# Landscaping Linear Mode Connectivity

Sidak Pal Singh, Linara Adilova, Michael Kamp, Asja Fischer, Bernhard Schölkopf, Thomas Hofmann



## How should the landscape be to show LMC (Frankle et al., 2019)?



Cross-sectional Sibling Landscape: Evolution of training error curves when forked at different epochs (0,1,2,15)

### Background

- Create two copies a network after some point in training and train them in an exactly similar manner, except for the random seed
- Frankle et, al., 2019 showed that:

### A Metaphor for LMC

"Imagine going down a mountain top with a friend in foggy weather. If you lose them towards the end, they are gonna be a short walk at the base. But, if near the start, they might actually end up on the other side of



when forked in the early phase the child networks are separated by a barrier; but otherwise are connected by a low-loss linear path

#### the mountain"

**Measuring Loss Barrier**  
The loss barrier 
$$\mathscr{B}(\alpha) = \mathscr{L}\left((1-\alpha)\theta_1^* + \alpha\theta_2^*\right) - (1-\alpha)\mathscr{L}(\theta_1^*) - \alpha\mathscr{L}(\theta_2^*)$$
  
when linearly interpolating  $\theta_1^*$  and  $\theta_2^*$  with weights  $1 - \alpha$  and  $\alpha$  is

$$\mathscr{B}(\alpha) \approx \frac{\alpha(1-\alpha)}{2} (\Delta\theta^*)^{\mathsf{T}} \Big( \alpha \nabla_{\theta}^2 \mathscr{L}(\theta_1^*) + (1-\alpha) \nabla_{\theta}^2 \mathscr{L}(\theta_2^*) \Big) \quad \Delta\theta^*$$

Qualitatively captures the empirically trend of LMC and layer wise LMC



#### Retrospective on how barriers arise



Product of local Landscapes  $\prod_{i=1}^{m} (\theta - \theta_i^*)^2$ 



Interplay of numerous nearby solutions and degenerate landscape geometry  $\begin{array}{c} 80 \\ 70 \\ 10 \\ 20 \\ 20 \\ 10 \\ 20 \\ 30 \\ 20 \\ 30 \\ 40 \\ 50 \\ 60 \end{array} \right)$ 

Final angle traced between model copies

Forking Epoch

#### during the course of training