



## Gamma Distribution PCA-Enhanced Feature Learning for Angle-Robust SAR Target Recognition

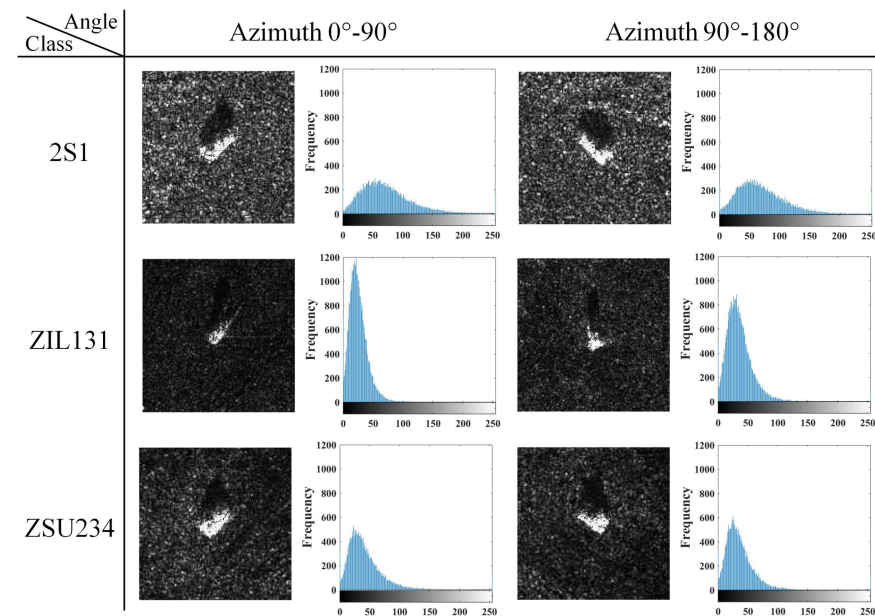
**Chong Zhang, Peng Zhang\*, Mengke Li\***



Reported by Chong Zhang



# Introduction



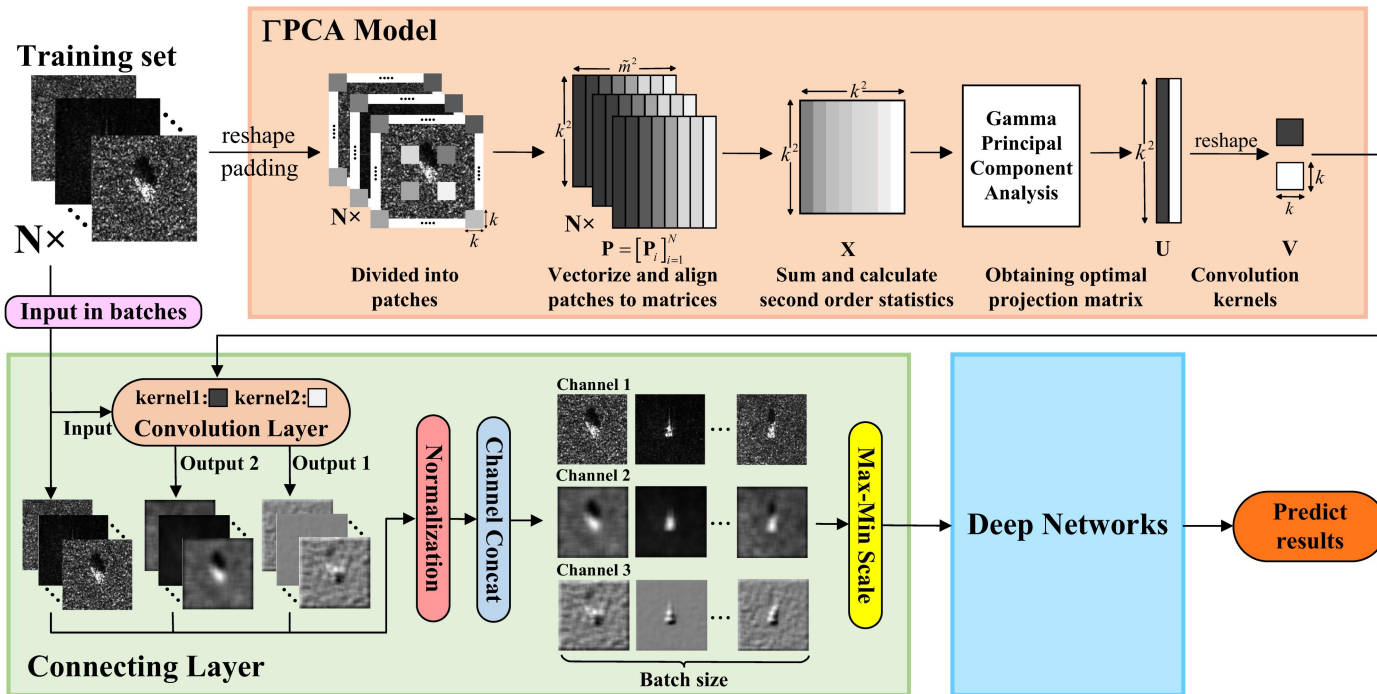
## Problem:

The imaging angle of synthetic aperture radar (SAR) significantly impact the scattering characteristics of targets, thereby angle-inadequate training samples leads to poor robustness of deep networks.

## Motivation:

- Fully consider the unique statistical characteristics of SAR data and capture the angle-invariant feature to alleviate deep model's sensitivity to angle variations.
- Simple, effective and readily compatible, so as to ensure its greater applicability.

## Model Framework



## Gamma Principal Component Analysis

Gamma distribution  
modeling



Objective function of  
GPCA



Optimization by MM  
algorithm

- Probability density function of Gamma distribution

$$f(x|\theta, \varphi) = \exp \left\{ \frac{x\theta + \log(-\theta)}{\varphi^2} + \frac{1}{\varphi^2} \log \frac{1}{\varphi^2} - \log \Gamma \left( \frac{1}{\varphi^2} \right) + \left( \frac{1}{\varphi^2} - 1 \right) \log x \right\}$$

- Optimization objective function



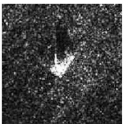
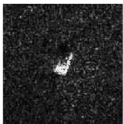
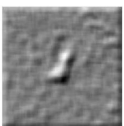
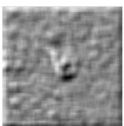
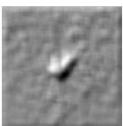
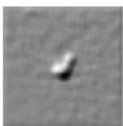
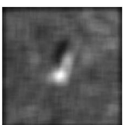
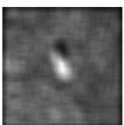
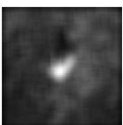
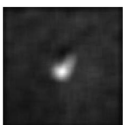
$$\max_{\substack{\boldsymbol{\mu} \in \mathbb{R}^d \\ \mathbf{U}^T \mathbf{U} = \mathbf{I}_k}} \sum_{i,j} \left\{ x_{ij} \left( [\mathbf{U} \mathbf{U}^T (\tilde{\boldsymbol{\eta}}_i(\theta) + \boldsymbol{\mu})]_j - \mu_j \right)^{-1} - \log \left( \mu_j - [\mathbf{U} \mathbf{U}^T (\tilde{\boldsymbol{\eta}}_i(\theta) + \boldsymbol{\mu})]_j \right) \right\}$$

- The modification of  $\mathbf{U}$  and  $\boldsymbol{\mu}$

$$\mathbf{U}^{(t+1)} = \arg \max_{(\mathbf{U})^T \mathbf{U} = \mathbf{I}_k} \text{tr} \left[ (\mathbf{U}^{(t)})^T \mathbf{F}^{(t)} (\mathbf{H}, \boldsymbol{\mu}^{(t)}, \mathbf{P}^{(t)}, \mathbf{Q}^{(t)}) \mathbf{U}^{(t)} \right]$$

$$\boldsymbol{\mu}^{(t+1)} = (\mathbf{1}_n^T \mathbf{Q}^{(t)} \mathbf{1}_n)^{-1} \left( \mathbf{H} \mathbf{U}^{(t)} (\mathbf{U}^{(t)})^T - \mathbf{P}^{(t)} \right)^T \mathbf{Q}^{(t)} \mathbf{1}_n$$

## ΓPCA Feature Extraction

Class Image	2S1 (Depression 15°)	2S1 (Depression 30°)	ZSU234 (Depression 15°)	ZSU234 (Depression 30°)
Original SAR image				
1 <sup>st</sup> Principal Component $\mathbf{o}_i^1$				
2 <sup>nd</sup> Principal Component $\mathbf{o}_i^2$				

Our proposed  $\Gamma$ PCA:

- Significantly alleviates the issue of angle sensitivity for SAR data.
- No additional computational overhead for network training.
- Easy to deploy into different networks.



# Experiments

Methods	A-R Test	AD-R Test	OA (%)
Resnet101	72.57±0.25	43.95±0.01	63.62±0.14
EfficientNet-B3	67.22±0.02	45.15±0.07	60.27±0.01
ViT-B/16	78.12±0.04	60.11±0.24	72.45±0.04
Swin-B	61.58±0.03	78.01±0.04	66.75±0.02
VisionLSTM-T	55.25±0.21	76.54±0.24	61.95±0.22
VisionMamba-T	62.37±0.06	66.67±0.03	63.68±0.04
MSNet-PIHA	68.08±0.03	55.75±0.06	64.20±0.03
WTConvNeXt-T	72.55±0.01	57.69±0.05	67.87±0.01
ΓPCA-Resnet (Ours)	73.79±0.02	52.96±0.05	<b>67.23±0.01</b>
ΓPCA-ViT (Ours)	78.48±0.01	70.67±0.16	<b>76.02±0.01</b>

Training set: Depression 17 Deg, Azimuth 0-90 Deg;  
Testing set: Depression 15 Deg & 30 Deg, Full-Azimuth.

Methods	A-R Test	AD-R Test	OA (%)
Resnet101	75.30±0.01	48.12±0.68	66.74±0.11
EfficientNet-B3	69.24±0.09	45.69±0.07	61.83±0.07
ViT-B/16	79.92±0.04	58.56±0.12	73.19±0.01
Swin-B	70.17±0.08	65.53±0.05	68.71±0.06
VisionLSTM	55.49±0.13	74.03±1.26	61.33±0.16
VisionMamba-T	66.21±0.02	72.98±0.02	68.34±0.01
MSNet-PIHA	65.36±0.02	56.55±0.11	62.59±0.03
WTConvNeXt-T	76.46±0.07	63.29±0.51	72.32±0.16
ΓPCA-Resnet (Ours)	71.34±0.04	66.21±0.02	<b>69.72±0.03</b>
ΓPCA-ViT (Ours)	80.15±0.01	64.48±0.05	<b>75.22±0.01</b>

Training set: Depression 17 Deg, Azimuth 180-270 Deg;  
Testing set: Depression 15 Deg & 30 Deg, Full-Azimuth.

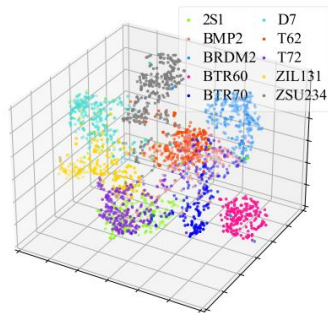
Comparison results on MSTAR dataset



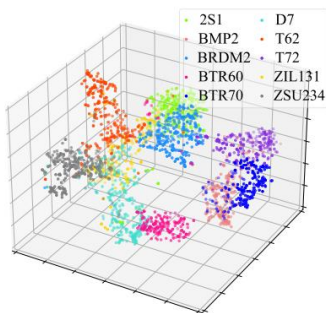
# Experiments

Methods	Overall Acc. (%)
Resnet101	97.33±0.02
$\Gamma$ PCA+Resnet101	<b>98.67±0.01</b>
ViT-B/16	97.05±0.03
$\Gamma$ PCA+ViT-B/16	<b>98.12±0.02</b>
Swin-B	97.41±0.01
$\Gamma$ PCA+Swin-B	<b>98.32±0.02</b>

- Comparison results on SAR-AIRcraft-1.0

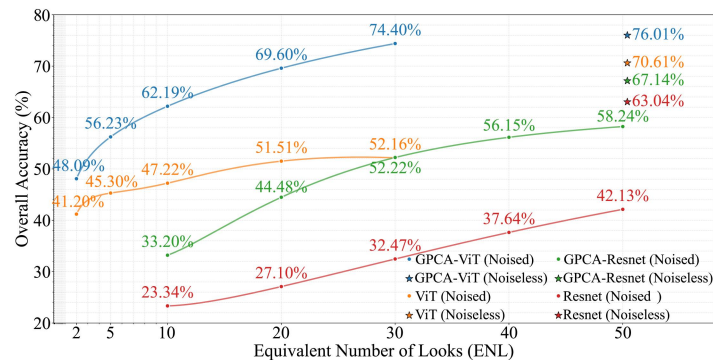


(a) ViT-B/16

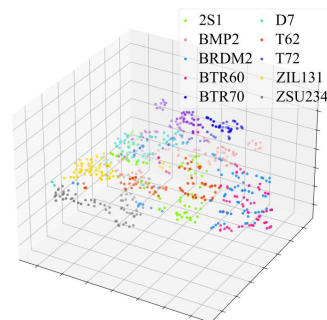


(b)  $\Gamma$ PCA-ViT

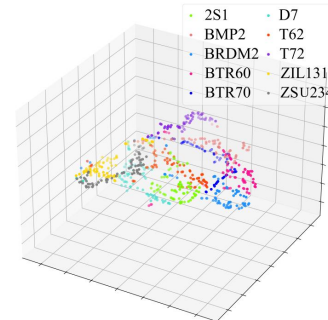
- ViT-B/16 vs.  $\Gamma$ PCA-ViT for feature extraction



- Anti-noise performance comparison



(a) Standard PCA



(b)  $\Gamma$ PCA

- Standard PCA vs.  $\Gamma$ PCA for feature extraction



# Conclusion

## Pros:

Simple yet effective and readily compatible:

- Significantly enhance the model's robustness to angle variations in the SAR ATR task;
- Can be seamlessly integrated into various deep models without introducing additional parameters.

## Cons:

Appropriate hyperparameters tuning:

- For different datasets, the size of the optimal mapping matrix and the fusion approaches between principal components influence final performance.





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# Thanks



- Code: <https://github.com/ChGrey/GammaPCA>
- Contact: pzhang@xidian.edu.cn; csmengkeli@gmail.com

