z}|C)$. The terms are color-coded: "Likelihood" is grey, "Posterior" is purple, and "Prior" is orange, with corresponding colored horizontal lines underneath the respective terms.



Cause for prior misspecification

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Likelihood Posterior Prior
 ~~$p(\mathbf{z}|C)$~~

approximated prior
(parameter coupling)



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$$-\mathcal{L}_{VI}(\theta, \phi) = \mathbb{E}_{q_\phi(\mathbf{z}|C, T)} \log p_\theta(Y_T | X_T, \mathbf{z}) - D_{KL}(q_\phi(\mathbf{z}|C, T) || p(\mathbf{z}|C))$$

Likelihood Posterior Prior
 $\cancel{q_\phi(\mathbf{z}|C)}$
approximated prior
(parameter coupling)

Prior misspecification: $q_\phi(\mathbf{z}|C) \neq p(\mathbf{z}|C), \forall \phi \in \Phi$



What if we don't have to trust this approximated prior?



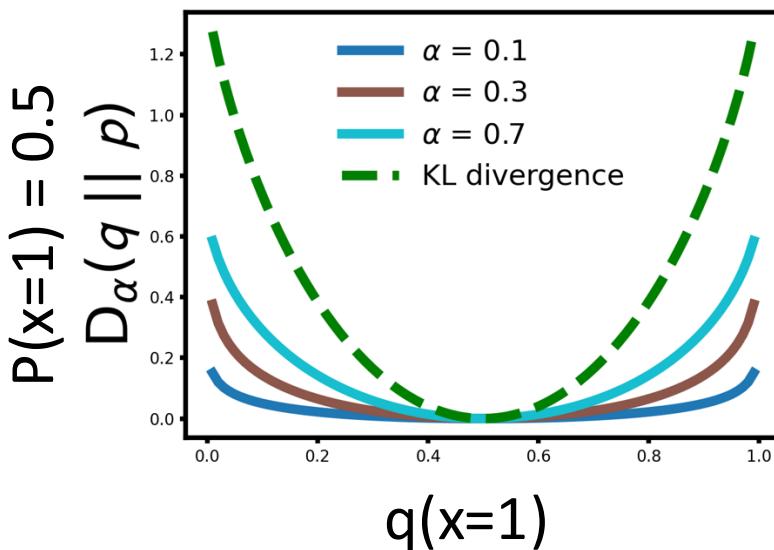
Renyi divergence (1961):

$$D_\alpha(q(\mathbf{z}) \parallel p(\mathbf{z})) = \frac{1}{\alpha - 1} \log \int q(\mathbf{z})^\alpha p(\mathbf{z})^{1-\alpha} d\mathbf{z}$$

- $D_\alpha \rightarrow D_{KL}$ as $\alpha \rightarrow 1$

Renyi divergence (1961):

$$D_\alpha(q(\mathbf{z}) || p(\mathbf{z})) = \frac{1}{\alpha - 1} \log \int q(\mathbf{z})^\alpha p(\mathbf{z})^{1-\alpha} d\mathbf{z}$$



- Smaller α , less trust on prior
- Larger α , more trust on prior



Renyi neural processes (RNP)

$$-\mathcal{L}_{RNP}(\theta, \phi) = \frac{1}{1 - \alpha} \log \mathbb{E}_{q_\phi(\mathbf{z}|C, T)} \left[\frac{p_\theta(Y_T | X_T, \mathbf{z}) q_\phi(\mathbf{z}|C)}{q_\phi(\mathbf{z}|C, T)} \right]^{1 - \alpha}$$



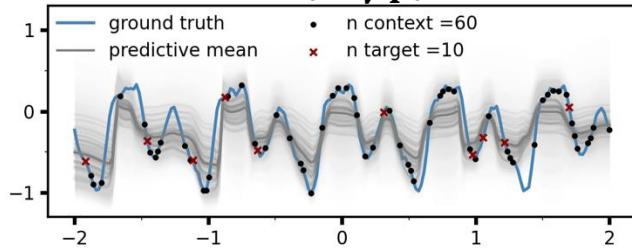
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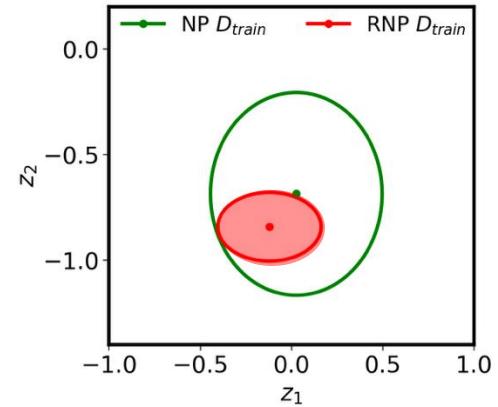
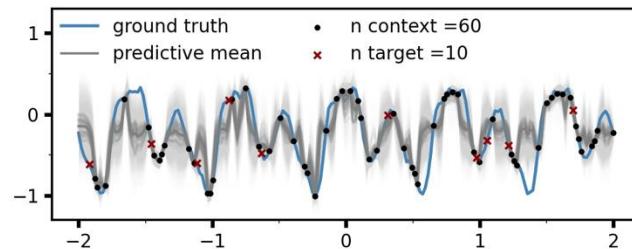
- $\alpha = 0$, $\mathcal{L}_{RNP} = \mathcal{L}_{ML}$ (maximizing likelihood estimation)
- $\alpha \rightarrow 1$, $\mathcal{L}_{RNP} = \mathcal{L}_{VI}$ (variational inference)
- $\alpha \in (0, 1)$, RNP mitigates the prior misspecification via α

Vanilla NP vs RNP on a GP-regression benchmark

NP (\mathcal{L}_{VI})



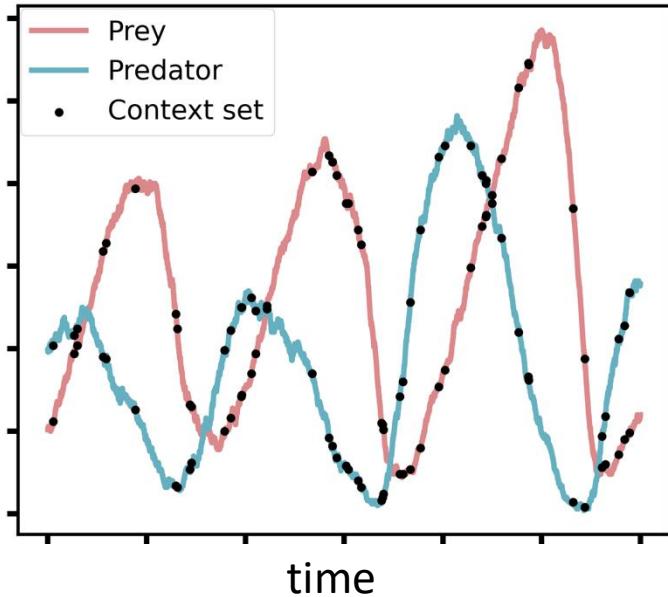
RNP ($\alpha = 0.7$)



Better variance
estimation

Simulation to real-world example

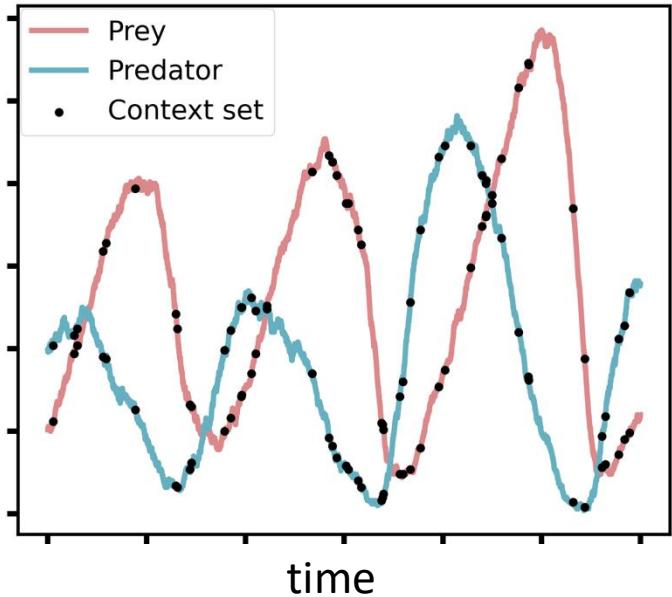
population



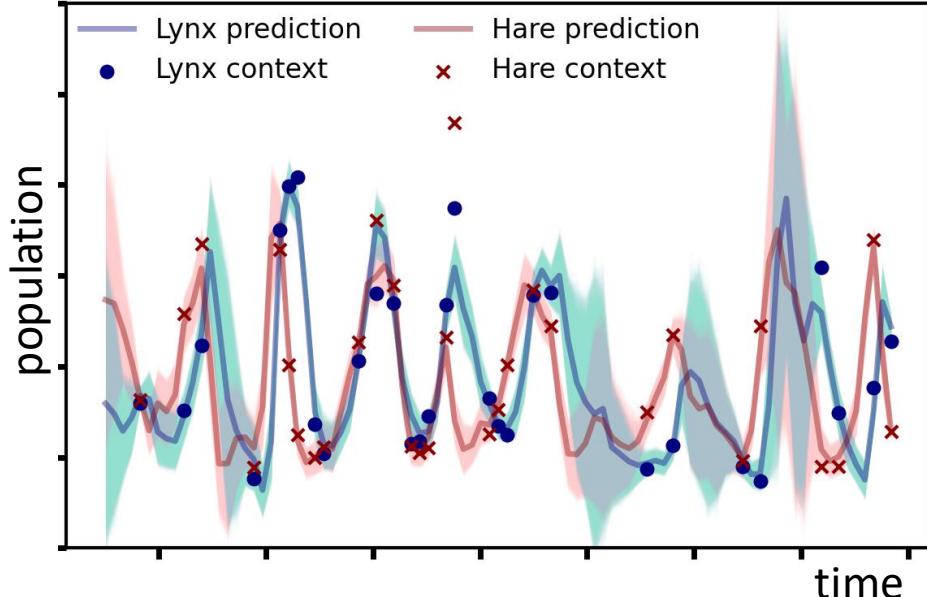
Sim: Prey-predator data D_{train}

Simulation to real-world example

population



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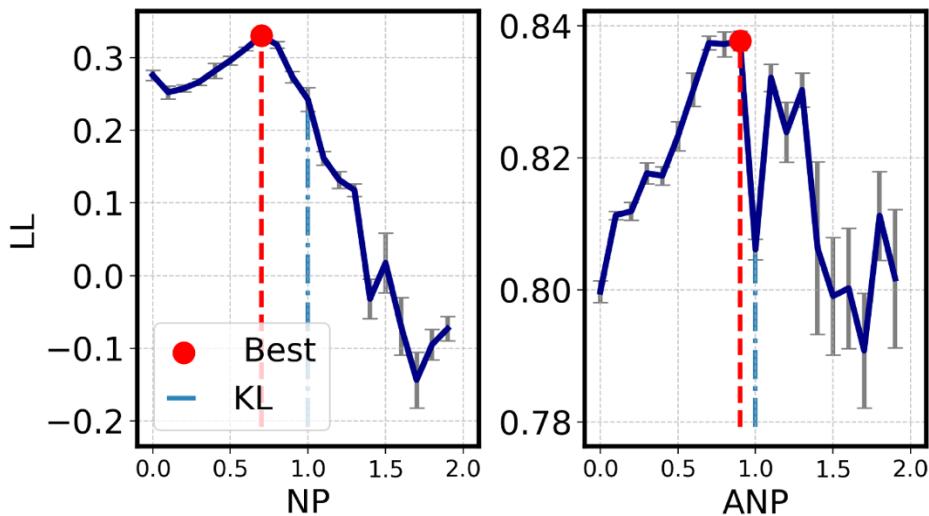
Real: Hare - lynx data D_{test}

$$LL_{RNP} = -3.63 \pm 0.09 \uparrow$$

$$LL_{NP} = -4.44 \pm 0.41$$

α tuning

GP regression



- $\alpha = 0.7$ empirically generalizes well
- Optimal α can be found using cross-validation
- Heuristic: annealing α from 1 to 0

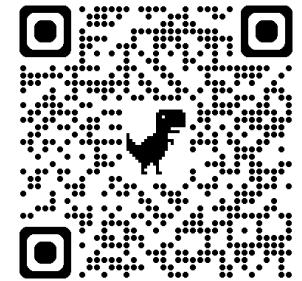


Renyi neural processes:

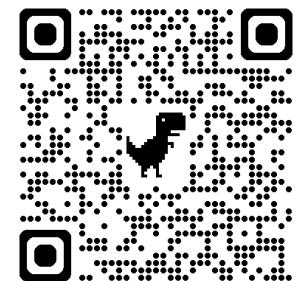
Our contributions:

- Identification of prior misspecification in neural processes
- Renyi divergence to mitigate prior misspecification
- Unification of two NP objectives with RNP

Paper



Github



Poster session 5 (ID 43943) (11 a.m. — 1:30 p.m. PDT)

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