# Doubly Robust Causal Effect Estimation under Networked Interference via Targeted Learning

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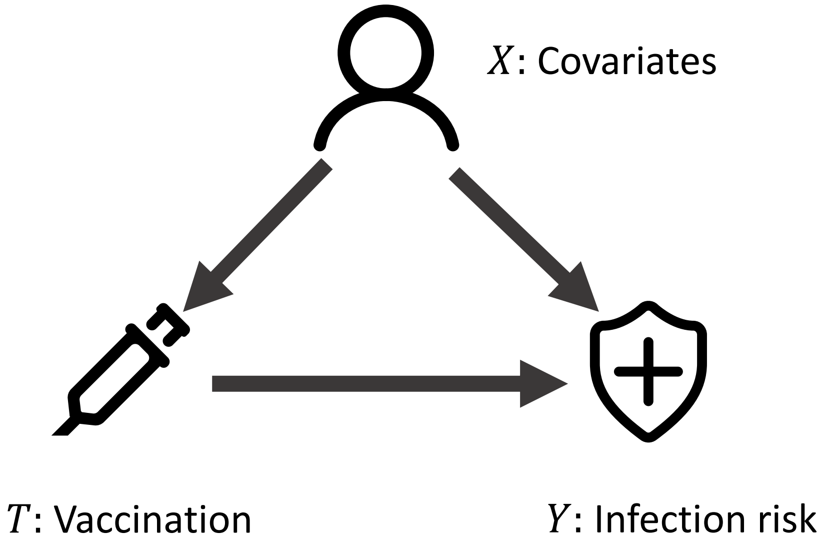
# Causal Effect

## What is causal effect

Causal effects: the impact of a treatment $T$ on the outcome $Y$

$$\psi(t) = \mathbb{E}\big[\mathbb{E}[Y(t)|X = x]\big]$$

$$\tau = \mathbb{E}\big[\mathbb{E}[Y(1) - Y(0)|X = x]\big]$$




A powerful tool to measure effectiveness of strategies.

E.g., how vaccination affect infection risk?

# Causal Effect under Networked Interference

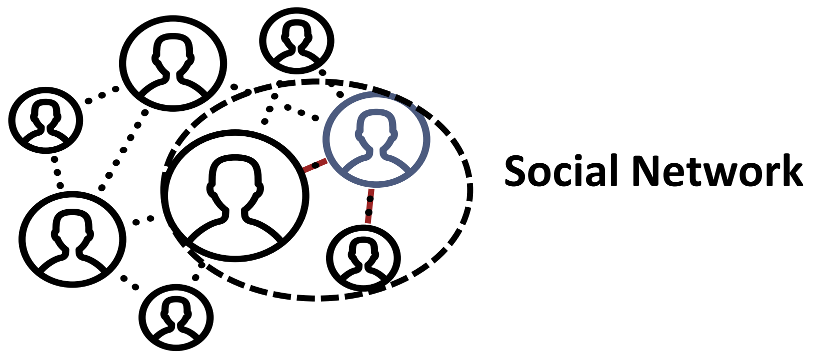
## Networked Interference

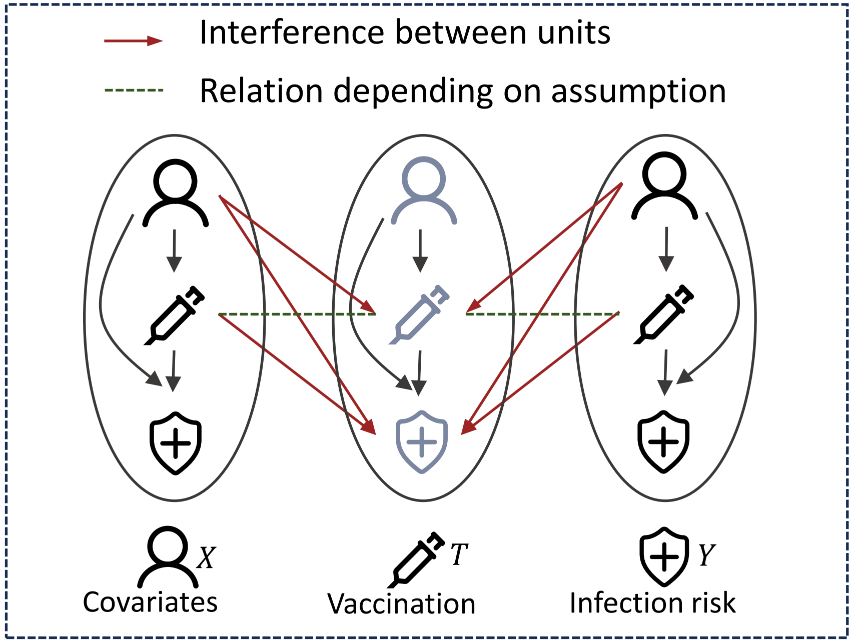
Networked Interference: units affect each other

Networked Causal Effects: the impact of a treatment $T$ and neighbors' s treatment $Z$ on the outcome $Y$

$$\psi(t,z) = \mathbb{E}\big[\mathbb{E}[Y(t,z)|x,x_{\mathcal{N}}]\big]$$

E.g., how much infection risk changes if a unit would receive vaccination and its neighbors would also receive vaccination?



# Causal Effect under Networked Interference

## Problem of existing method

To estimate networked effects $\psi(t,z)=\mathbb{E}[Y(t,z)]$, Forastiere et al. (2021) propose Generalized Propensity Score (GPS)[1]:

$$g(t,z|x,x_{\mathcal{N}})=p(t,z|x,x_{\mathcal{N}})$$

Depending on different assumptions, GPS can be decomposed in different ways:

$$g(t,z|x,x_{\mathcal{N}})=g_1(t|x,x_{\mathcal{N}})g_2(z|t,x,x_{\mathcal{N}}) \qquad \text{with } t\perp z|x,x_{\mathcal{N}}$$

$$g(t,z|x,x_{\mathcal{N}})=g_1(t|x,x_{\mathcal{N}})g_2(z|x,x_{\mathcal{N}}) \qquad \text{with } t\perp z|x,x_{\mathcal{N}}$$

$$g(t,z|x,x_{\mathcal{N}})=g_1(t|x,z)g_2(z|x,x_{\mathcal{N}}) \qquad \text{with } t\perp x_{\mathcal{N}}|x,z$$

Model misspecification occur with wrong assumption.

[1] Forastiere L, Airoldi E M, Mealli F. Identification and estimation of treatment and interference effects in observational studies on networks[J]. Journal of the American Statistical Association, 2021, 116(534): 901-918.

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Dependin

**Need: Networked effect estimator that is robust to model misspecification**

$$g(t,z|x,x_{\mathcal{N}}) = g_1(t|x,x_{\mathcal{N}})g_2(z|x,x_{\mathcal{N}}) \qquad \text{with } t \perp z|x,x_{\mathcal{N}}$$

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# Causal Effect under Networked Interference

## Efficient Influence Curve (EIC)

What kind of estimator is robust to model misspecification?

Theorem 4.2 For $t \in \{0,1\}$ and $z \in [0,1]$, the efficient influence curve of $\psi(t,z)$ is:

$$\varphi(t,z,X,X_{\mathcal{N}};\mu,g,\psi) = \left(\frac{\mathbb{I}_{T,Z}(t,z)}{g(t,z|X,X_{\mathcal{N}})}\right)(y-\mu(t,z,X,X_{\mathcal{N}})) + \mu(t,z,X,X_{\mathcal{N}}) - \psi(t,z),$$

where $\mu(t,z,X,X_{\mathcal{N}}) \coloneqq \mathbb{E}[Y|t,z,X,X_{\mathcal{N}}]$ and $g(t,z|X,X_{\mathcal{N}}) \coloneqq \mathbb{E}[t,z|X,X_{\mathcal{N}}]$.

Lemma 4.3 (Double Robustness Property) For $t \in \{0,1\}$ and $z \in [0,1]$, if models $\hat{g},\hat{\mu}$ solve EIC, $\mathbb{P}\varphi = 0$, then the estimator $\hat{\psi}$ for $\psi$ is doubly robust. Further, if $\|\hat{g}-g\|_\infty = O_p(r_1(n))$ and $\|\hat{\mu}-\mu\|_\infty = O_p(r_2(n))$, we have:

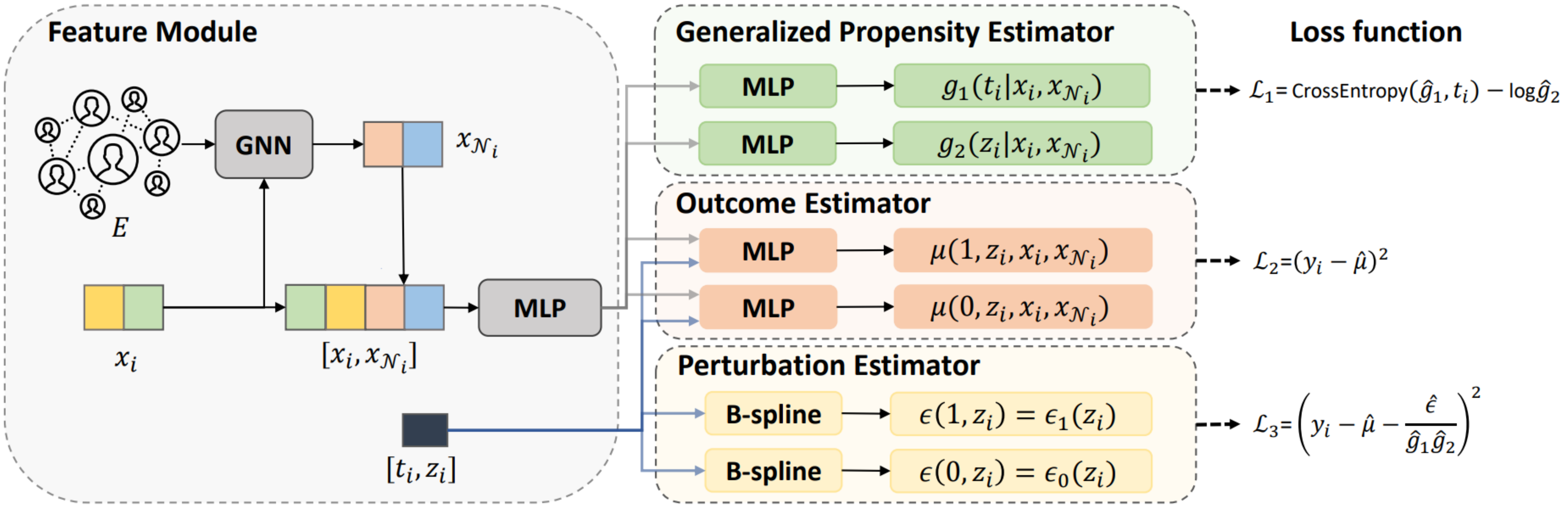
$$\sup_{t,z\in\mathcal{T},\mathcal{Z}} |\mathbb{P}\varphi(t,z,X,X_{\mathcal{N}};\hat{\mu},\hat{g},\psi)| = O_p(r_1(n))r_2(n)))$$

Goal: Designing an estimator for $\psi$ solving EIC to achieve DR property.

# Causal Effect under Networked Interference

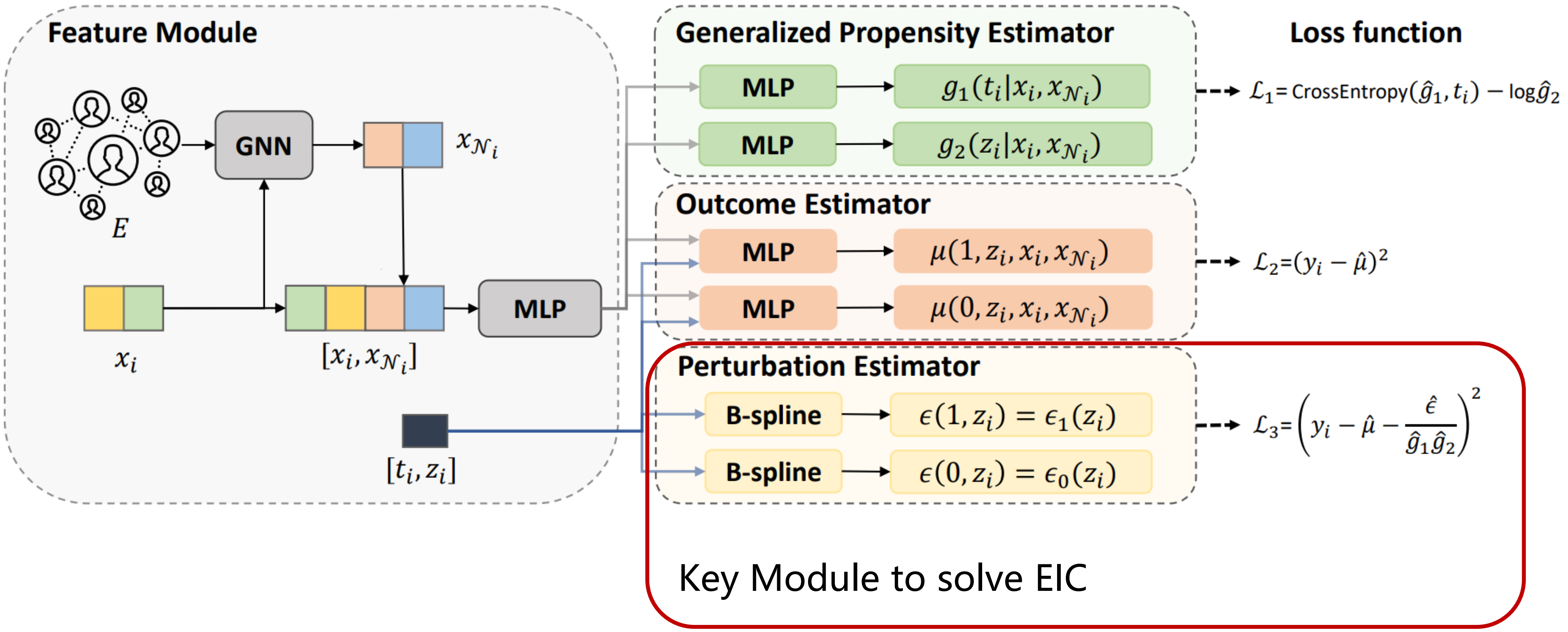
## Overview of Estimator TNet

We design a one-step estimator using NNs.



# Causal Effect under Networked Interference

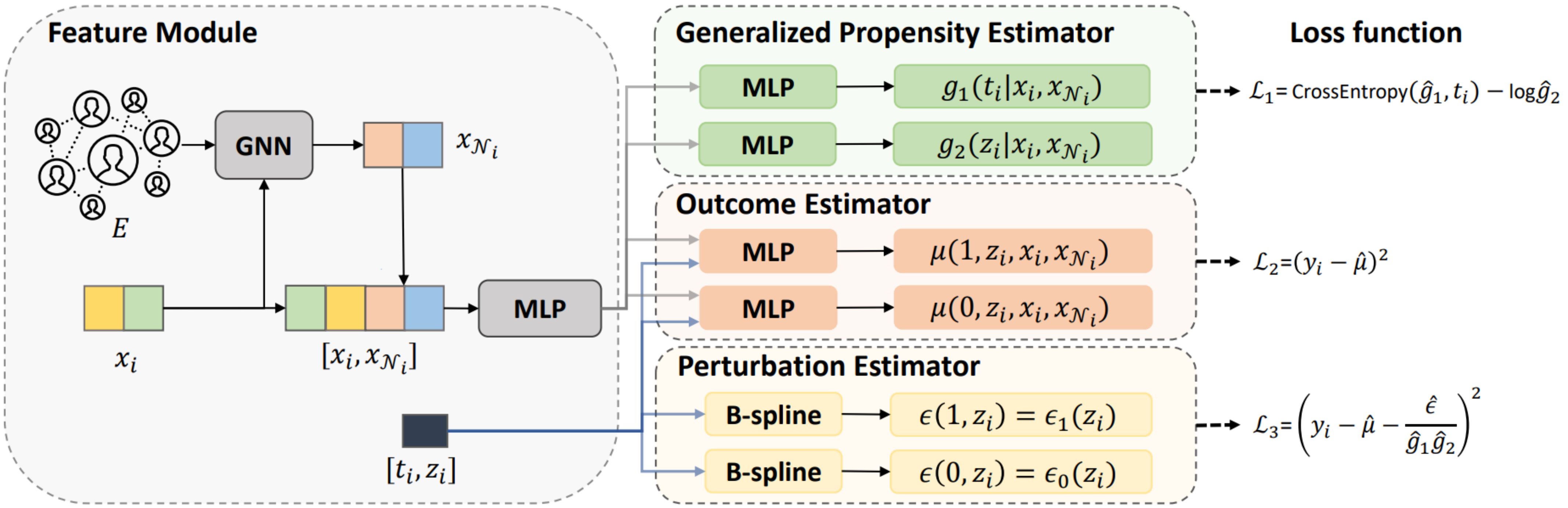
## Overview of Estimator TNet




Final estimator is $\hat{\mu} + \frac{\hat{\epsilon}}{\hat{g}_1 \hat{g}_2}$

# Causal Effect under Networked Interference

## How to achieve DR property




Key insight:

$$0 = \frac{\partial(\mathcal{L})}{\partial \epsilon}\Big|_{\epsilon=\hat{\epsilon}} = \frac{\partial(\mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3)}{\partial \epsilon}\Big|_{\epsilon=\hat{\epsilon}} = 2\beta \Sigma_{i=1}^{n} \varphi(t, z, x_i, x_{\mathcal{N}_i}; \hat{\mu}, \hat{g}, \psi).$$

which means our estimator solve EIC $\mathbb{P}\varphi = 0$, achieving DR property according to Lemma 4.3.

# Causal Effect under Networked Interference

## Theoretical Analysis of Estimator TNet

How fast convergence rate TNet can achieve?

Theorem 6.1 Under mild assumptions, we have

$$\|\hat{\psi} - \psi\|_{L^2} = O_p\left(n_0^{-1/3}\sqrt{n_0} + n_1^{-1/3}\sqrt{n_1} + r_1(n)r_2(n)\right),$$

where $\|\hat{g} - g\|_\infty = O_p(r_1(n))$, $\|\hat{\mu} - \mu\|_\infty = O_p(r_2(n))$.

- First term $n_0^{-1/3}\sqrt{n_0} + n_1^{-1/3}\sqrt{n_1}$ is the rate achieved in term of $\epsilon$.
- Second term $r_1(n)r_2(n)$ is the product of convergences rates of nuisances function.

Second term again shows DR property:

$$r_1(n)r_2(n) = o(1) \text{ when } r_1(n) = o(1) \text{ or } r_2(n) = o(1).$$

# Causal Effect under Networked Interference

## Experimental Results

Verifying the effectiveness of TNet: achieving the lowest prediction errors

*Table 1.* Experimental results on BC(homo) Dataset. The top result is highlighted in bold, and the runner-up is underlined.

| Metric | setting | effect | CFR+z | GEst | ND+z | NetEst | RRNet | NDR | TNet(w/o. $\mathcal{L}_3$) | TNet |
|---|---|---|---|---|---|---|---|---|---|---|
| $\varepsilon_{average}$ | Within Sample | AME | $0.1010_{\pm0.0678}$ | $0.1512_{\pm0.1073}$ | $0.0868_{\pm0.0757}$ | $0.1257_{\pm0.1343}$ | $\underline{0.0877}_{\pm0.0565}$ | $0.5033_{\pm0.0080}$ | $0.1056_{\pm0.0690}$ | $\mathbf{0.0481_{\pm0.0365}}$ |
| | | ASE | $0.1956_{\pm0.0582}$ | $0.1860_{\pm0.0225}$ | $0.2140_{\pm0.0287}$ | $0.0347_{\pm0.0169}$ | $\underline{0.0227}_{\pm0.0165}$ | $0.2464_{\pm0.0042}$ | $0.1337_{\pm0.0139}$ | $\mathbf{0.0180_{\pm0.0183}}$ |
| | | ATE | $0.2802_{\pm0.1814}$ | $0.1342_{\pm0.0785}$ | $0.3742_{\pm0.1041}$ | $0.1229_{\pm0.0583}$ | $0.0907_{\pm0.0662}$ | $\mathbf{0.0284_{\pm0.0149}}$ | $0.2467_{\pm0.0520}$ | $\underline{0.0533}_{\pm0.0405}$ |
| | Out-of Sample | AME | $0.1011_{\pm0.0681}$ | $0.1534_{\pm0.1025}$ | $0.0901_{\pm0.0750}$ | $0.1258_{\pm0.1350}$ | $0.0879_{\pm0.0561}$ | / | $0.1081_{\pm0.0671}$ | $\mathbf{0.0481_{\pm0.0364}}$ |
| | | ASE | $0.1969_{\pm0.0581}$ | $0.1859_{\pm0.0228}$ | $0.2127_{\pm0.0279}$ | $0.0322_{\pm0.0173}$ | $\underline{0.0225}_{\pm0.0167}$ | / | $0.1238_{\pm0.0094}$ | $\mathbf{0.0179_{\pm0.0183}}$ |
| | | ATE | $0.2792_{\pm0.1826}$ | $0.1298_{\pm0.0782}$ | $0.3688_{\pm0.1040}$ | $0.1238_{\pm0.0568}$ | $\underline{0.0911}_{\pm0.0667}$ | / | $0.2358_{\pm0.0503}$ | $\mathbf{0.0532_{\pm0.0405}}$ |
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| | | ITE | $0.3033_{\pm0.1562}$ | $0.1848_{\pm0.0635}$ | $0.3780_{\pm0.1031}$ | $0.1278_{\pm0.0583}$ | $\underline{0.1010}_{\pm0.0587}$ | / | $0.2783_{\pm0.0493}$ | $\mathbf{0.0560_{\pm0.0383}}$ |
| | Out-of Sample | IME | $0.1254_{\pm0.0572}$ | $0.2031_{\pm0.0749}$ | $0.1195_{\pm0.0658}$ | $0.1412_{\pm0.1246}$ | $\underline{0.0953}_{\pm0.0524}$ | / | $0.1487_{\pm0.0574}$ | $\mathbf{0.0506_{\pm0.0351}}$ |
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Verifying the effectiveness of perturbation module $\mathcal{L}_3$, which makes TNet doubly robust.

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# Thank you!

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