

From Geometry to Causality:  
Ricci Curvature and the Reliability of Causal Inference on Networks

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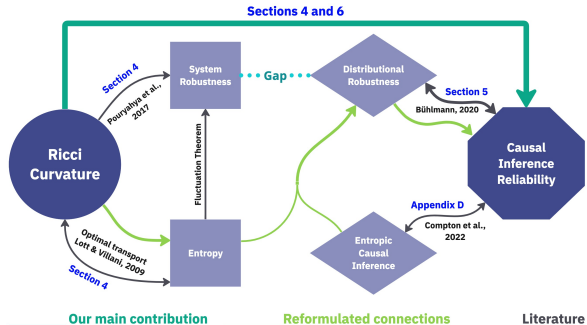
**ICML**  
International Conference  
On Machine Learning



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- We connect graph geometry with causal inference; then predict the reliability of causal estimates



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Establish a theoretical connection between graph Ricci curvature and causal inference reliability

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Empirically demonstrate that Ricci curvature indicates accuracy of causal estimates

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### ■ Methodological contribution :

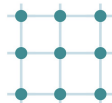
Propose Ricci flow adjustment to improve causal estimation by flattening the network, reducing error

- Ricci curvature

- measures how much the local geometry deviates from being Euclidean
- can be extended to graphs to capture the extent of dispersion/volume change through an edge
- is linked to *system robustness*



Tree-like  
Hyperbolic (-)



Grid  
Euclidean (0)



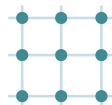
Dense  
Spherical (+)

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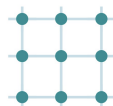
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- Learning causal parameters  $\leftrightarrow$  *distributionally robust* estimation
- The two notions of robustness are different, but we prove a connection :

More robust system  $\Leftarrow$  Positive Ricci curvature

$\Rightarrow$  Higher distributional robustness

$\Rightarrow$  More reliable causal estimates

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Positive Ricci curvature makes accurate causal estimation  
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# Curvature and Causal Parameter Estimation Confidence

- **Main theoretical result :**

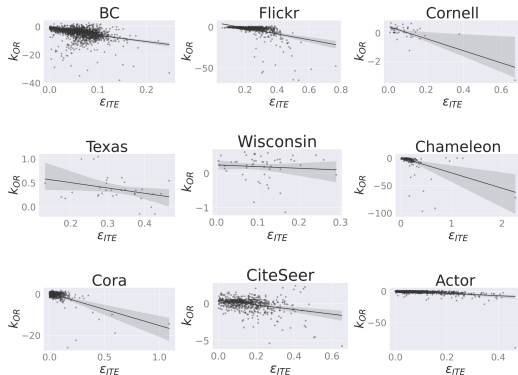
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- **Empirical validation :**

Ricci curvature is negatively correlated with treatment effect estimation error on empirical networks



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# Improve Causal Inference on Networks using the Ricci flow

- Most empirical networks are sparse  $\implies$  Many edges have negative curvature
- Flatten the network, using graph Ricci flow, before causal effect estimation
- This preprocessing reduces error, and can be combined with any causal inference method

	BC	Flickr	Cornell	Texas	Wisconsin	chameleon	Cora	CiteSeer	Actor
T-Learner+RF	0.328	0.462	0.192	0.414	0.463	0.372	0.232	0.386	0.238
X-Learner+RF	5.612	5.745	5.928	3.827	3.815	3.709	8.626	5.606	5.231
TARNet	0.969	1.024	0.705	1.028	0.711	1.212	0.679	0.638	0.796
CFR	0.895	0.960	0.806	1.038	0.849	0.926	0.570	0.620	0.735
T-Learner+GNN	4.178	9.630	5.125	4.437	0.559	16.715	0.285	0.529	7.912
X-Learner+GNN	4.627	3.933	20.461	1.995	16.244	329.959	3.165	4.428	4.296
NetDeconf	1.092	1.251	0.900	1.137	0.952	1.207	0.791	0.752	0.895
f-TLearner+GNN	3.268	2.762	4.370	3.106	0.466	7.764	0.263	0.494	3.896
f-XLearner+GNN	4.222	3.859	17.395	2.020	20.815	251.290	3.053	3.919	3.967
f-NetDeconf	1.088	1.245	0.900	1.143	0.954	1.200	0.810	0.767	0.898
NetEst	0.069	0.213	0.165	0.330	0.147	0.247	0.082	0.176	0.094
f-NetEst (ours)	0.033	0.208	0.127	0.308	0.142	0.230	0.078	0.165	0.088

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- This gives a computational tool to evaluate reliability of causal estimates
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- This gives a computational tool to evaluate reliability of causal estimates
  - without any ground truth
  - independent of the causal inference method
- Based on this, we propose a method to reduce causal estimation error. The proposed method
  - improves the accuracy of causal estimates
  - is a preprocessing that can be combined with any causal inference method
  - cannot target specific neighborhoods
  - is just a preprocessing and is not updated during training
  - **importantly** : alters the graph by weighting the edges. Is this change conceptually acceptable within the context of the problem in hand ?



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Thank You!

