Emergent Symbolic Mechanisms Support Abstract Reasoning in Large Language Models

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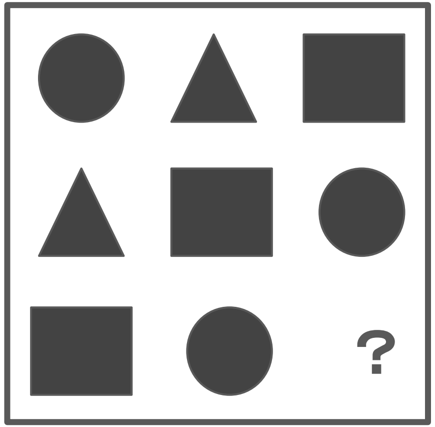
https://github.com/yukang123/LLMSymbMech

Introduction



Raven's Progressive Matrix

Abstract Reasoning

- Large Language Models (LLMs) have shown impressive performance on various human reasoning tasks including abstract (analogical) reasoning[*].
- Some studies questioned the robustness of LLMs' reasoning abilities[^].

structured human-like symbolic processing Mechanism?

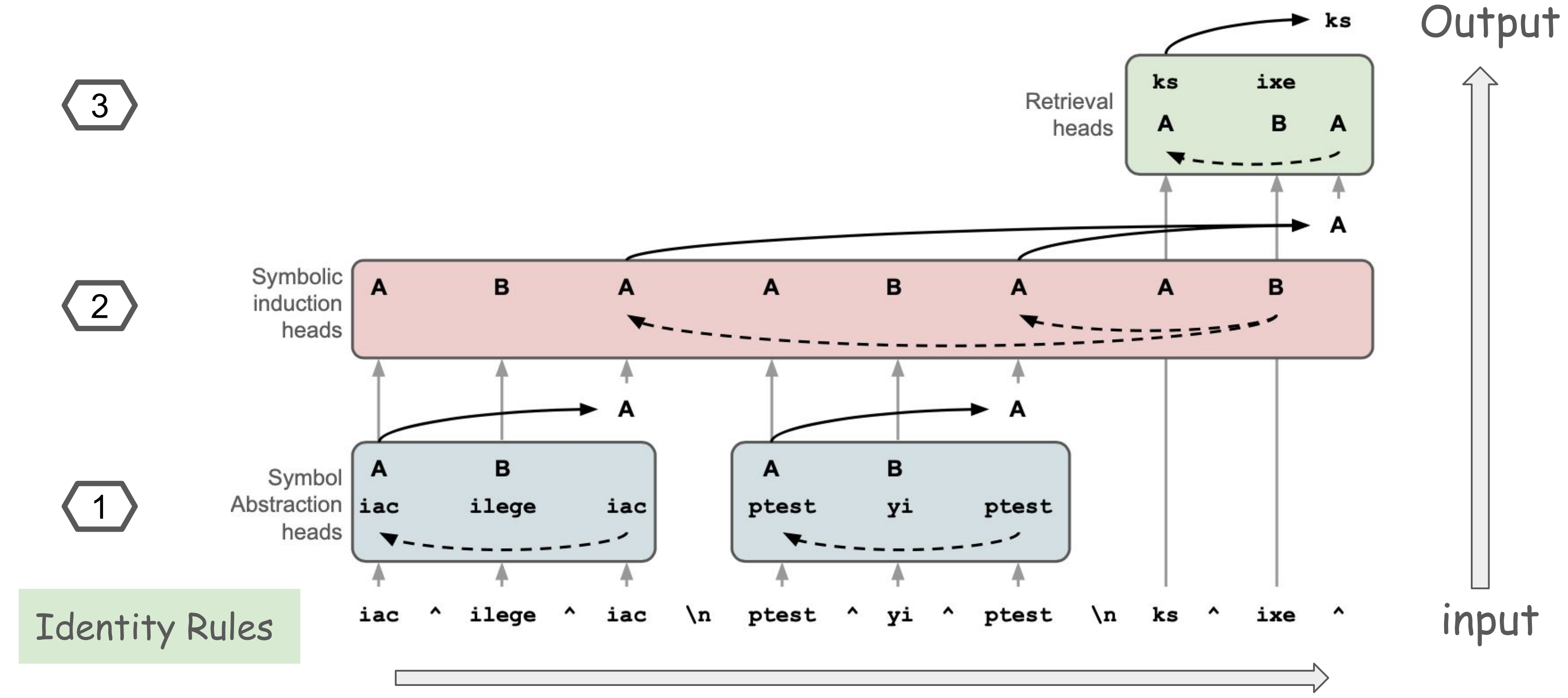
OR

Statistically approximating training data ?

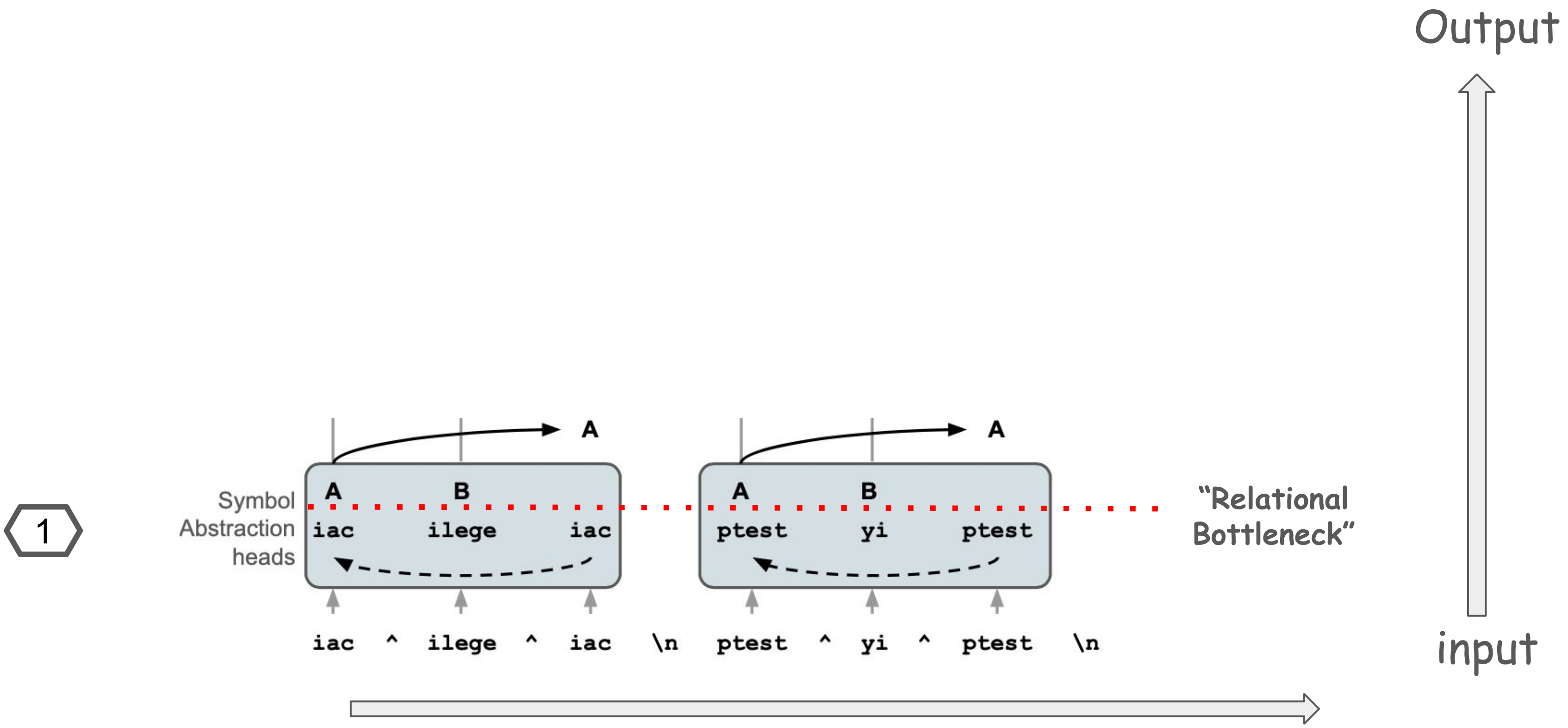
[*] Taylor, Webb, et al,. "Emergent analogical reasoning in large language models." *Nature Human Behaviour* 7.9 (2023): 1526-1541.

[^] Lewis, M. and Mitchell, M. Evaluating the robustness of analogical reasoning in large language models. arXiv:2411.14215, 2024.

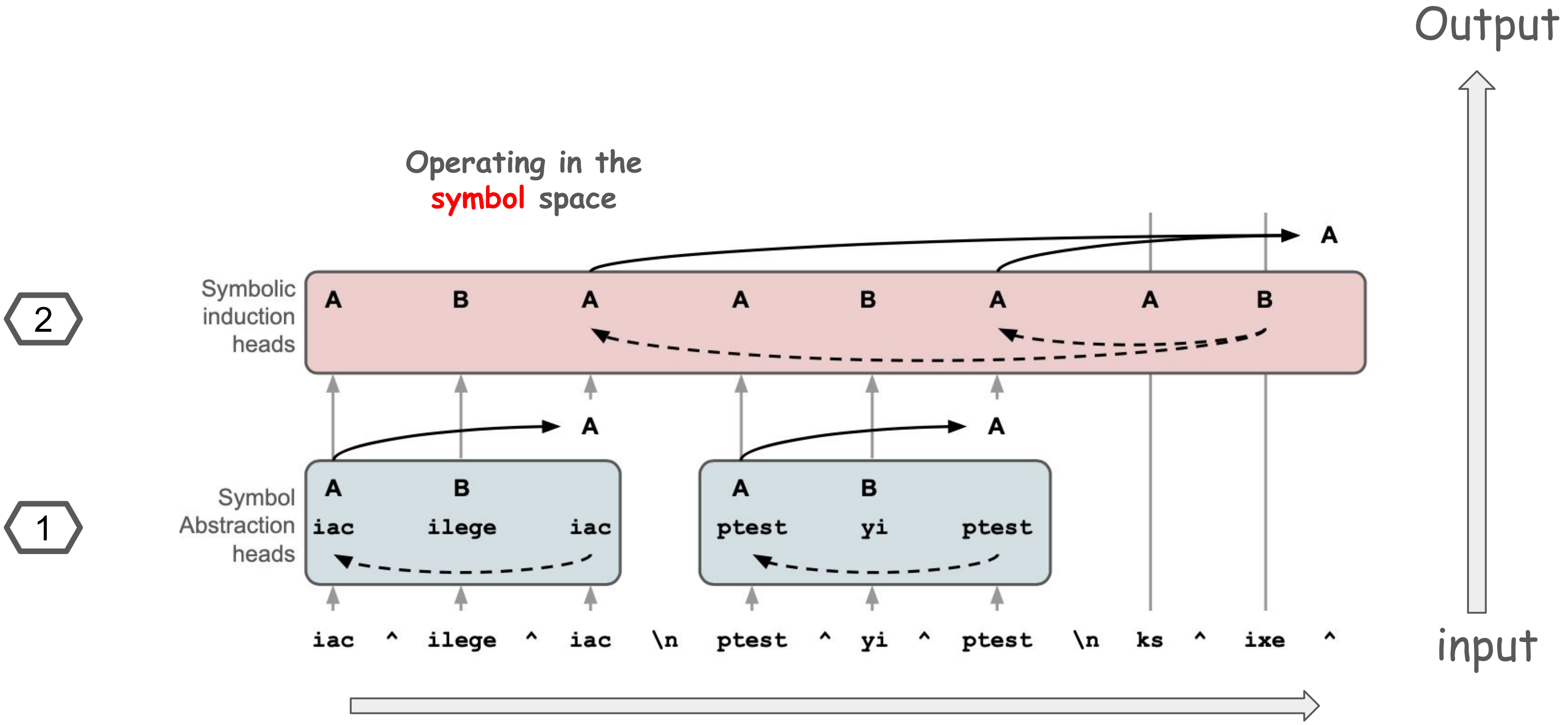
Our Findings : **Emergent** Symbolic Processing Mechanism in LLMs



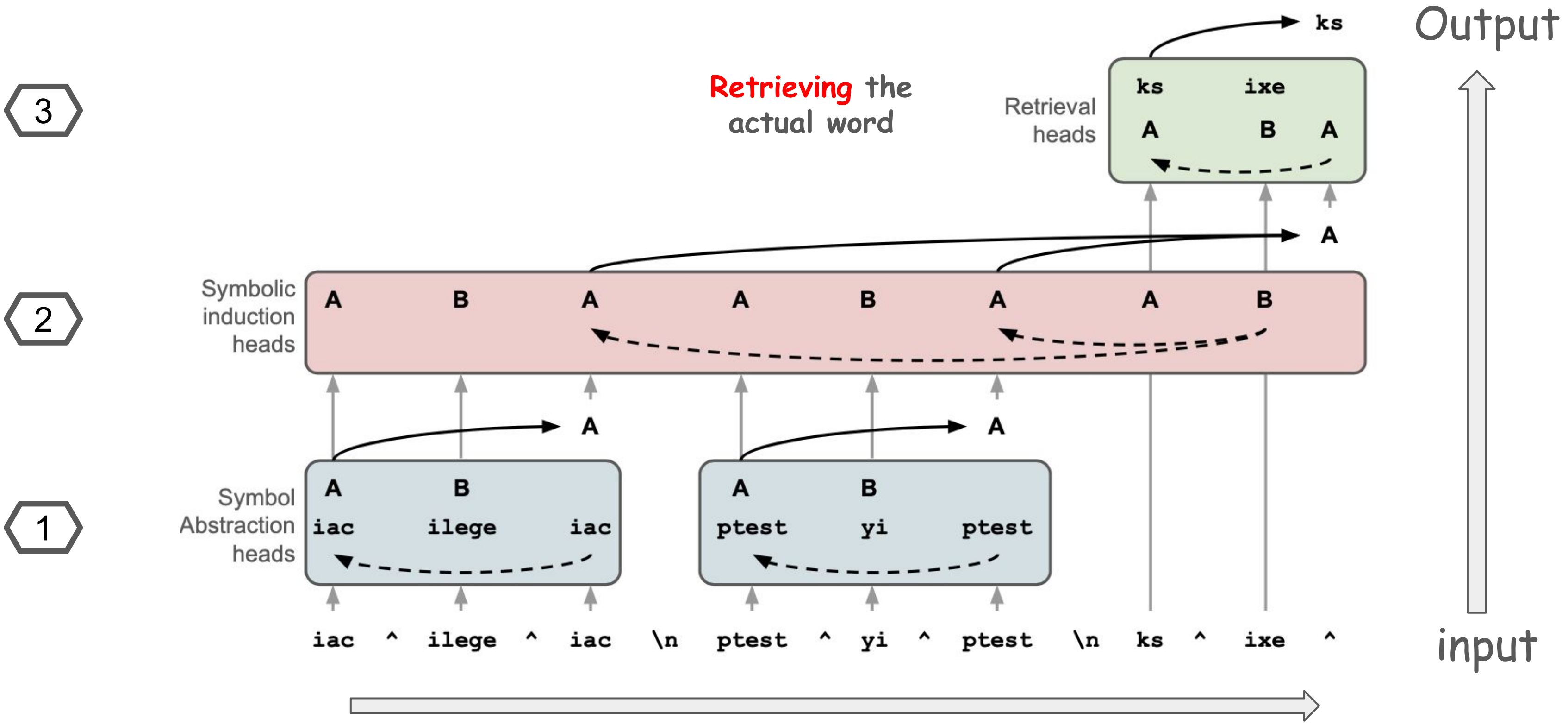
Emergent Symbolic Processing Mechanism in LLMs



Emergent Symbolic Processing Mechanism in LLMs



Emergent Symbolic Processing Mechanism in LLMs



Our Analyses: (please refer to the paper)

1. Causal Mediation Analyses (CMA)
2. Attention Analyses
3. Representation Similarity Analyses (RSA)
4. Ablation Studies
5. Comparison with Induction Heads and Function Vectors

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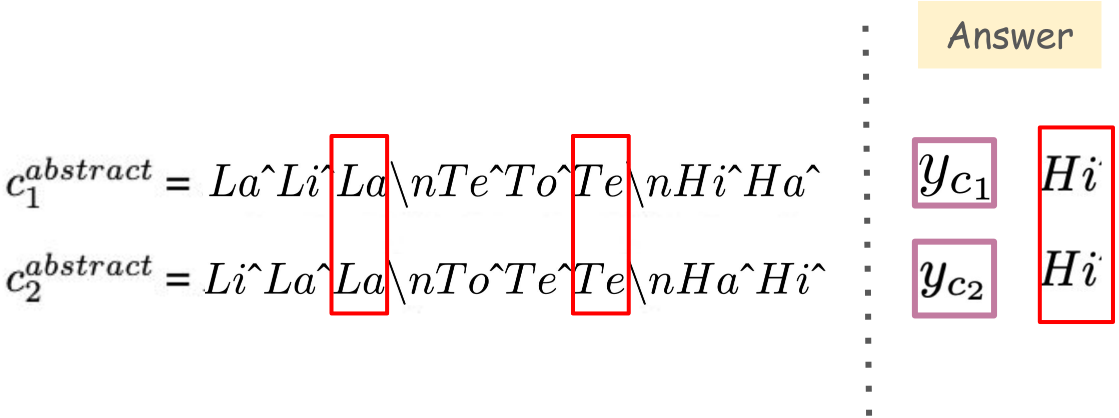
Causal Mediation Analyses (CMA)

1. Design a context pair (c_1, c_2) to isolate either abstract symbols or literal tokens

Abstract Context Pair : test whether the embedding represents **abstract symbols**

(used for *symbol abstraction heads* and *symbolic induction heads*)



Answer

$$c_1^{abstract} = La\hat{}Li\hat{}La\backslash nTe\hat{}To\hat{}Te\backslash nHi\hat{}Ha\hat{} \qquad y_{c_1} \quad Hi$$

$$c_2^{abstract} = Li\hat{}La\hat{}La\backslash nTo\hat{}Te\hat{}Te\backslash nHa\hat{}Hi\hat{} \qquad y_{c_2} \quad Hi$$

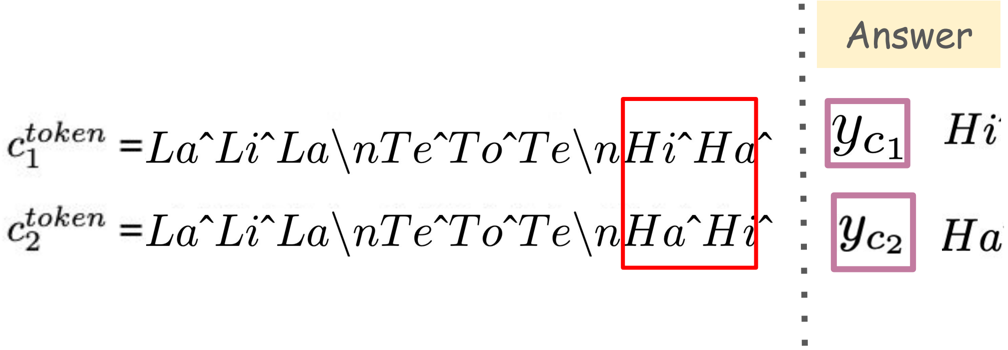
Same Token, Different Symbols/Rules (A vs B)

Causal Mediation Analyses (CMA)

1. Design a context pair (c_1, c_2) to isolate either abstract symbols or literal tokens

Token Context Pair : test whether the embedding represents **literal tokens**

(used for *Retrieval heads*)



$$c_1^{token} = La\hat{\ }Li\hat{\ }La\backslash nTe\hat{\ }To\hat{\ }Te\backslash n\boxed{Hi\hat{\ }Ha}\hat{\ }$$

$$c_2^{token} = La\hat{\ }Li\hat{\ }La\backslash nTe\hat{\ }To\hat{\ }Te\backslash n\boxed{Ha\hat{\ }Hi}\hat{\ }$$

Same Rule, Different Tokens

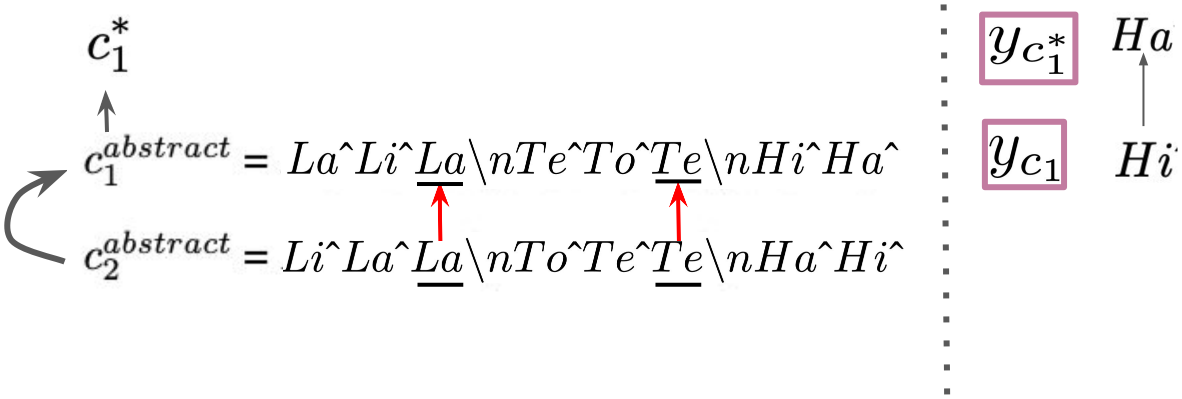
Causal Mediation Analyses

2. Activation Patching at certain token positions:

   Replace attention head outputs in context c_1 with the corresponding activations from c_2

symbol abstraction heads

Answer



c_1^*

$c_1^{abstract} = La\hat{\ }Li\hat{\ }\underline{La}\backslash nTe\hat{\ }To\hat{\ }\underline{Te}\backslash nHi\hat{\ }Ha\hat{\ }$

$c_2^{abstract} = Li\hat{\ }La\hat{\ }\underline{La}\backslash nTo\hat{\ }Te\hat{\ }\underline{Te}\backslash nHa\hat{\ }Hi\hat{\ }$

$\boxed{y_{c_1^*}}$ Ha

$\boxed{y_{c_1}}$ Hi

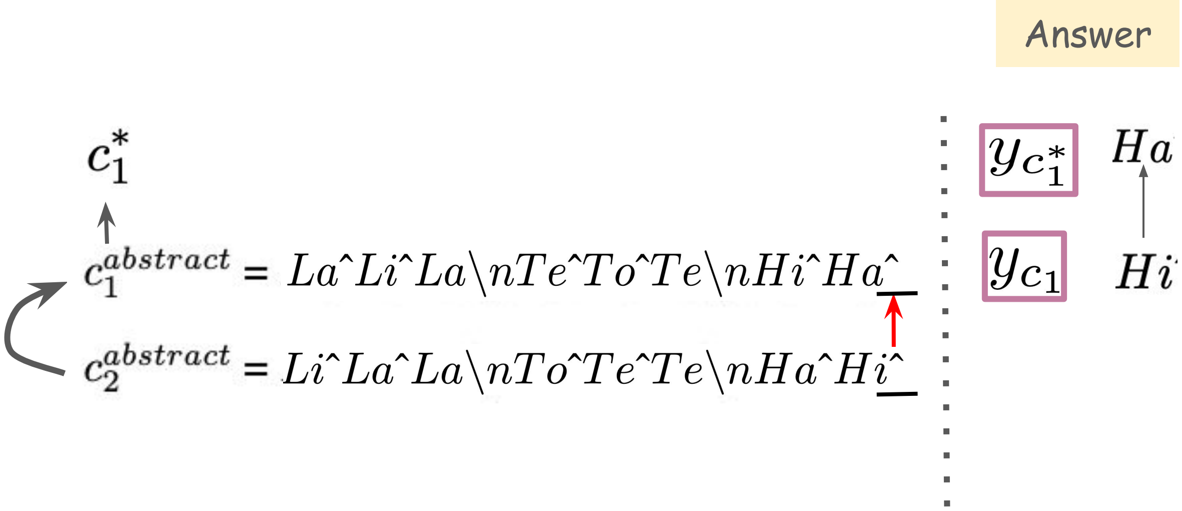
If activations represent the abstract variable (B in c_2) ……

Causal Mediation Analyses

2. Activation Patching at <u>certain token positions</u>:

Replace attention head outputs in context c_1 with the corresponding activations from c_2

symbolic induction heads


Answer
c_1^*
$c_1^{abstract}$ = La^Li^La\nTe^To^Te\nHi^Ha^
$c_2^{abstract}$ = Li^La^La\nTo^Te^Te\nHa^Hi^
$y_{c_1^*}$
y_{c_1}
Ha
Hi

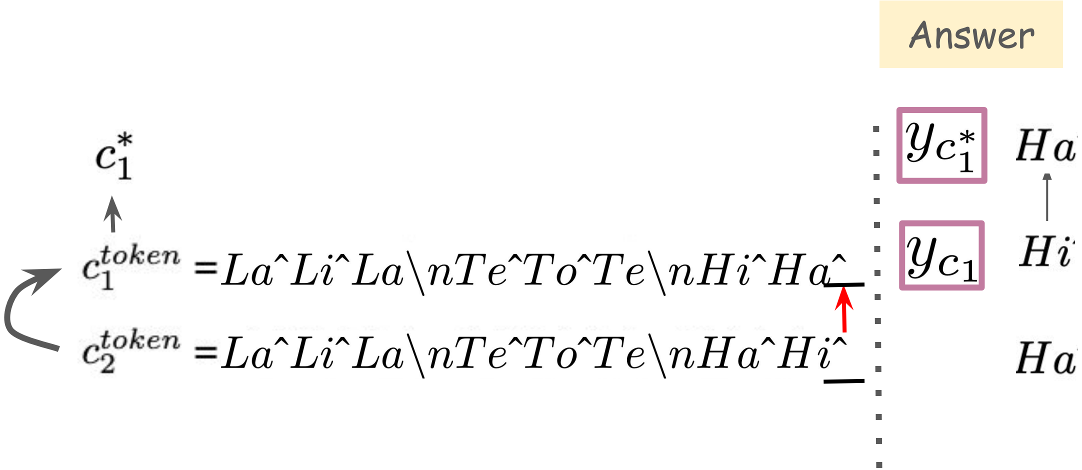
If activations represent the abstract variable (B in c_2) ……

Causal Mediation Analyses

2. Activation Patching at <u>certain token positions</u>:

Replace attention head outputs in context c_1 with the corresponding activations from c_2

Retrieval heads

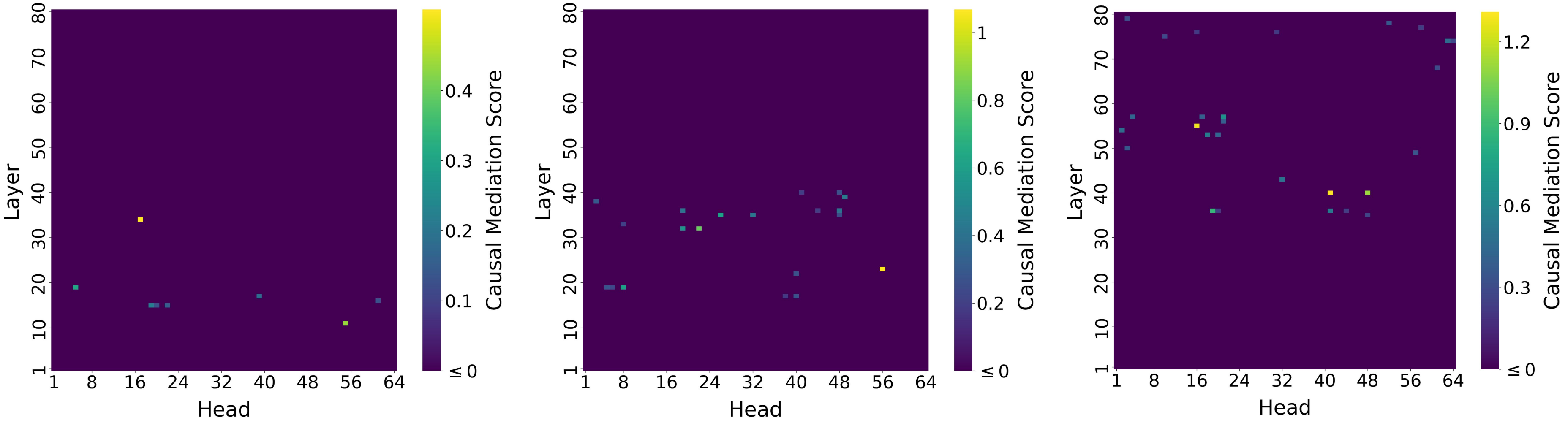

Answer
c_1^*
c_1^{token} = $La\hat{}Li\hat{}La\backslash nTe\hat{}To\hat{}Te\backslash nHi\hat{}Ha\hat{}$_
c_2^{token} = $La\hat{}Li\hat{}La\backslash nTe\hat{}To\hat{}Te\backslash nHa\hat{}Hi\hat{}$_
$y_{c_1^*}$
Ha
y_{c_1}
Hi
Ha

If activations represent the literal token for the answer (*Ha* in c_2) ……

Causal Mediation Analyses

3. Measure whether the causal effects on c_1 comply with the hypotheses

Llama-3.1 70B



Symbol Abstraction Head

Symbolic Induction Head

Retrieval Head

- Defined a score based on the changes in output logits for the answers as a measure of causal effects.
- Conducted multi-hypothesis permutation tests to select significant ones

Our Analyses: (please refer to the paper)

❖ Experiments with Llama-3.1 70B on identity rule tasks

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Our Analyses: (please refer to the paper)

- Evaluating **more LLMs** on identity rule tasks
  - Tested *13* models across *4* model families (GPT-2, Gemma-2, Qwen2.5, Llama-3.1)
  - Found similar robust symbolic mechanisms in *3* model families with GPT-2 as an exception
  - GPT-2 models showed low generation accuracy while symbol abstraction heads rarely emerged.
- Studying **more** complex abstract **reasoning tasks**
  - *Letter string analogies* and *verbal analogies*
  - Identified similar three-stage symbolic processing structures through CMA.
  - Different tasks may involve different attention heads to implement symbol processing.

Please Check out our paper!