Local Causal Structure Learning in the Presence of Latent Variables

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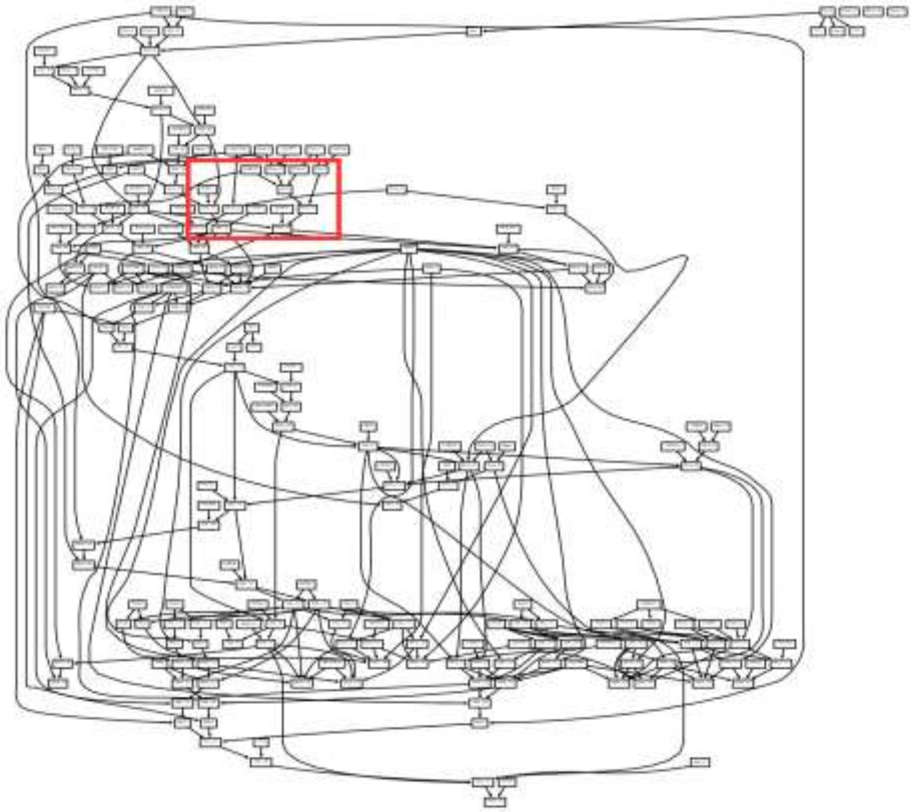
Background

	1	2	…	n
$\mathbf{V_1}$	V_{11}	V_{12}	…	V_{1n}
$\mathbf{V_2}$	V_{21}	V_{22}	⋮	V_{2n}
$\mathbf{V_3}$	V_{31}	V_{32}	…	V_{3n}
⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮
$\mathbf{V_t}$	V_{t1}	V_{t2}	…	V_{tn}
⋮	⋮	⋮	…	⋮
⋮	⋮	⋮	⋮	⋮
$\mathbf{V_q}$	V_{q1}	V_{q2}	…	V_{qn}

Observational dataset

Causal Discovery



Causal Graph

Open Problem:
How to learn parents and children of a target from observational data that may include latent variables?

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	⋮	⋮	⋮	⋮	⋮
Target	$\mathbf{V_t}$	V_{t1}	V_{t2}	…	V_{tn}
	⋮	⋮	⋮	…	⋮
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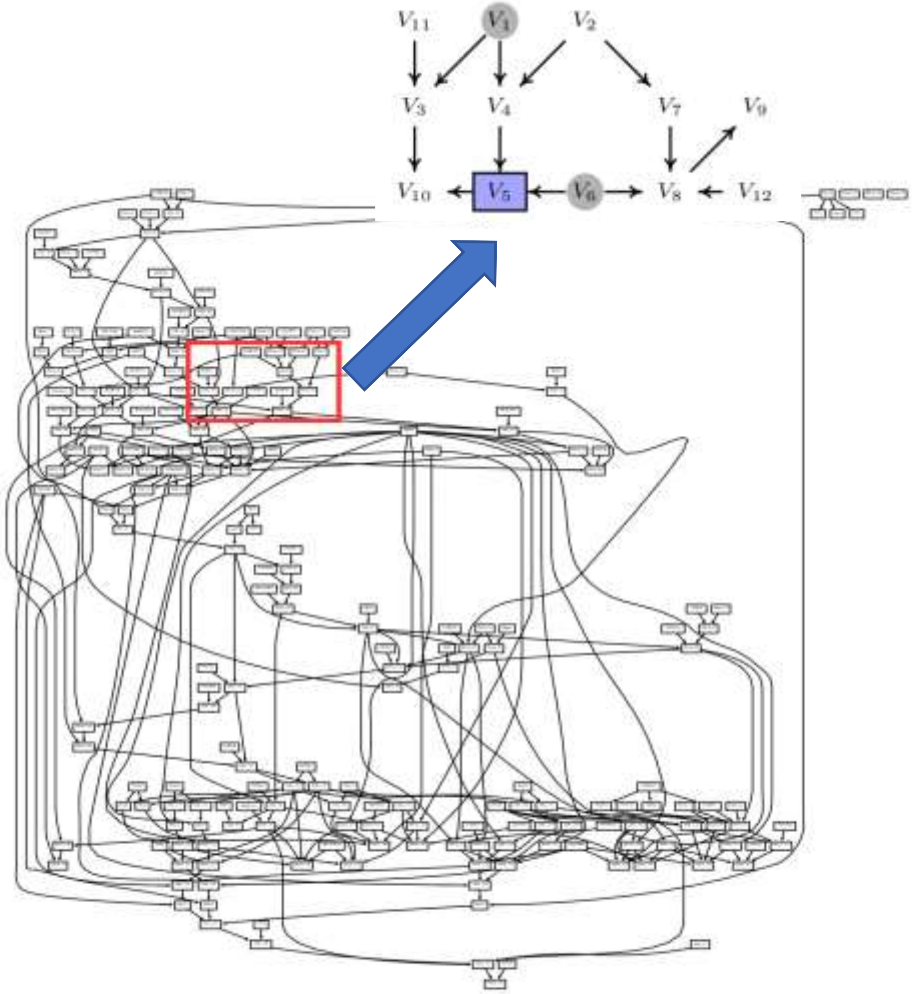

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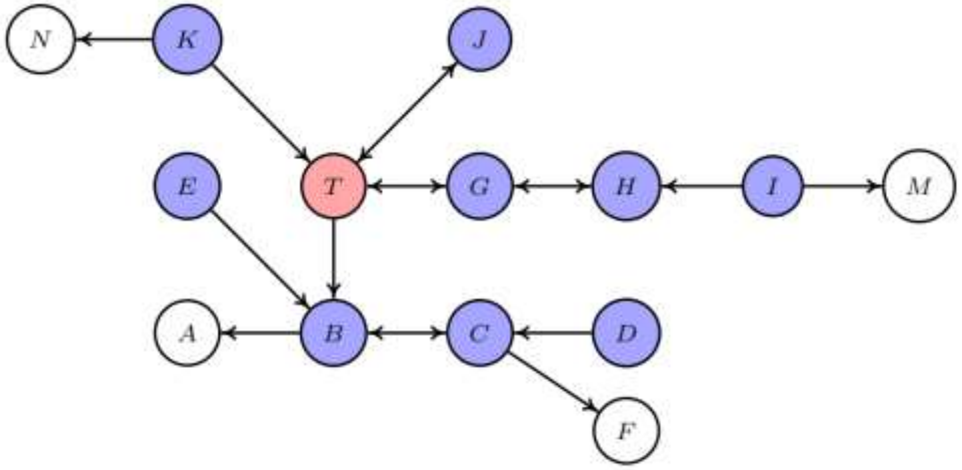
Main Concept

Markov Blanket

In a MAG, the Markov blanket of a vertice T, noted as MMB(T), consists of

(1) Parents of T; Children of T;

(2) A set of variables that for $\forall\, V_i$ within the set, V_i is not adjacent to T and has a collider path to T.



The illustrative example for MMB, where T is the target of interest and the blue nodes belong to $MMB(T)$.

Standard Assumption: Causal Markov condition; Causal Faithfulness condition.

No selection bias.

Basic Idea

(1) Finding MMB and learning the local structure over MMB;

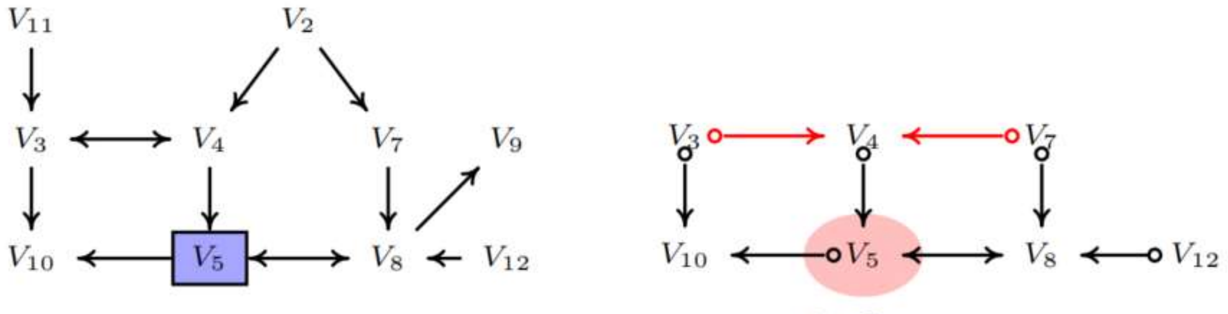
(2) Saving the **correct** information in the local structure;

(3) Repeating (1) and (2), **until** we get the local structure around T equivalent to the structure identified through global learning methods.

How to obtain correct causal information locally when latent variables exist?

- Q1: What causal information of **m-separation** in local structure learning is **consistent** with those in global learning?

- Q2: What causal information of **V-structures** in local structure learning is **consistent** with those in global learning?



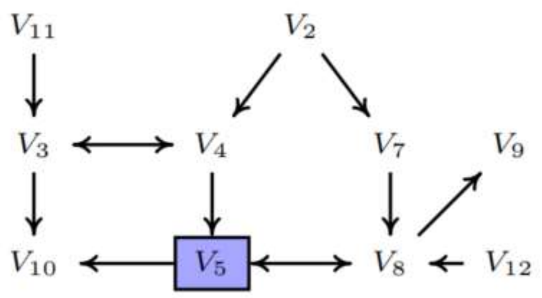
(a) Underlying MAG (b) $\mathcal{L}_{MMB+(V_5)}$

Not all the information learned by the local structure learning is correct.

What causal information of **m-separation** in local structure learning is **consistent** with those in global learning?

Theorem 1:
Let T be any node in **O**, and X be a node in $MMB(T)$. Then T and X are m-separated by a subset of $\boldsymbol{O}\backslash\{T, X\}$ if and only if they are m-separated by a subset of $MMB(T)\backslash\{X\}$.

Theorem 1 implies that the existence of an edge connecting T to any other node X ∈ MMB(T) can be equivalently determined through both the full distribution of **O** and the marginal distribution of $MMB^{+}(T)$.



(a) Underlying MAG

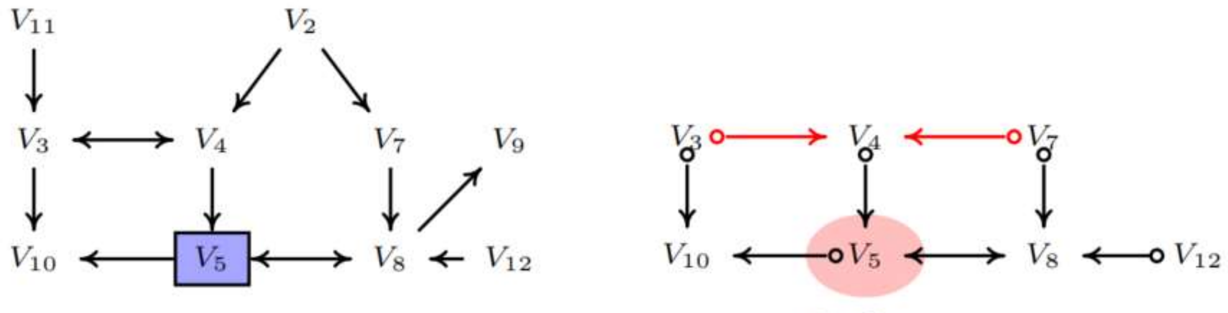
What causal information of V-structures in local structure learning is **consistent** with those in global learning?

Theorem 2:

(Fully Correct V-structures). Consider a sub-MAG of $\mathbf{M'}$ over $MMB^{+}(T)$. Let V_a, V_b be two nodes in $MMB(T)$. The following statements hold.

S1. The V-structure $V_a * \rightarrow$ T $\leftarrow * \; V_b$ that identified by the marginal distribution of $MMB^{+}(T)$ are true V-structures in the ground-truth MAG **M**.

S2. The V-structure T$*\!\!\rightarrow V_a \leftarrow\!\!* \; V_b$ can be successfully identified by the marginal distribution of $MMB^{+}(T)$.

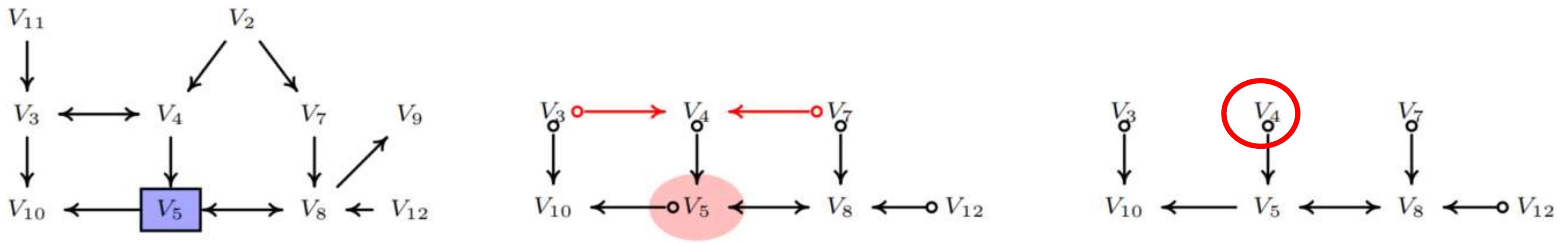


(a) Underlying MAG

(b) $\mathcal{L}_{MMB^{+}(V_5)}$

MMB-by-MMB Algorithm

- **Step 1:** Finding a MAG Markov blanket MMB(T) of the target T and learning the local structure $\boldsymbol{L}_{\boldsymbol{MMB+(T)}}$
- **Step 2:** Putting the edges connected to T and the V-structures containing T in $\boldsymbol{L}_{\boldsymbol{MMB+(T)}}$ to $\boldsymbol{P}$, according to Theorem 1 and Theorem 2.
- **Step 3:** Orienting maximally the edge marks in $\boldsymbol{P}$ using the standard orientation rules of Zhang (2008b).



(a) Underlying MAG (b) $\mathcal{L}_{MMB+(V_5)}$ (c) Updated $\mathcal{P}$ after learning $\mathcal{L}_{MMB+(V_5)}$

Three Steps 1 ~ 3 are **repeated sequentially** until our local learning structure is consistent with the global one.

How to design a stop criteria to ensure that our local learning structure is consistent with the global one?

Theorem 3:
Let T be the target node of interest within **O** and ***Waitlist*** represent the collection of nodes that need to be checked by Theorem 1 and Theorem 2. If any of the subsequent rules are met, the local structure identified for T, encompassing its direct causes and effects, will be equivalent to the structure identified through global learning methods.

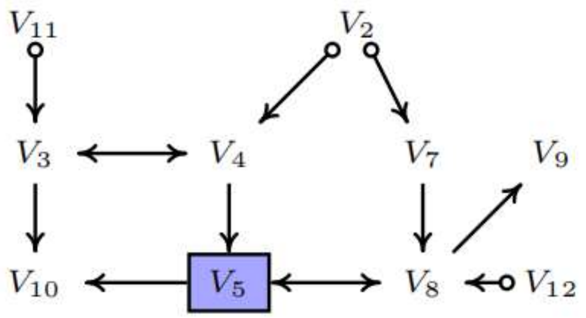
R1. The structures around the target T are all determined.

R2. The ***Waitlist*** is empty.

R3. All paths from the target T, which include undirected edges (connected to the target T), are blocked by edges $*\!\rightarrow$.

Example 1

- **Step 1:** Finding a MAG Markov blanket MMB(T) of the target T and learning the local structure $\boldsymbol{L_{MMB+(T)}}$.



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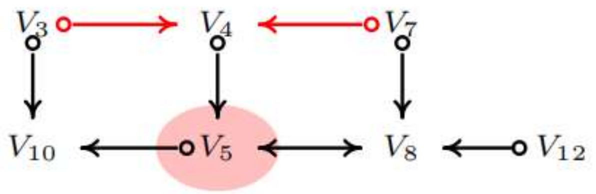

$L_{\mathrm{MMB}^+(V_5)}$

- **Step 3:** Orienting maximally the edge marks in $\boldsymbol{P}$ using the standard orientation rules of Zhang (2008b).

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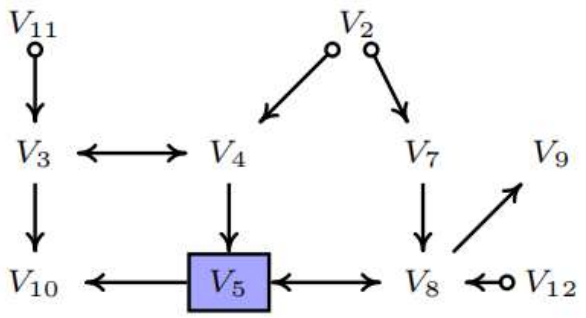

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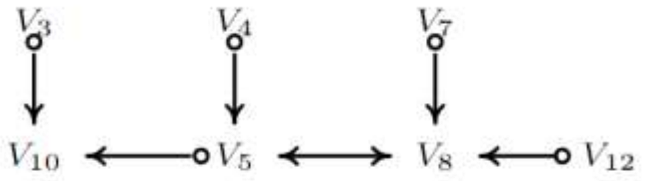

Updated $\boldsymbol{P}$ after learning $L_{\mathrm{MMB}^{+}(V_5)}$

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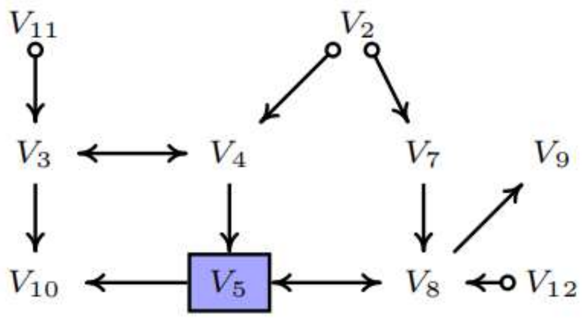

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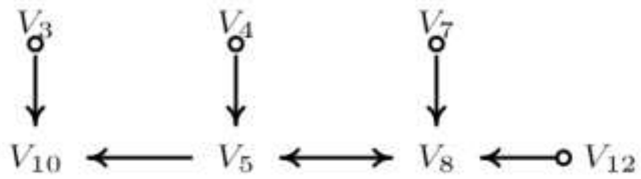

Updated $\boldsymbol{P}$ after orienting

- **Step 3:** Orienting maximally the edge marks in $\boldsymbol{P}$ using the standard orientation rules of Zhang (2008b).

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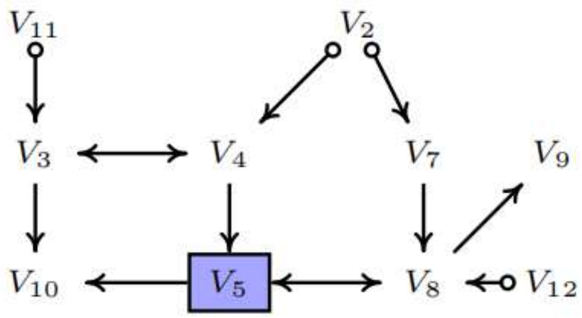

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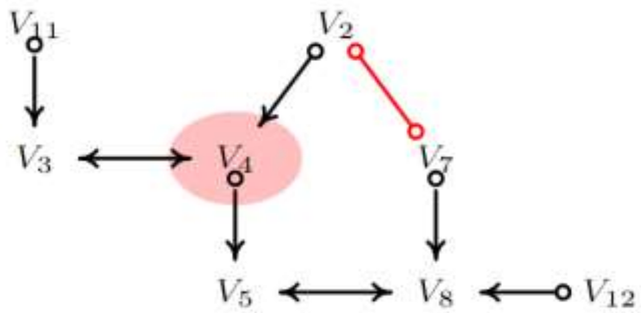

$L_{\mathrm{MMB}^{+}(V_4)}$

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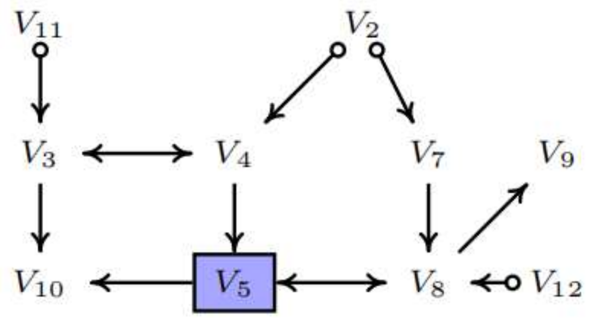

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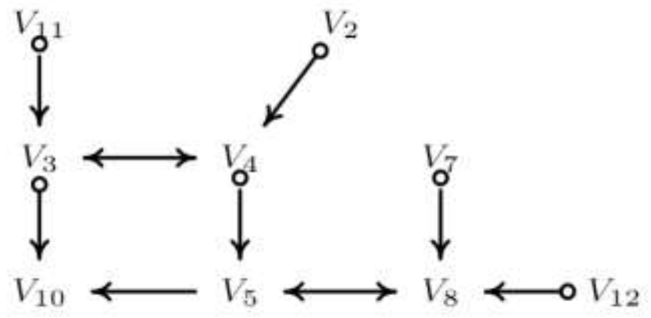

Updated $\boldsymbol{P}$ after learning $L_{\mathrm{MMB}^+(V_4)}$

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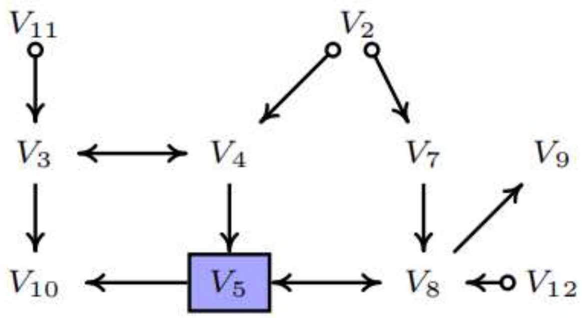

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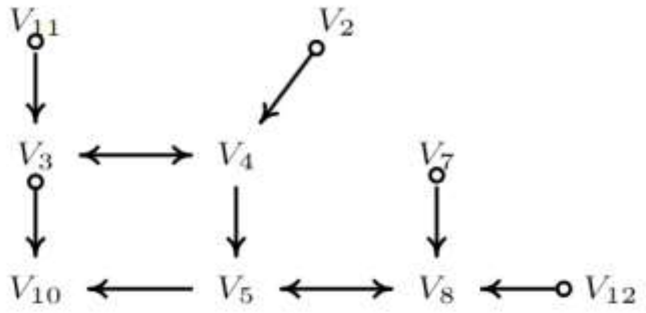

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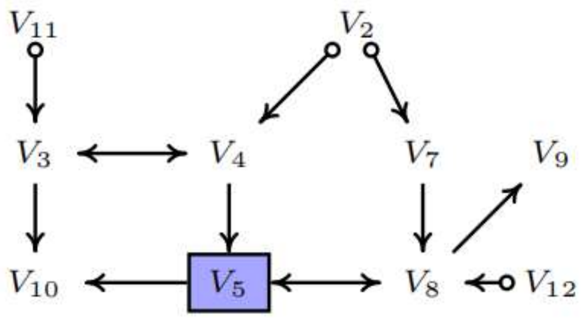

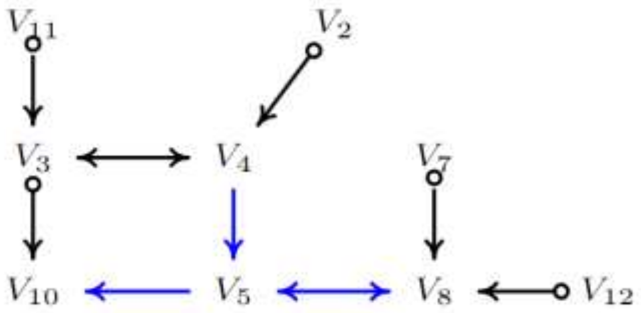

The final local ***P***

Conclusion

- Provide theories for local structure learning in the presence of unobserved confounds.
- Introduce a novel local causal discovery algorithm called MMB-by-MMB, which is designed to be effective in models with latent variables.
- Future work: How can background knowledge be utilized to further aid in identifying causes and effects within local structures?

**Thank you
for your attention!**