

Online Isolation Forest

Filippo Leveni [†]✉, Guilherme Weigert Cassales [‡], Bernard Pfahringer [‡], Albert Bifet [‡],
Giacomo Boracchi [†]



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Online Anomaly Detection



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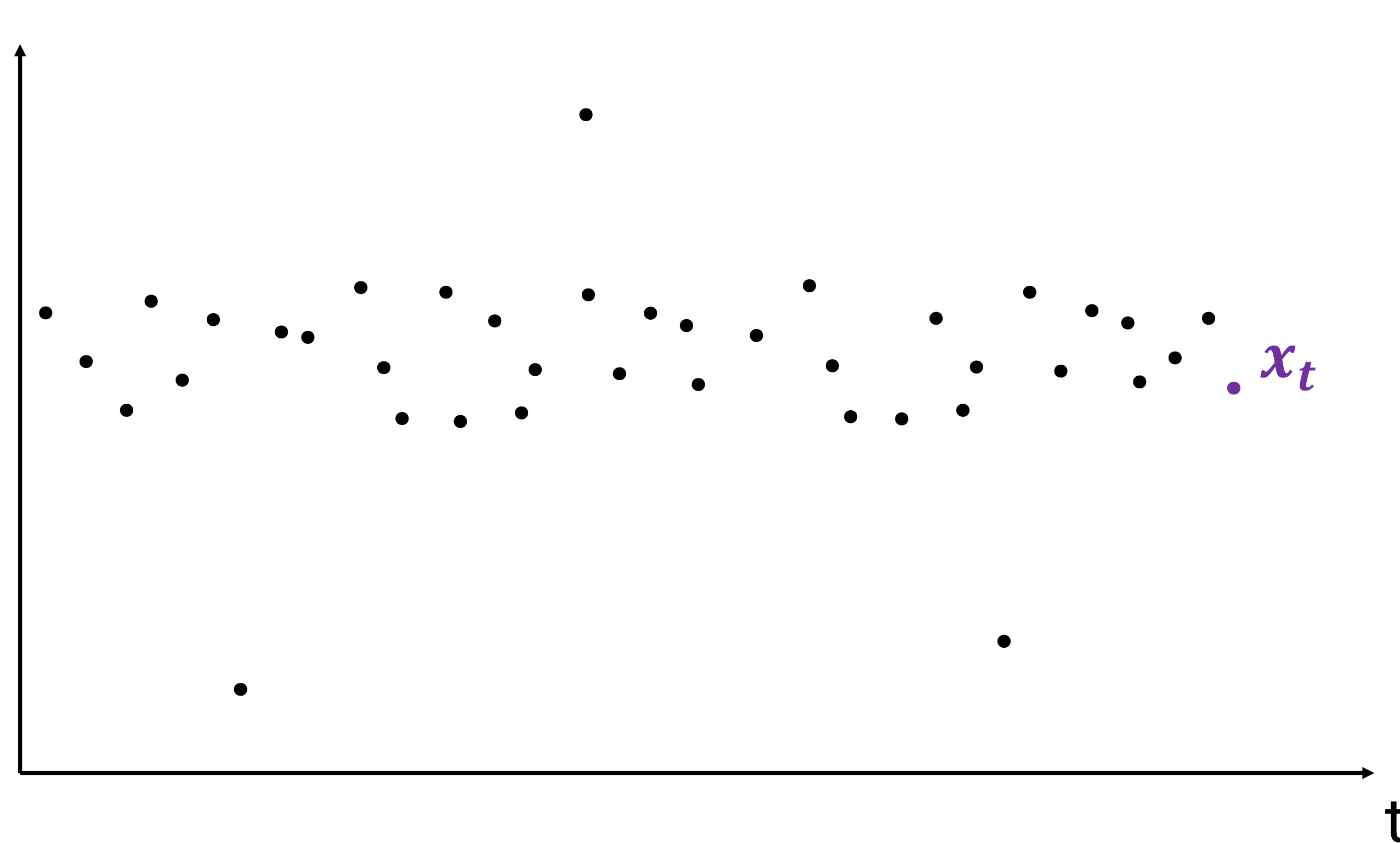
Online Anomaly Detection



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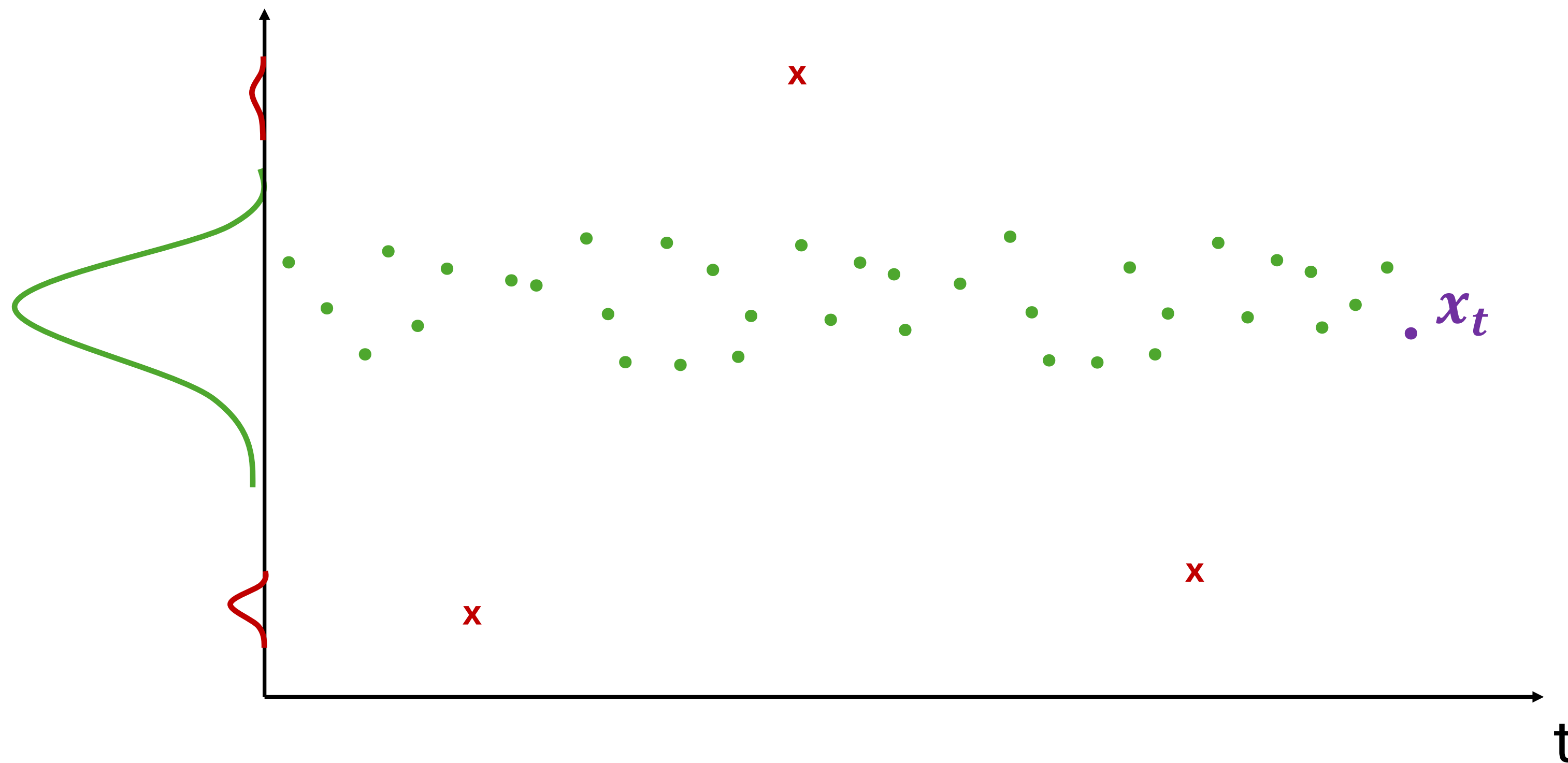


Online Anomaly Detection



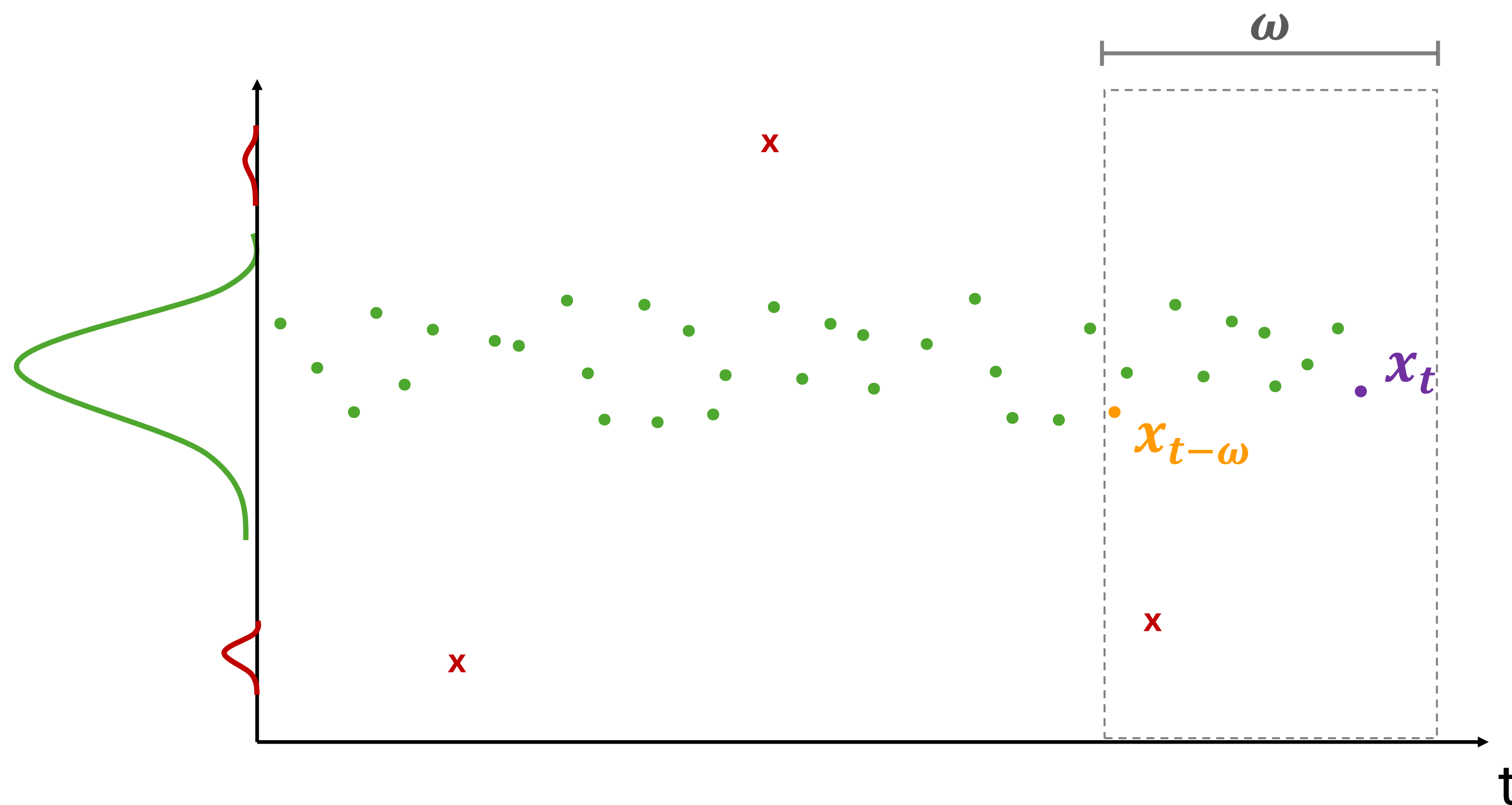
Given a point x_t

Online Anomaly Detection



Given a point x_t , identify whether it comes from Φ_0 or Φ_1 , under the assumption that **anomalies** are “few” and “different”.

Online Anomaly Detection

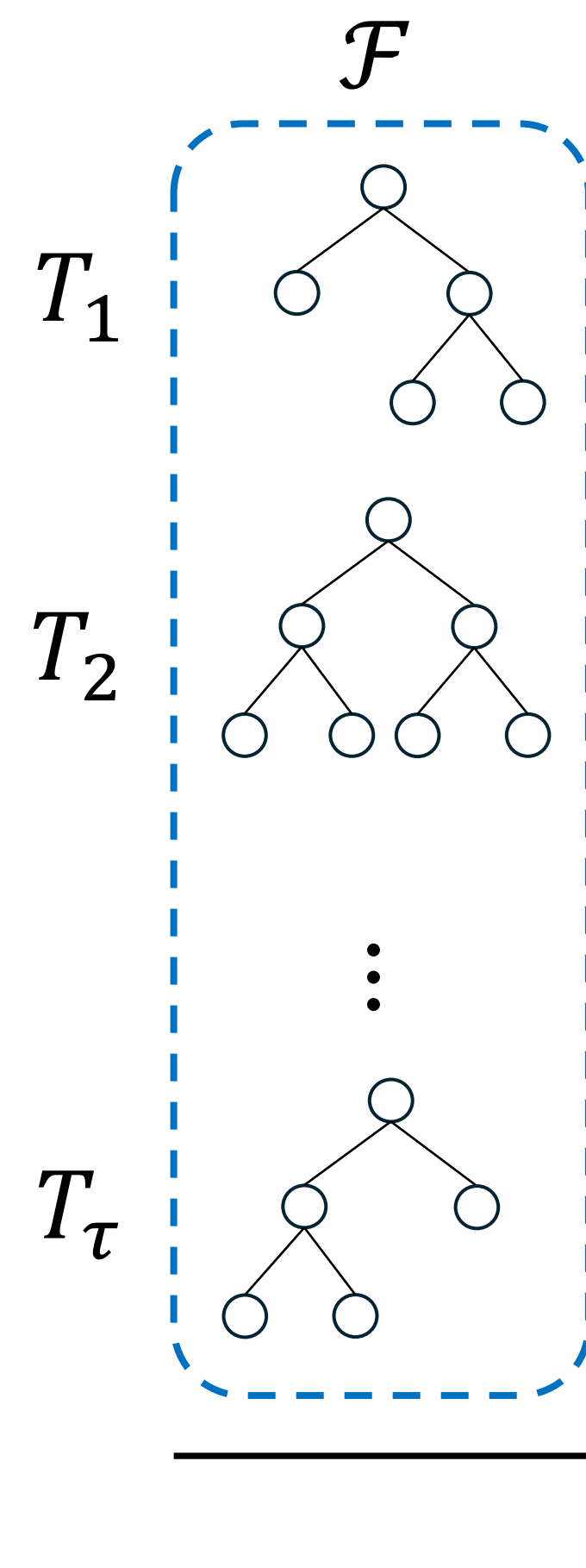


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Requirements:

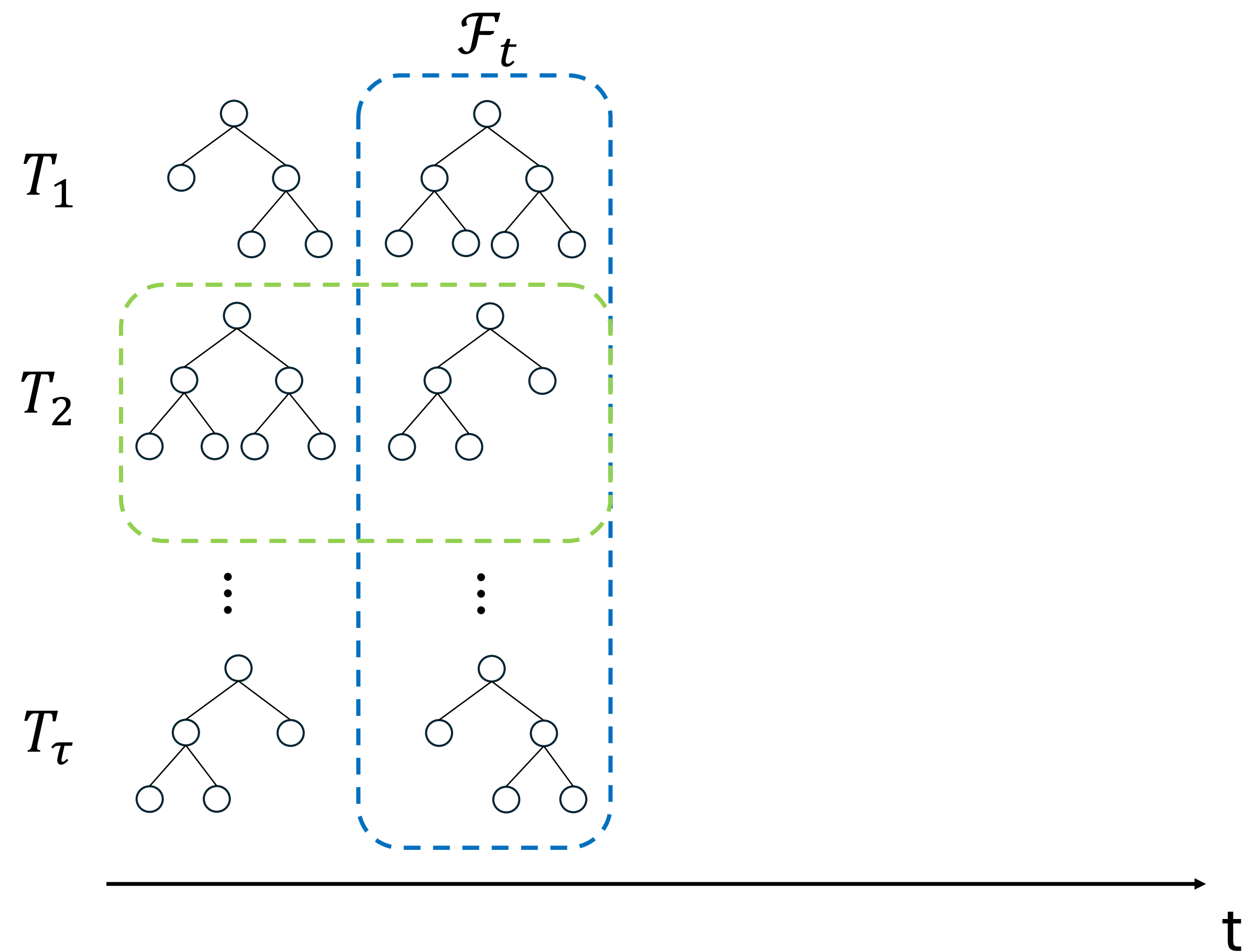
- store in memory only a subset $x_{t-\omega}, \dots, x_t$ at each time instant t .
- classification time of x_t must be as small as possible.

Solution idea



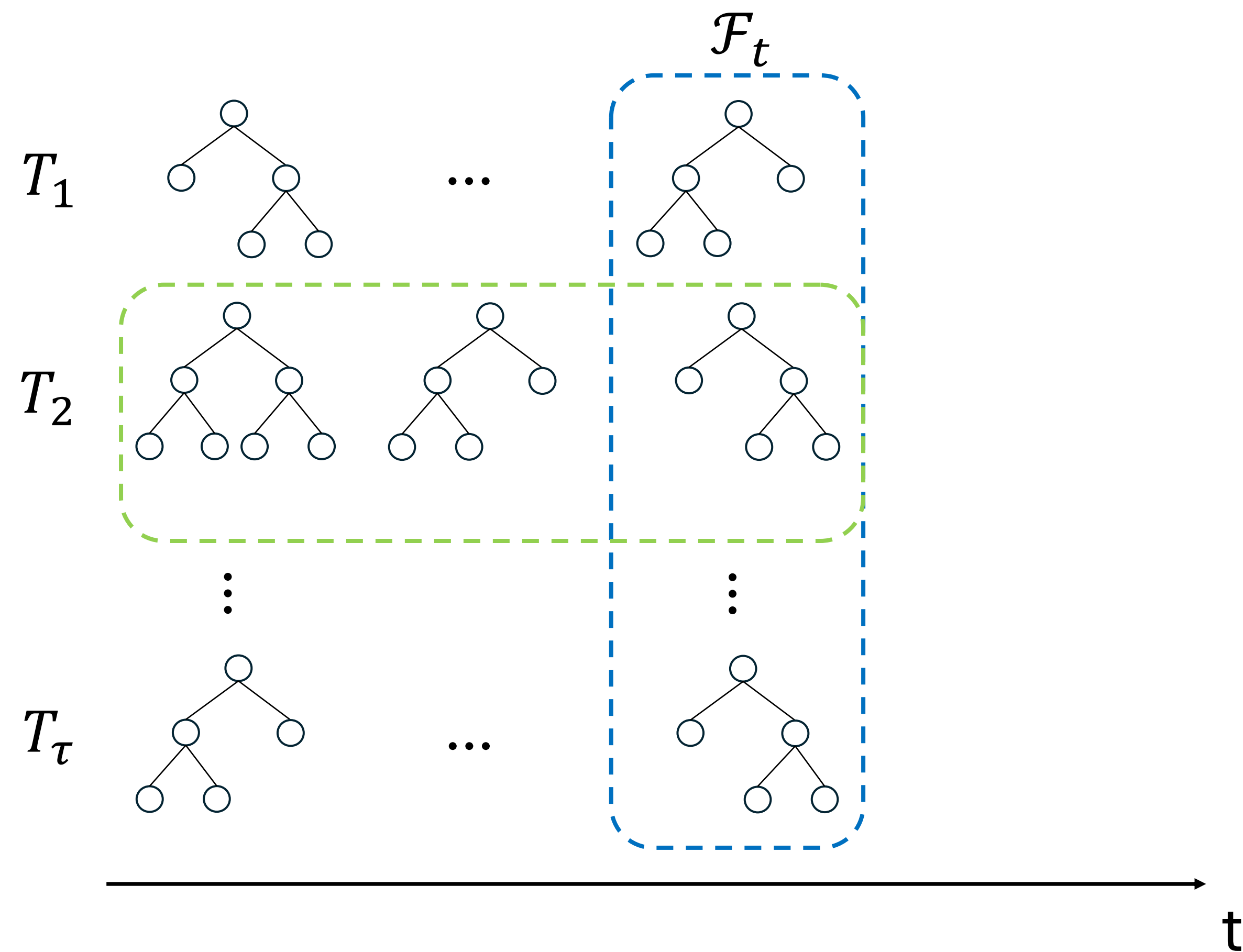
1) Build an ensemble $\mathcal{F}$ of trees

Solution idea



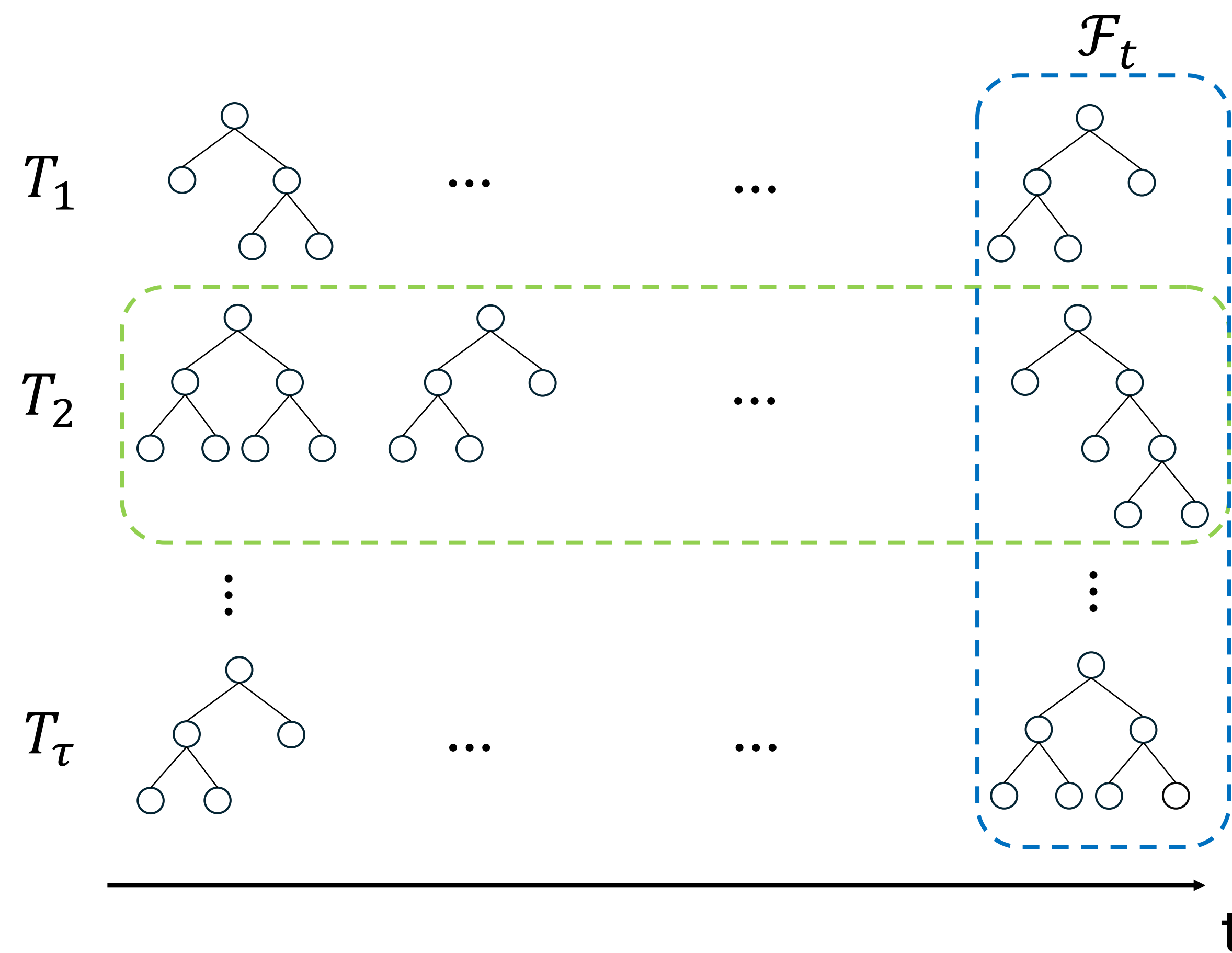
- 1) Build an ensemble $\mathcal{F}$ of trees that continuously **expand and contract their structure** at each time instant t

Solution idea



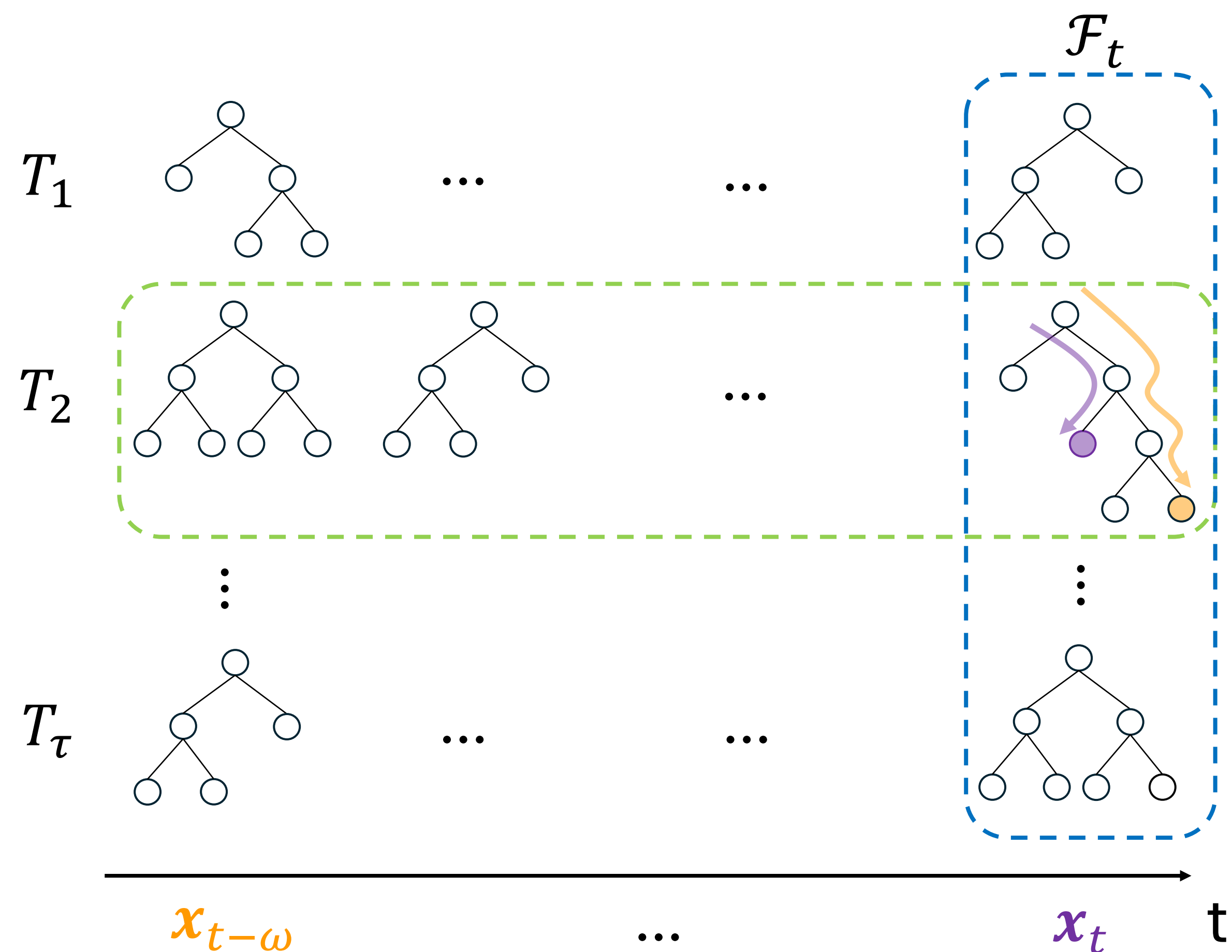
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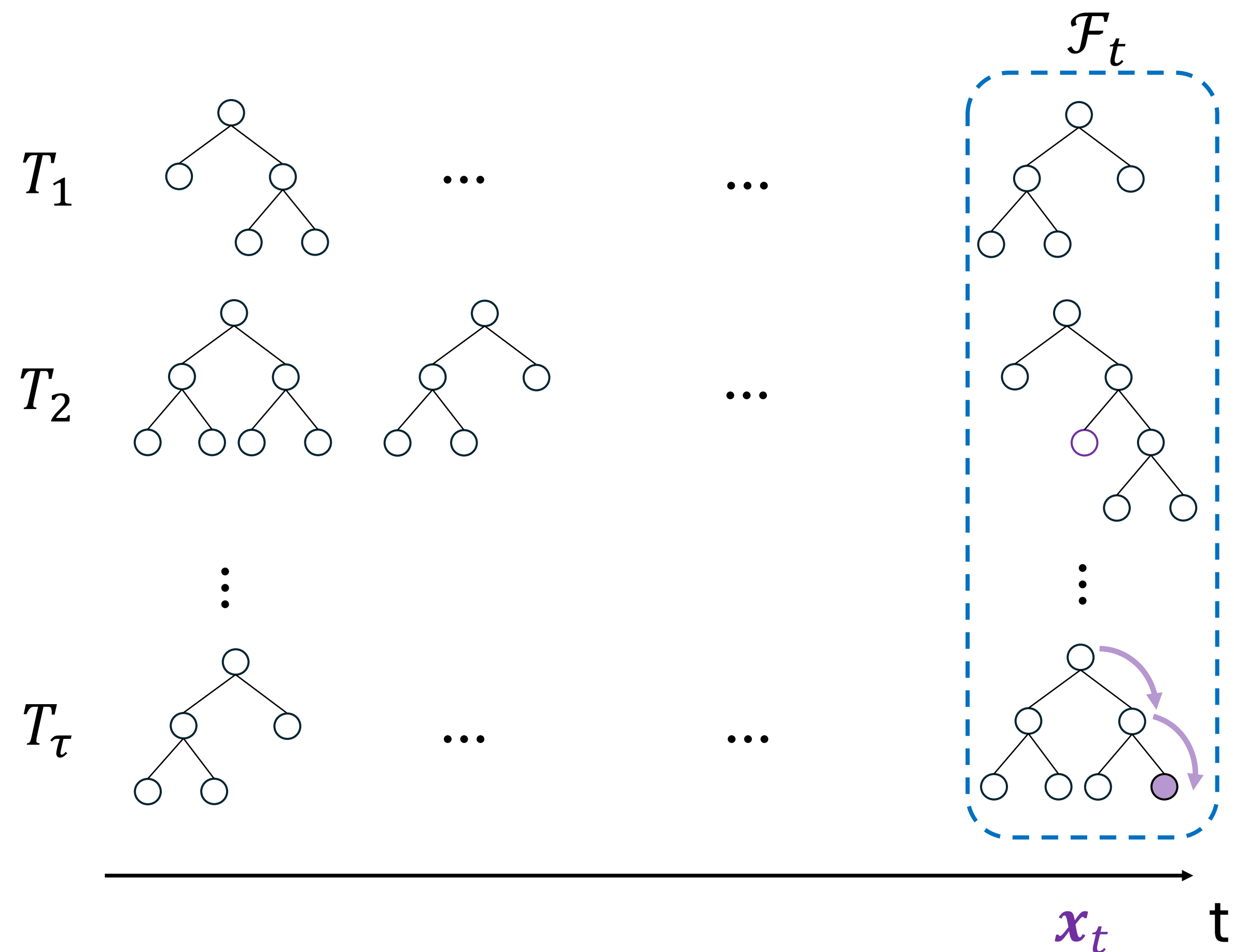
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- 1) Build an ensemble $\mathcal{F}$ of trees that continuously **expand and contract their structure** at each time instant t , by learning the new sample x_t , and forgetting the old sample $x_{t-\omega}$.

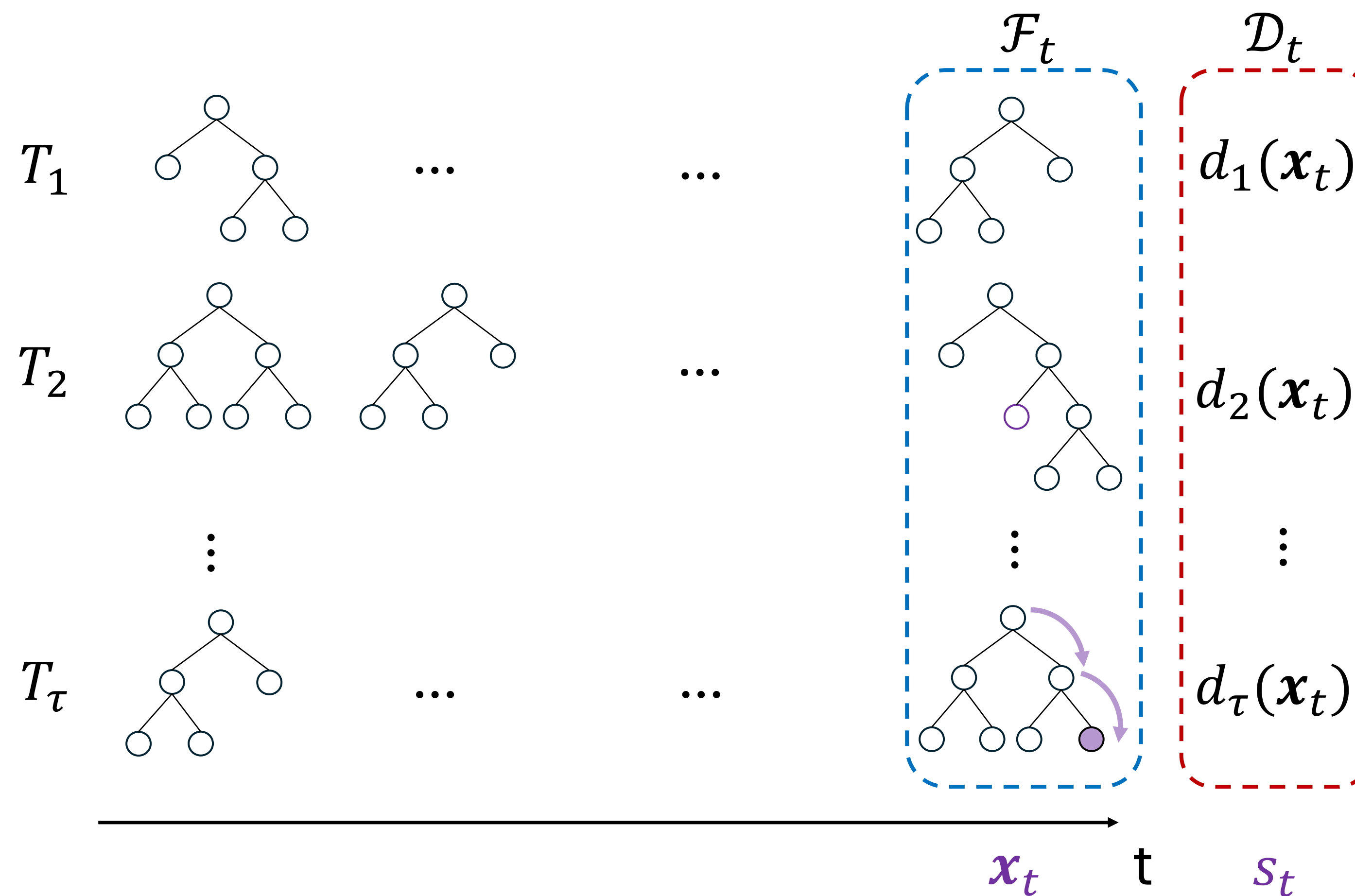
Solution idea



- 1) Build an ensemble $\mathcal{F}$ of trees that continuously **expand and contract their structure** at each time instant t , by learning the new sample x_t , and forgetting the old sample $x_{t-\omega}$.
- 2) Compute anomaly score s_t for x_t as a function of the average depth $E(\mathcal{D}_t)$ of all the leaves where x_t falls in each tree:

$$s_t = 2^{-\frac{E(\mathcal{D}_t)}{c(\omega, \eta)}}$$

Solution idea



- 1) Build an ensemble $\mathcal{F}$ of trees that continuously **expand and contract their structure** at each time instant t , by learning the new sample $\mathbf{x}_t$, and forgetting the old sample $\mathbf{x}_{t-\omega}$.
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Solution idea



T_1

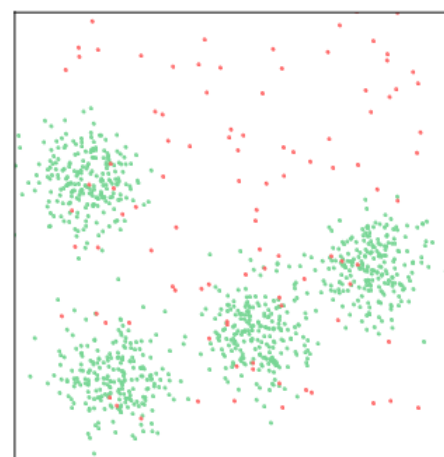
T_2

T_τ

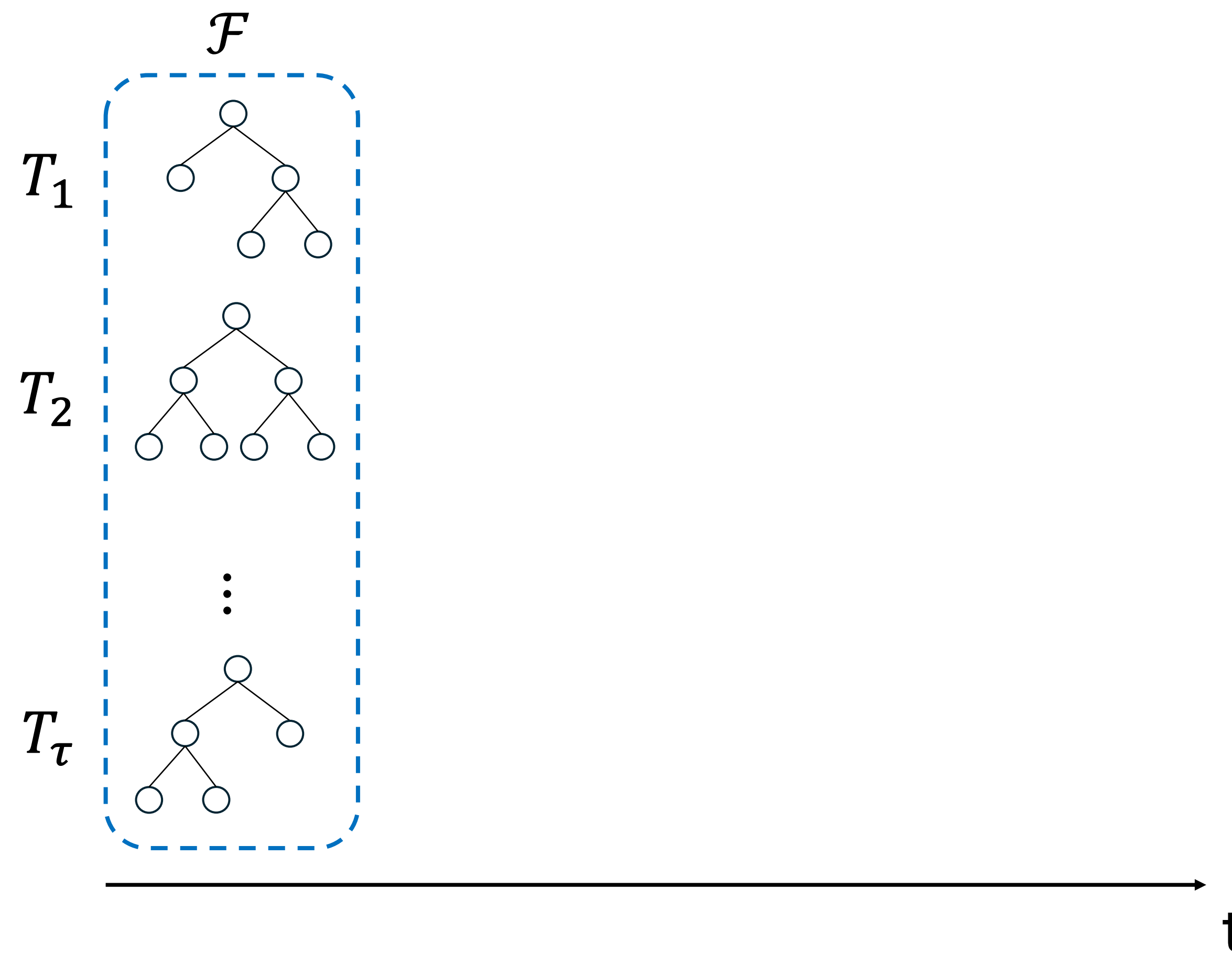
→ t

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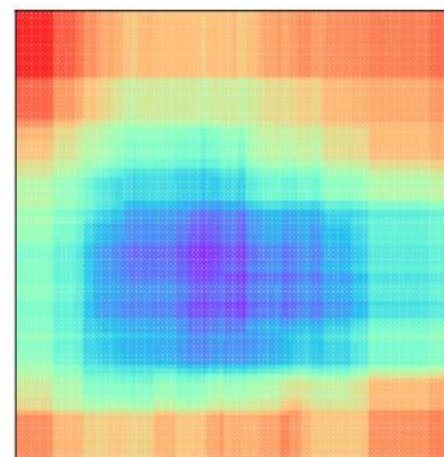
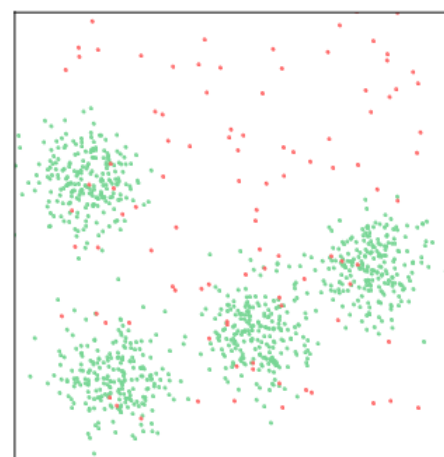


Solution idea

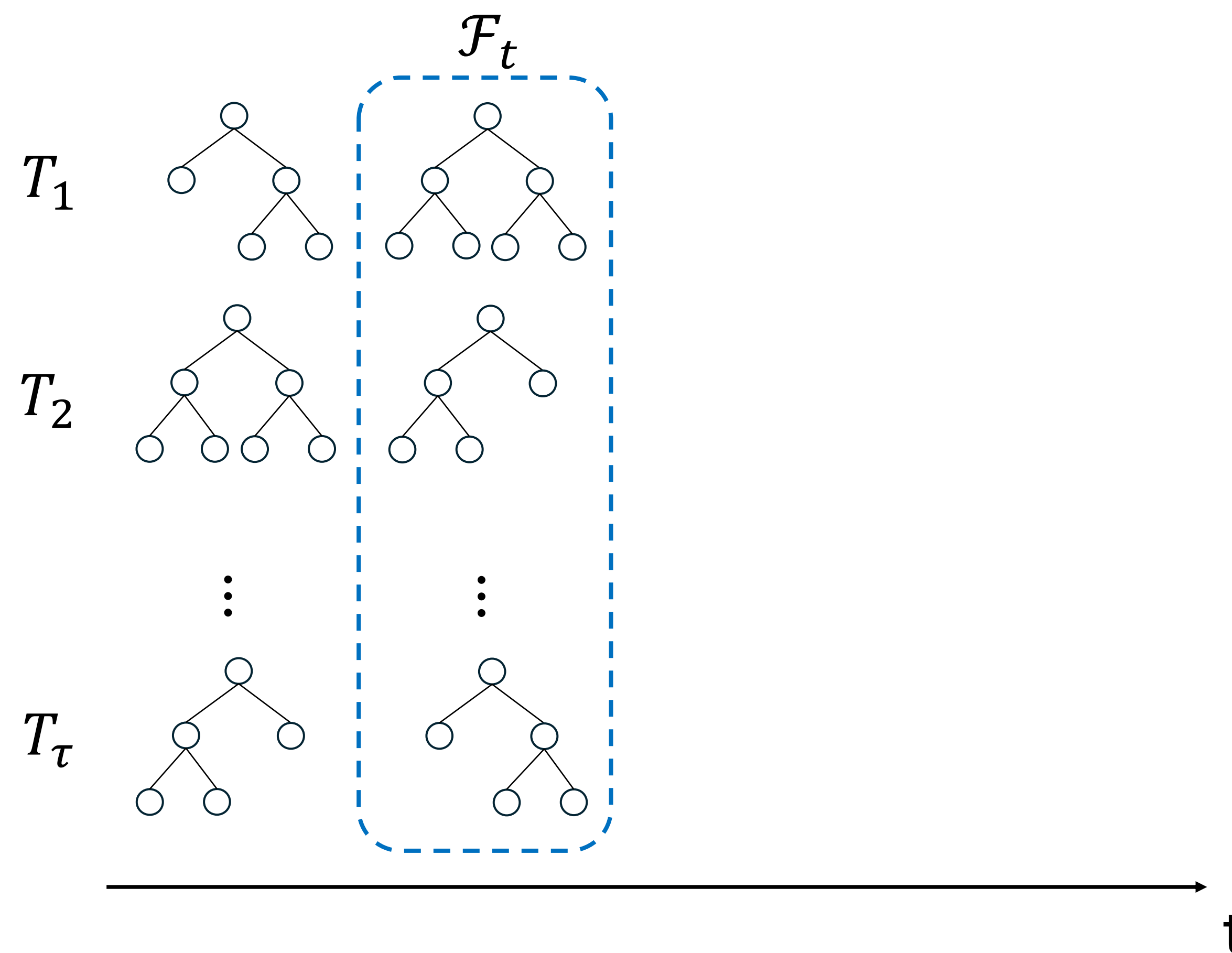


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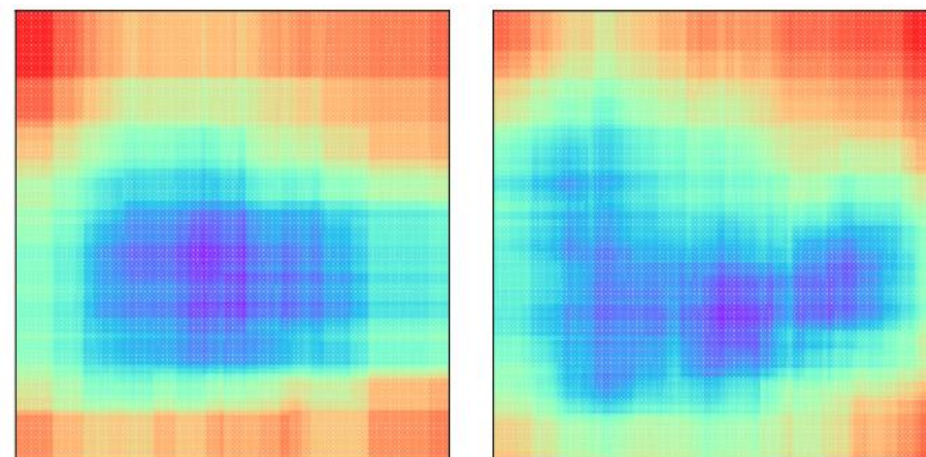
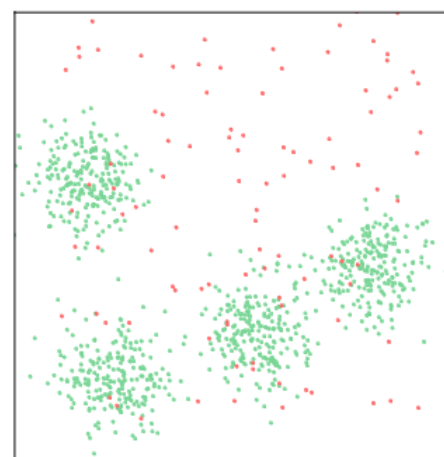


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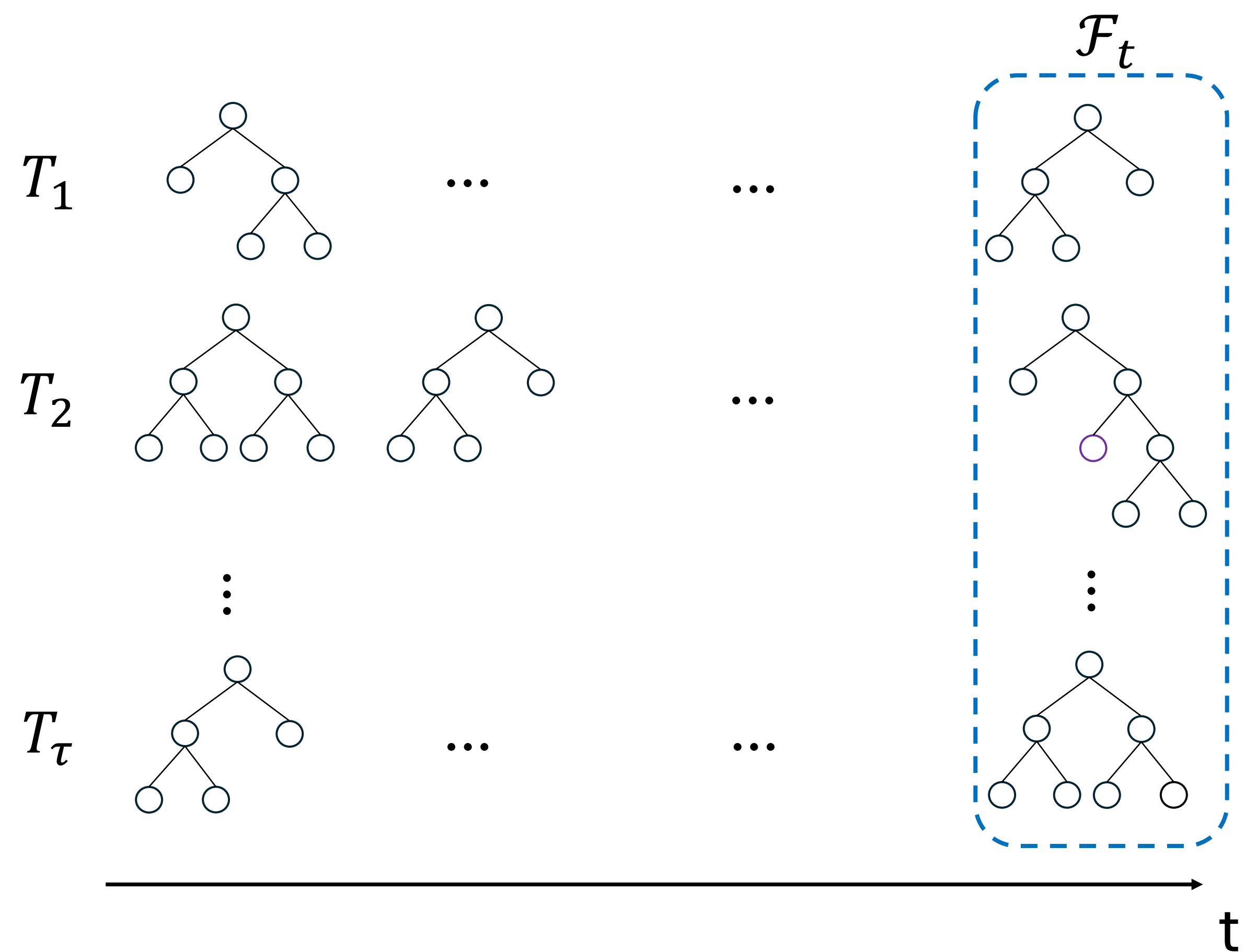


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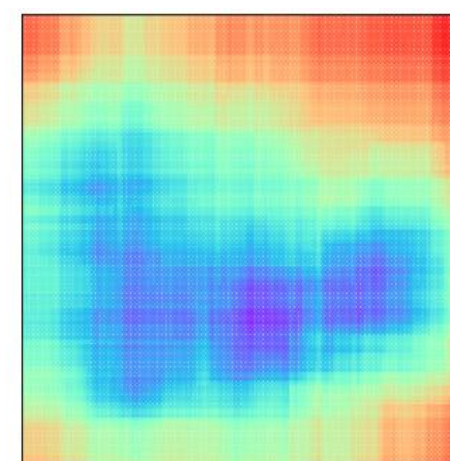
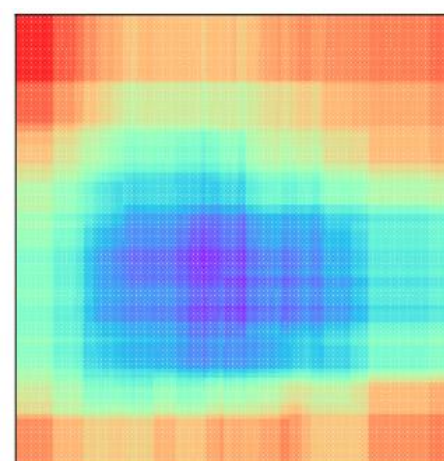
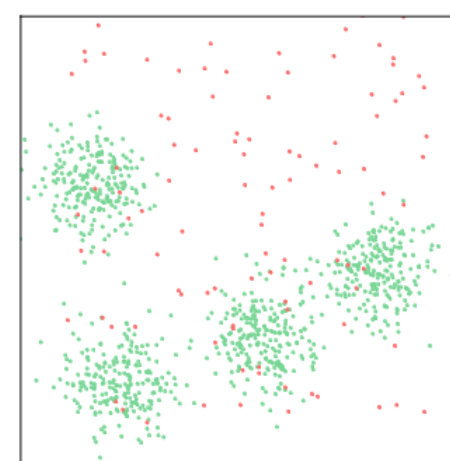


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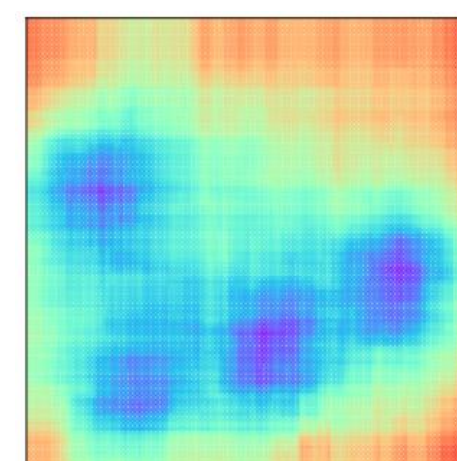


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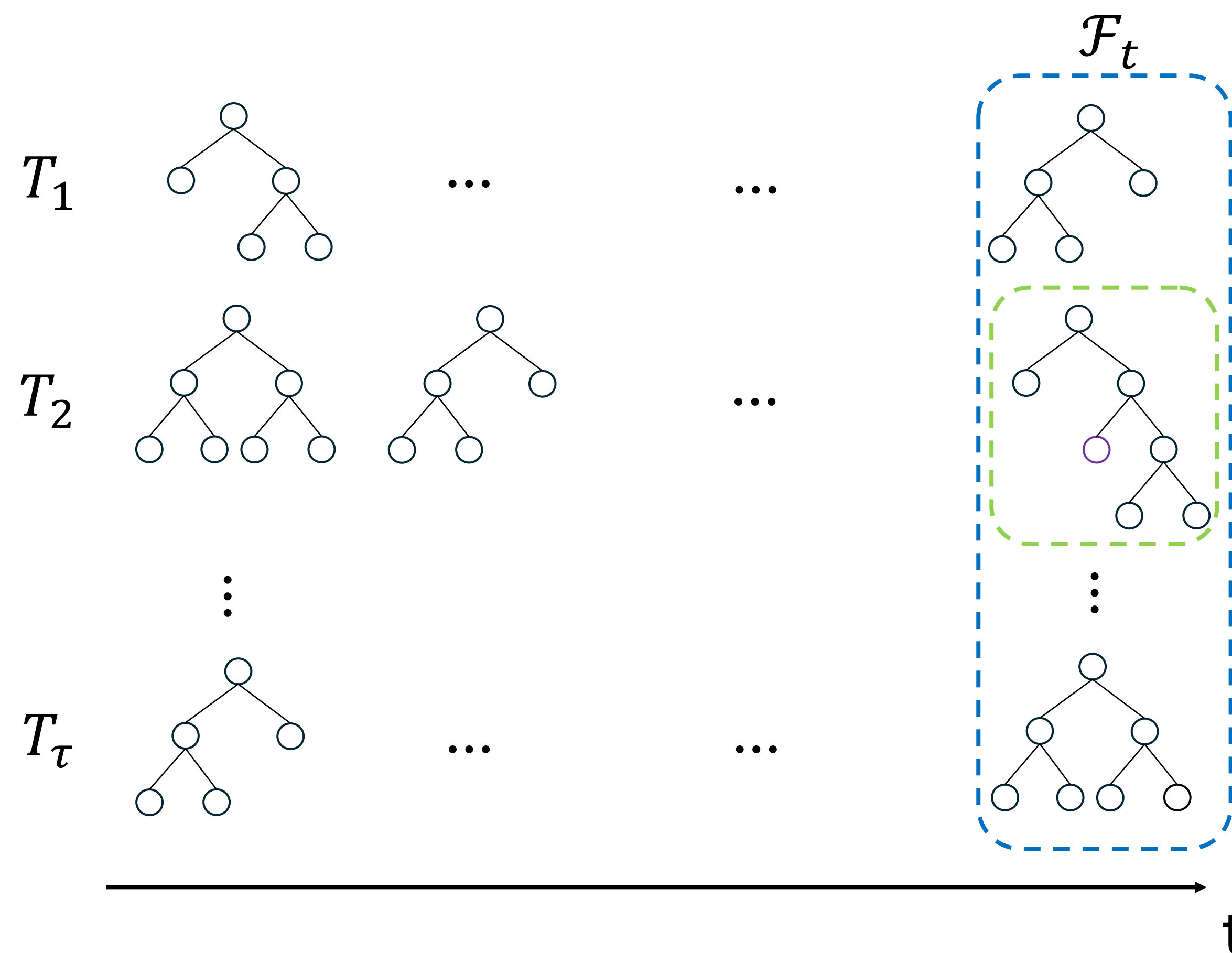
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...

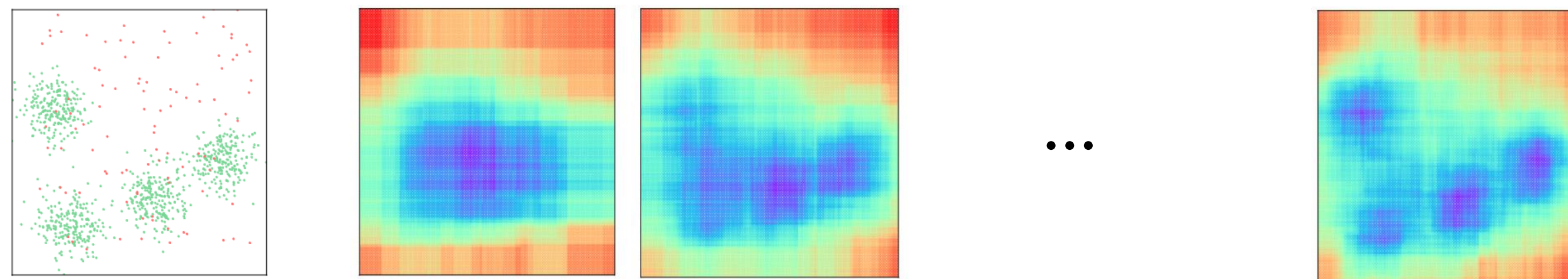


Solution idea

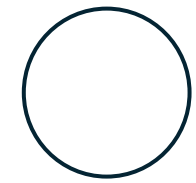
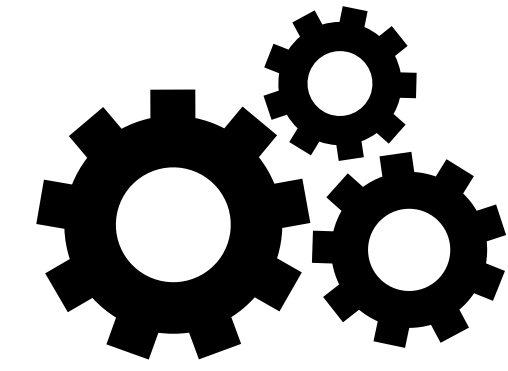


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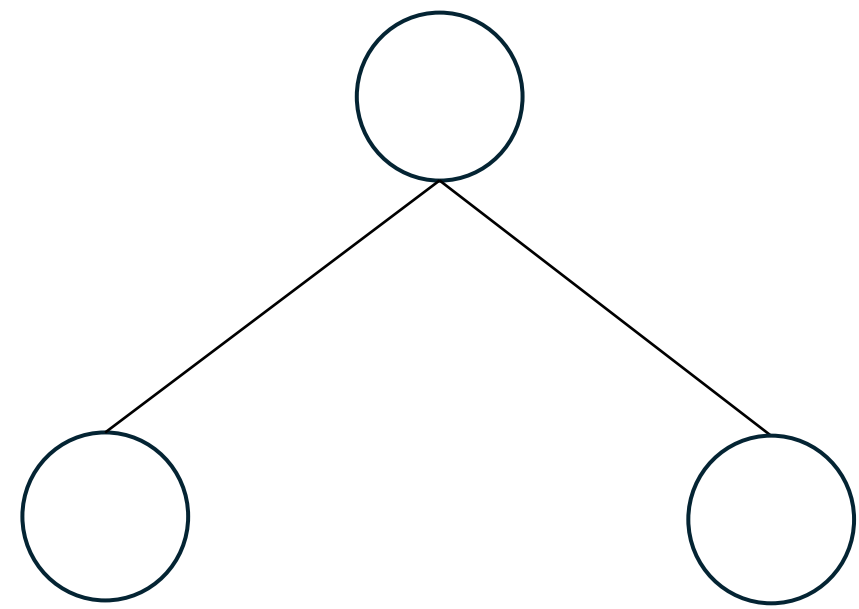
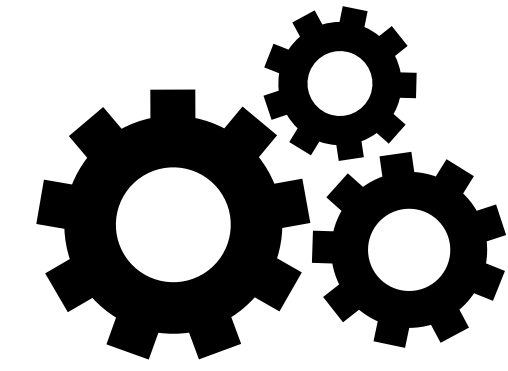


Online Isolation Tree



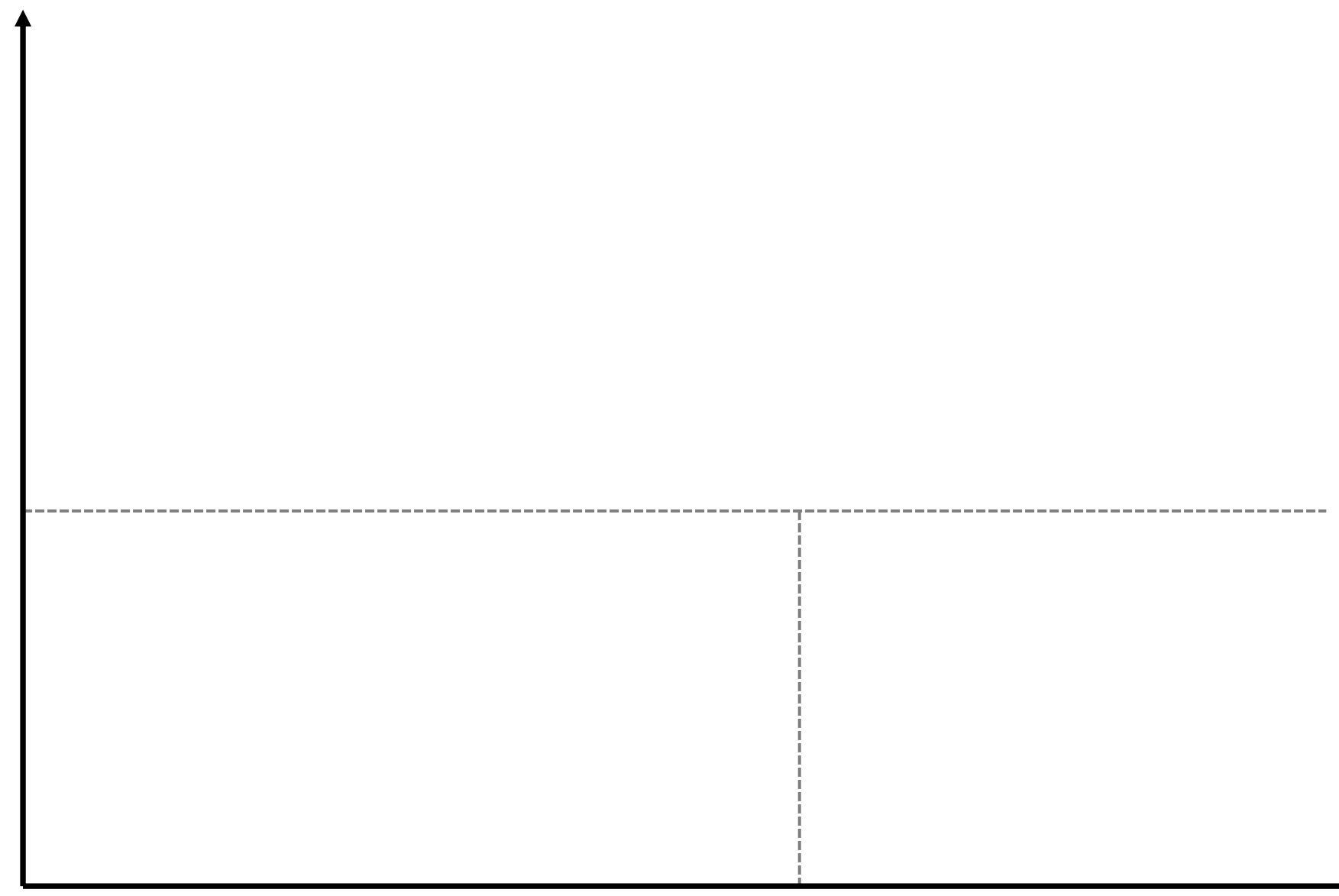
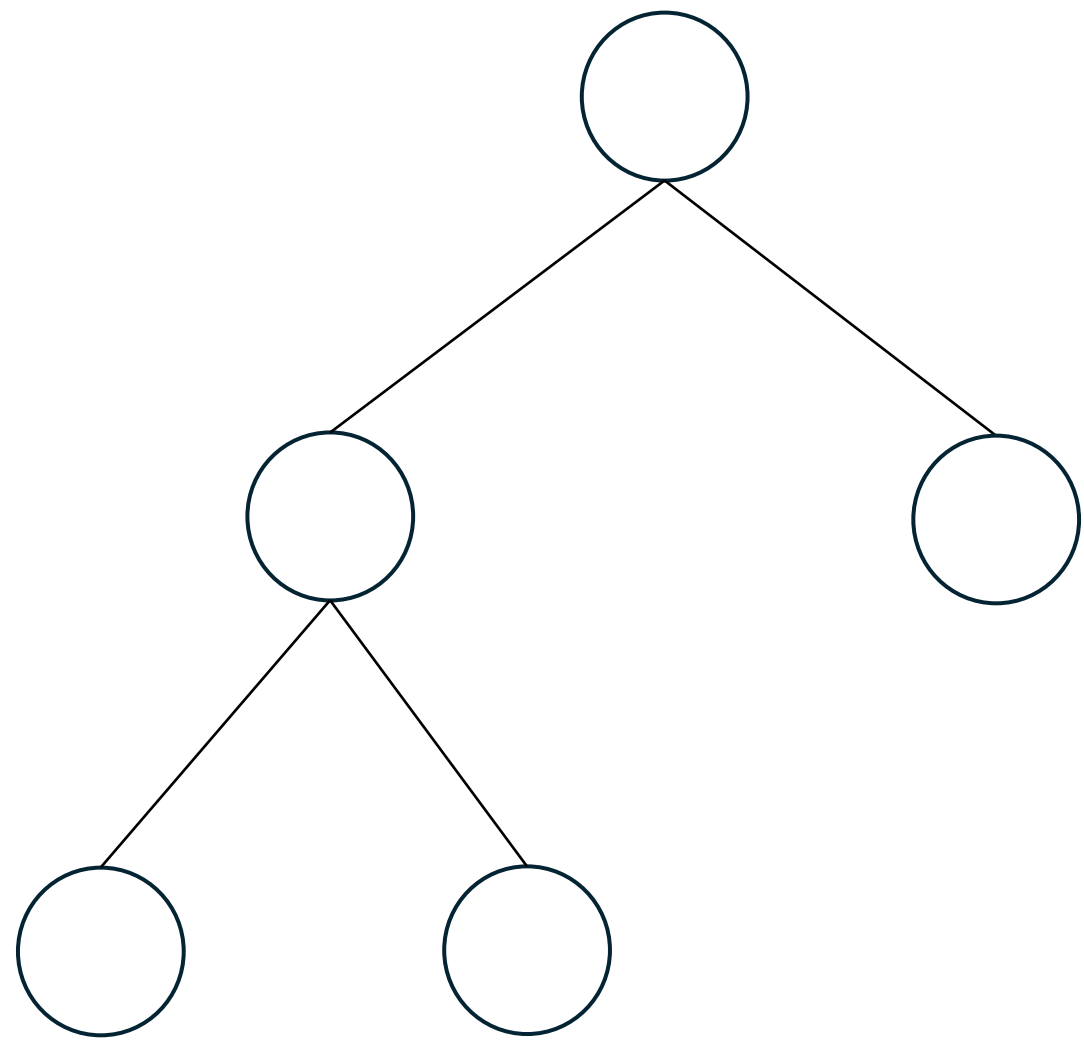
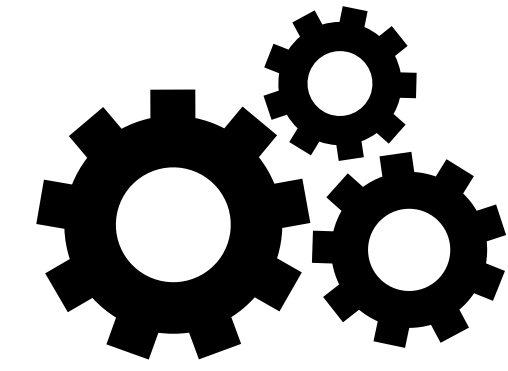
Online Isolation Tree is a ***d*-dimensional histogram** built by recursively splitting the input space $\mathbb{R}^d$.

Online Isolation Tree



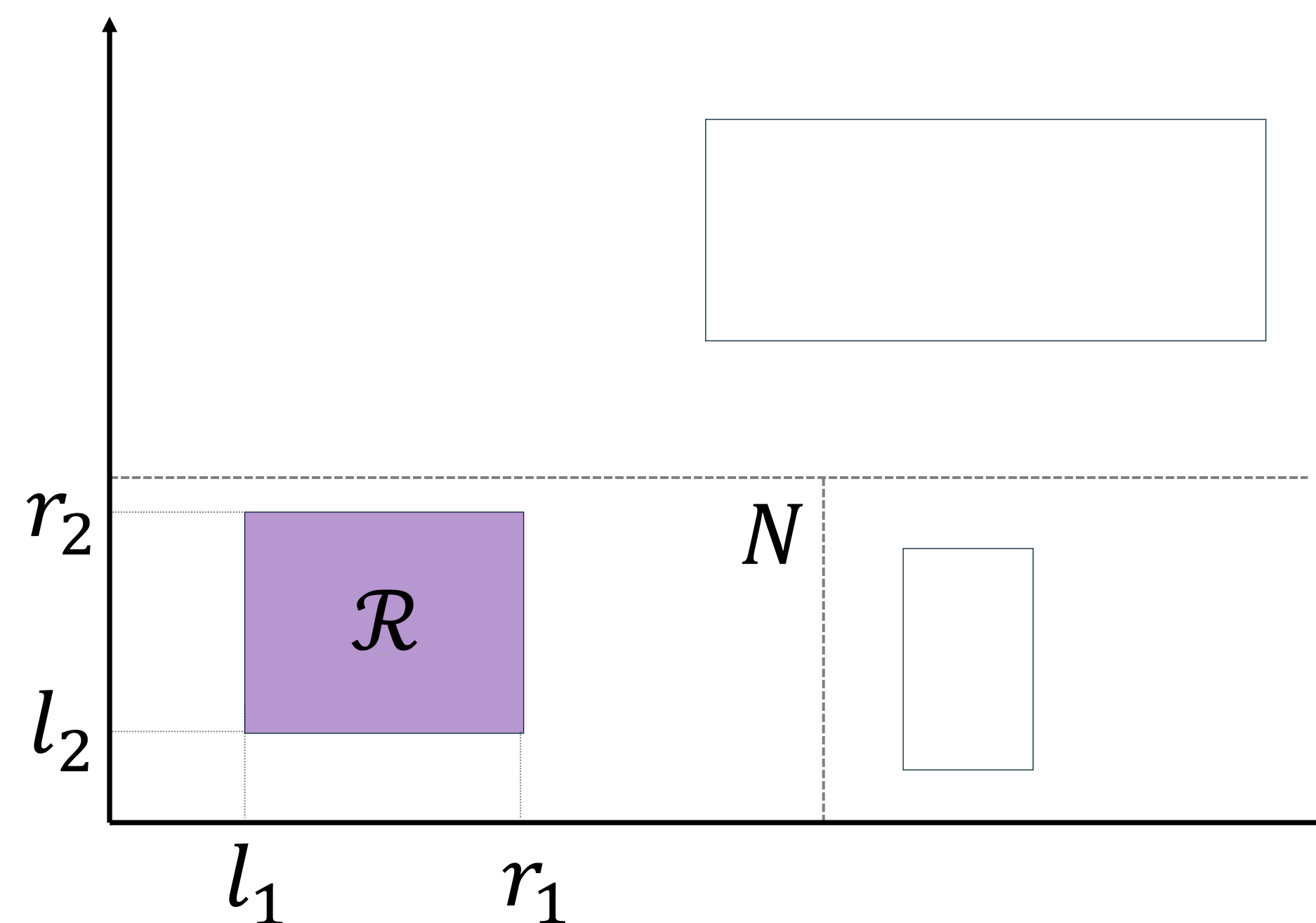
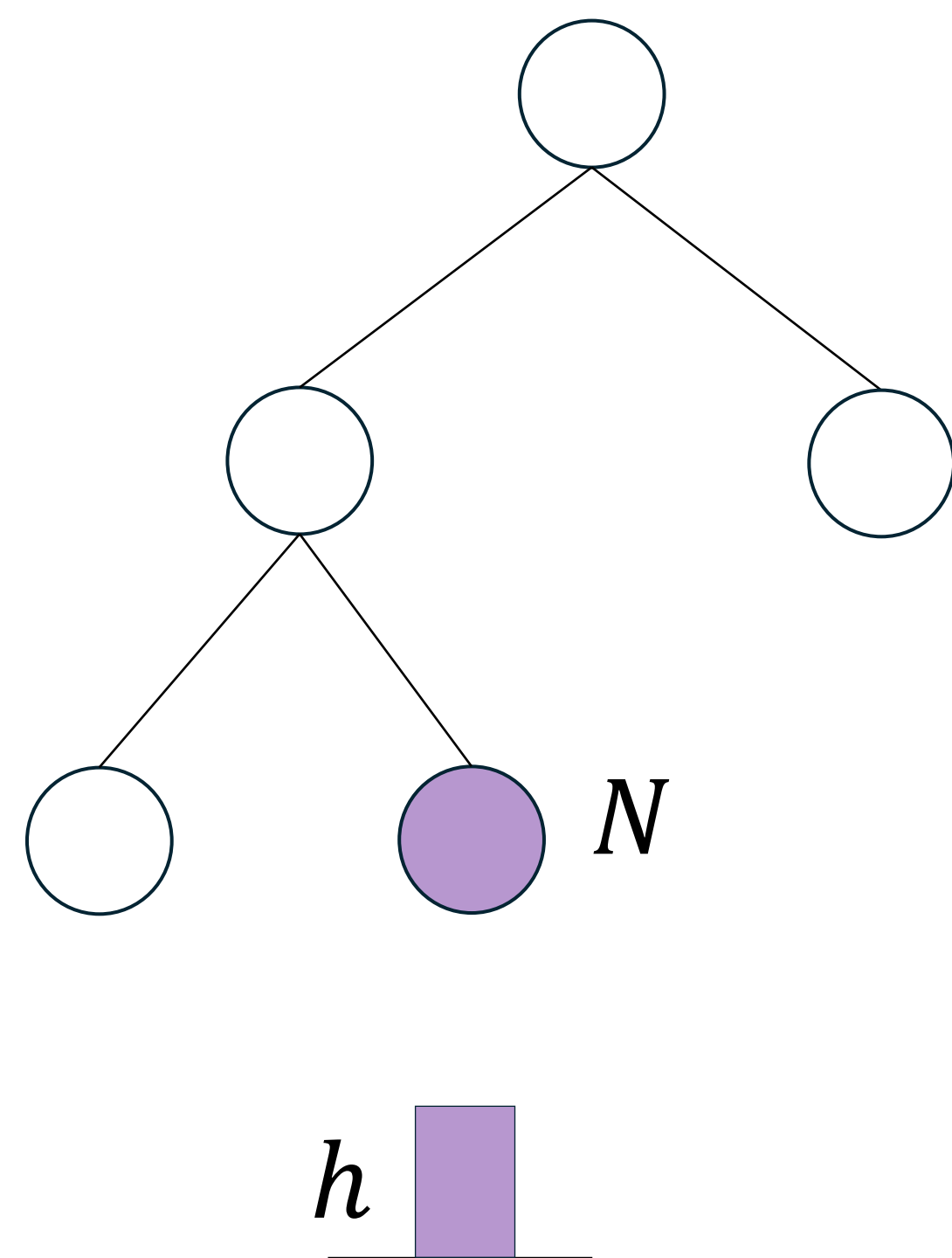
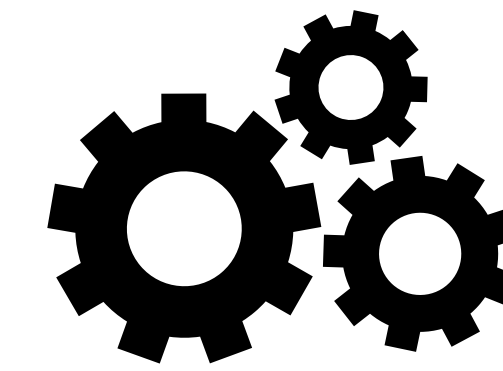
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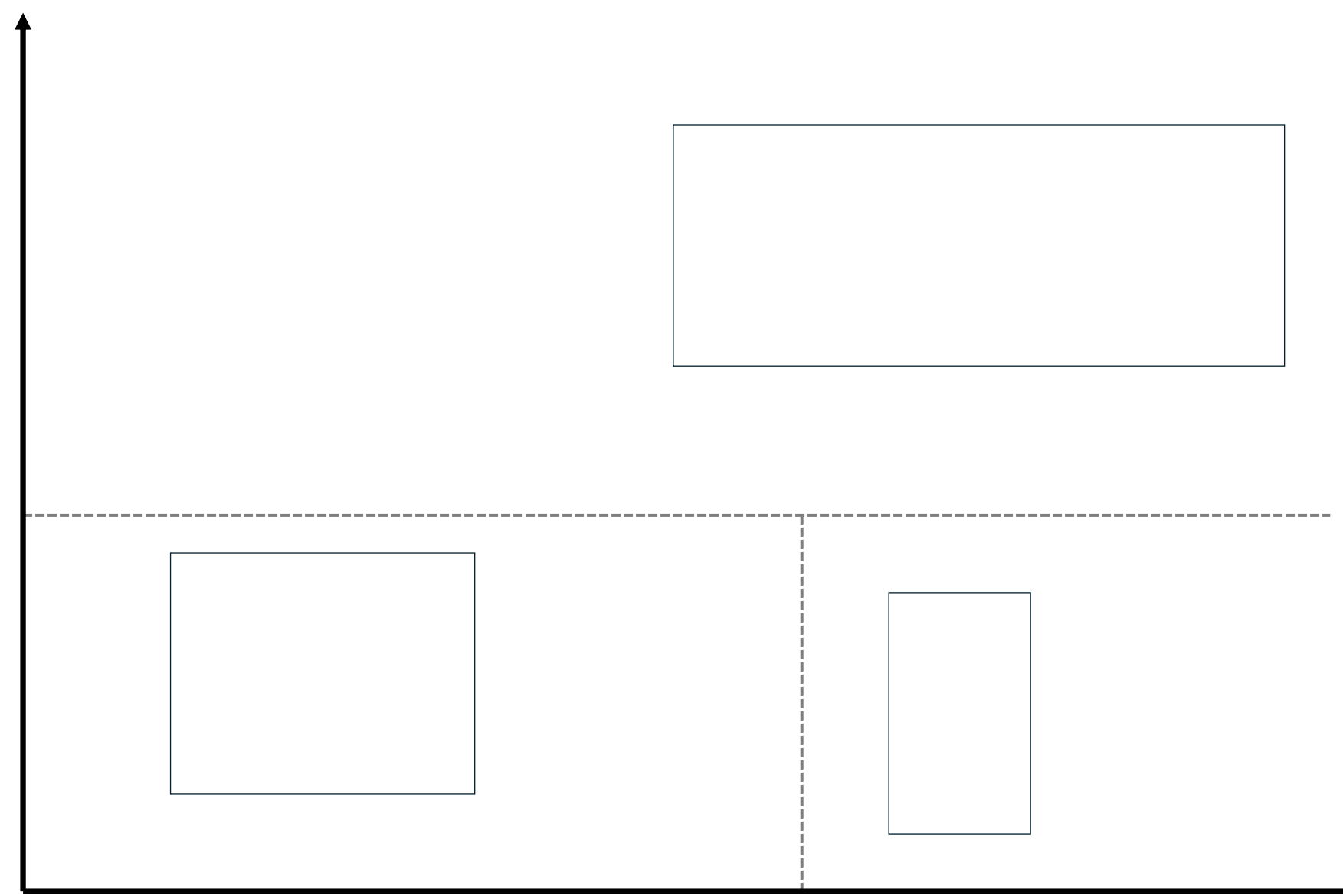
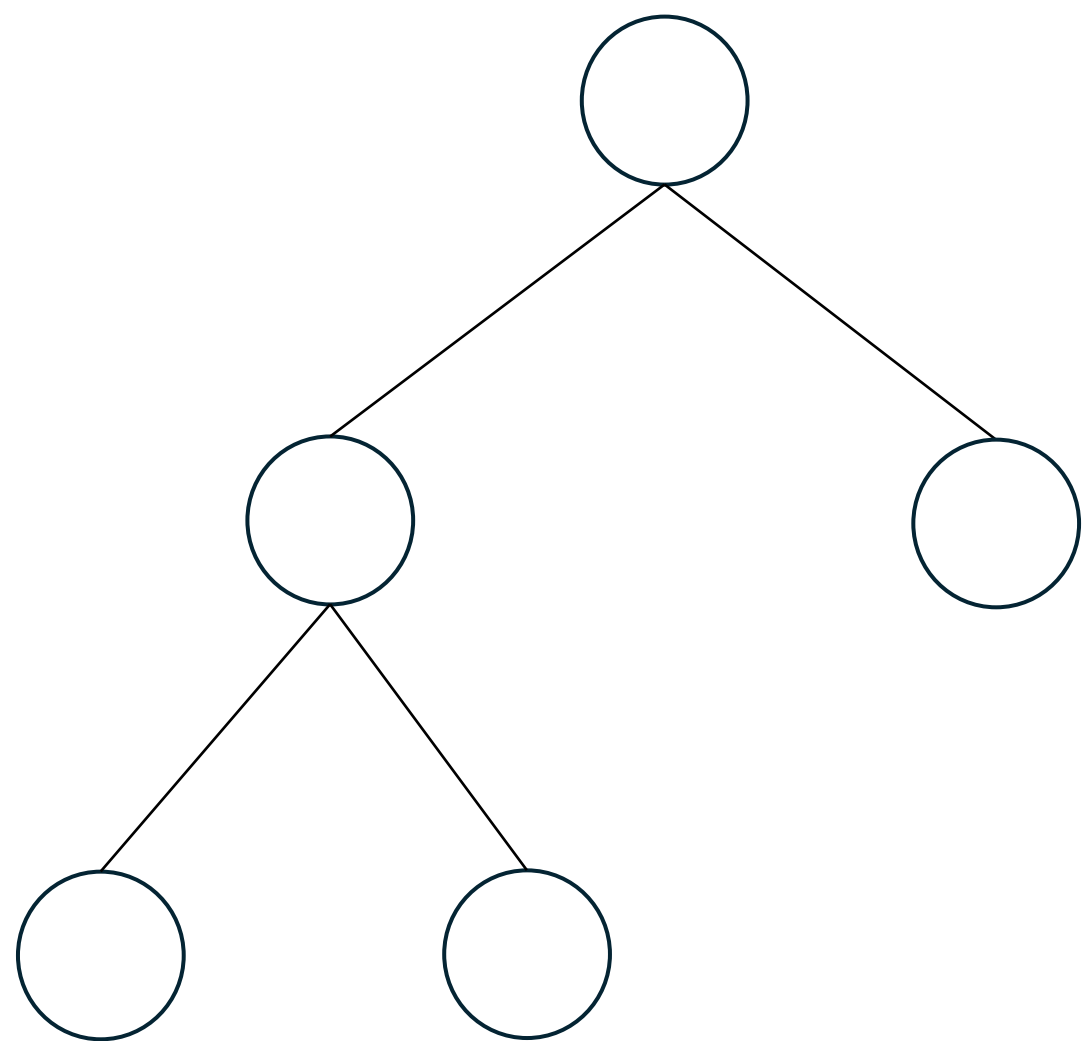


Online Isolation Tree is a *d*-dimensional **histogram** built by recursively splitting the input space $\mathbb{R}^d$.

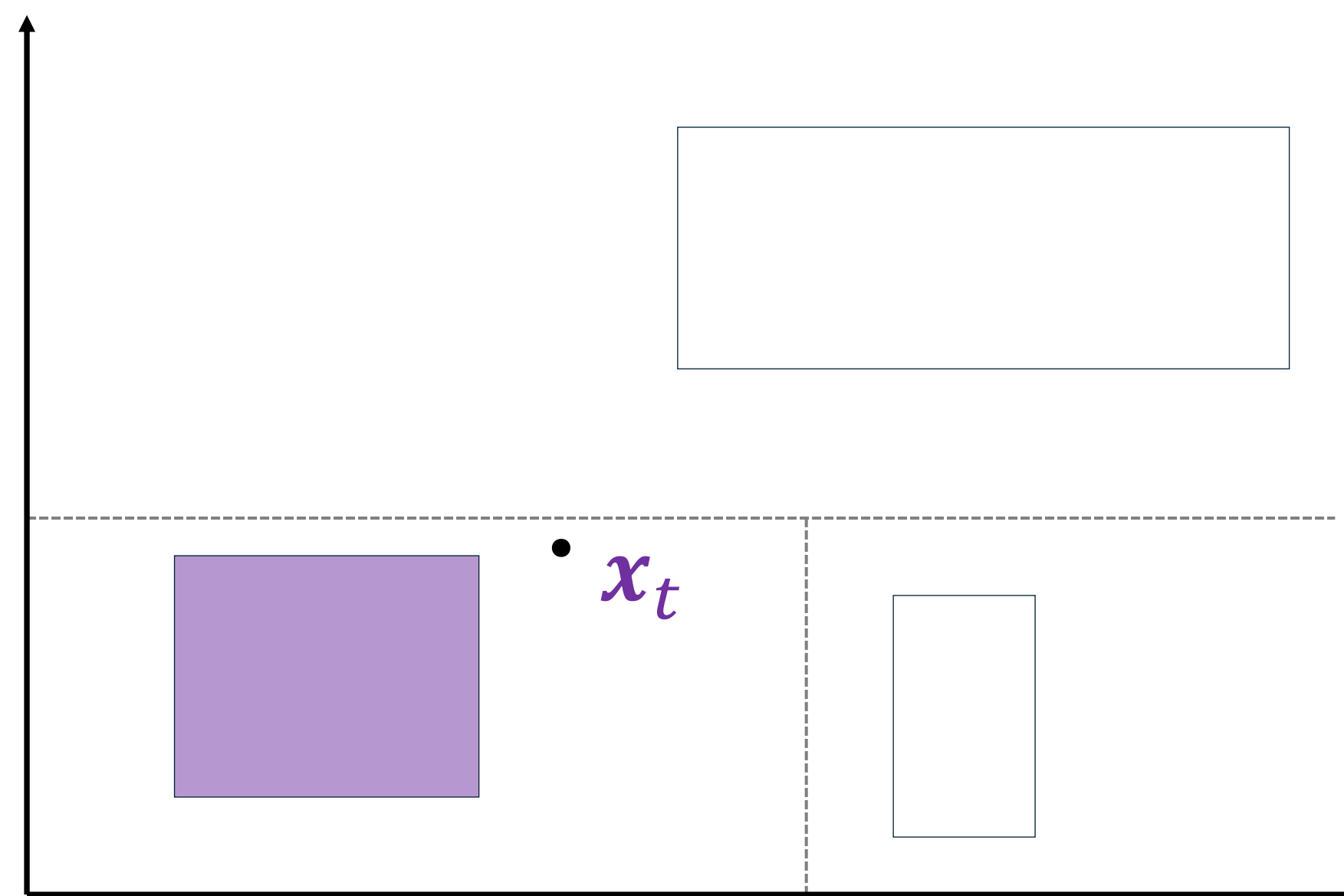
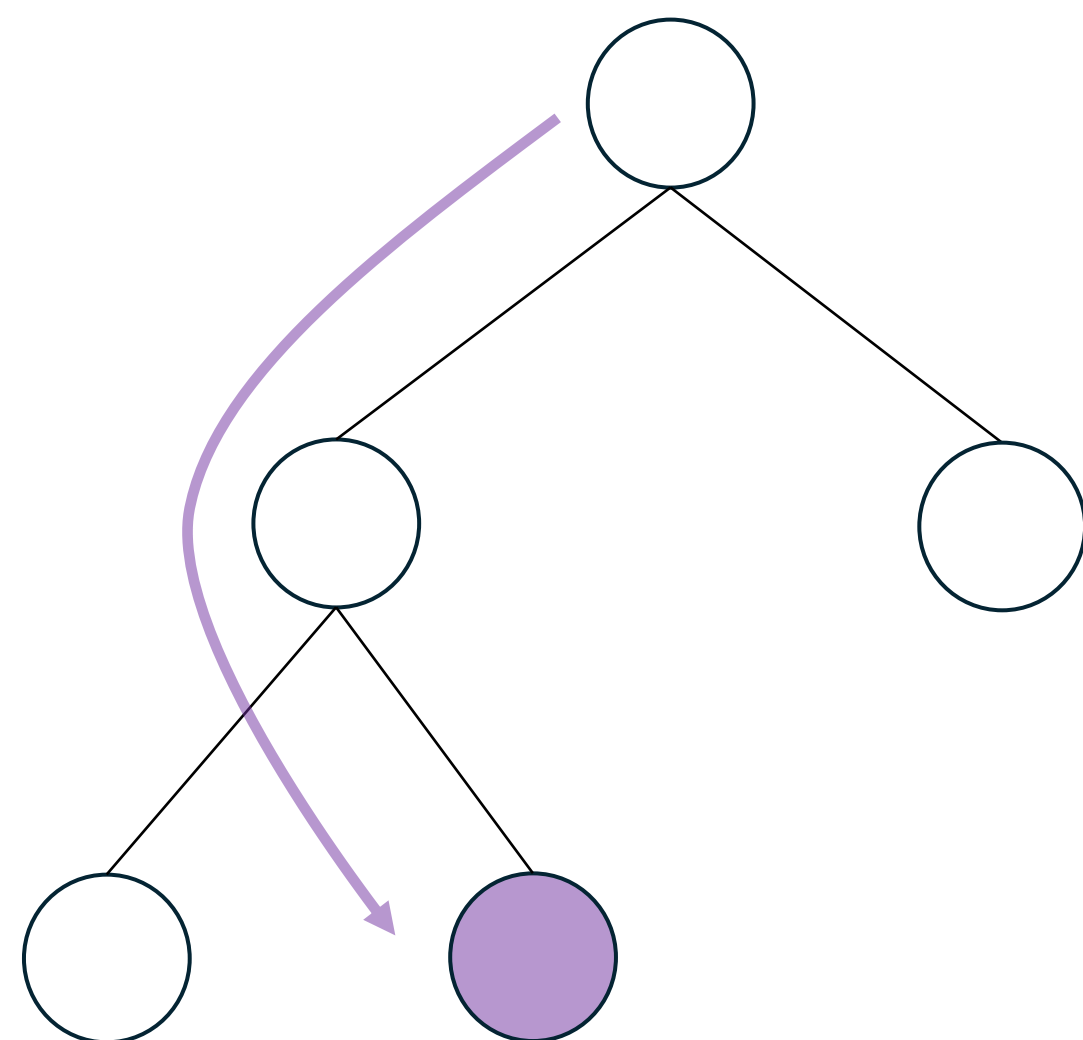
Each node *N* is characterized as $N = (h, \mathcal{R})$, where:

- *h* is the number of points that crossed it, that is the bin height,
- $\mathcal{R} = \times_{i=1}^d [l_i, r_i]$ is the minimal bounding box that encloses them, that is the support of the bin.

Learning procedure

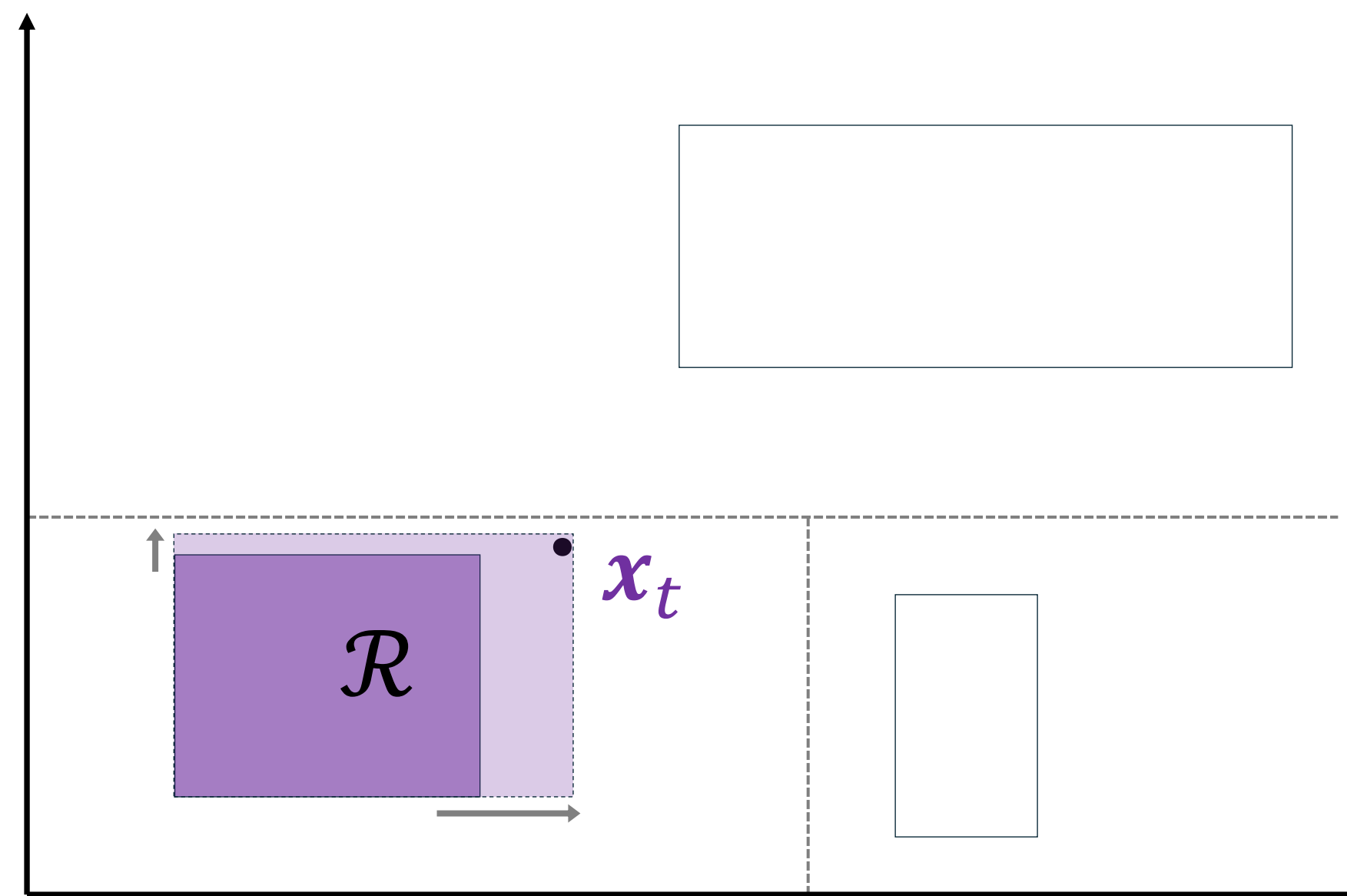
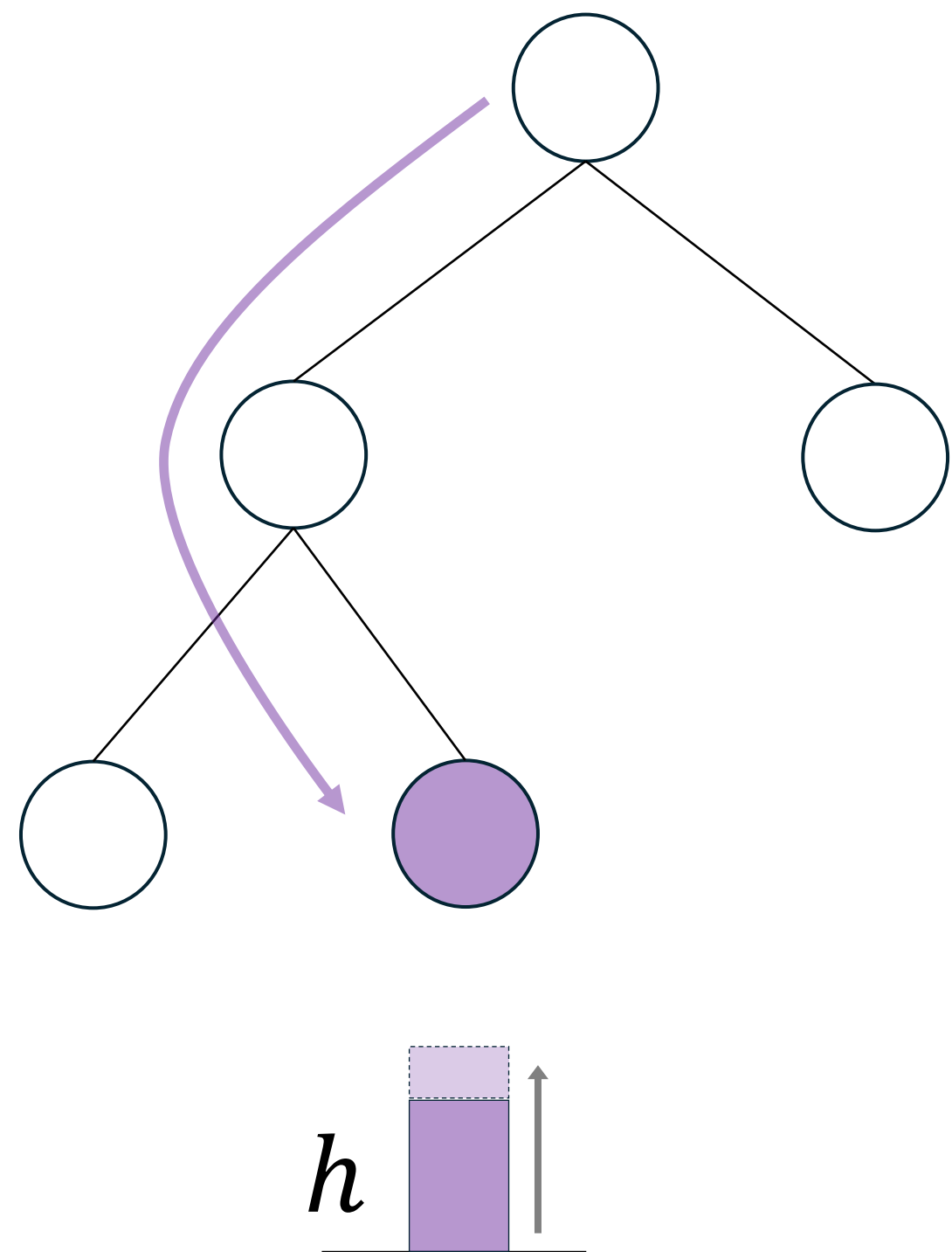


Learning procedure



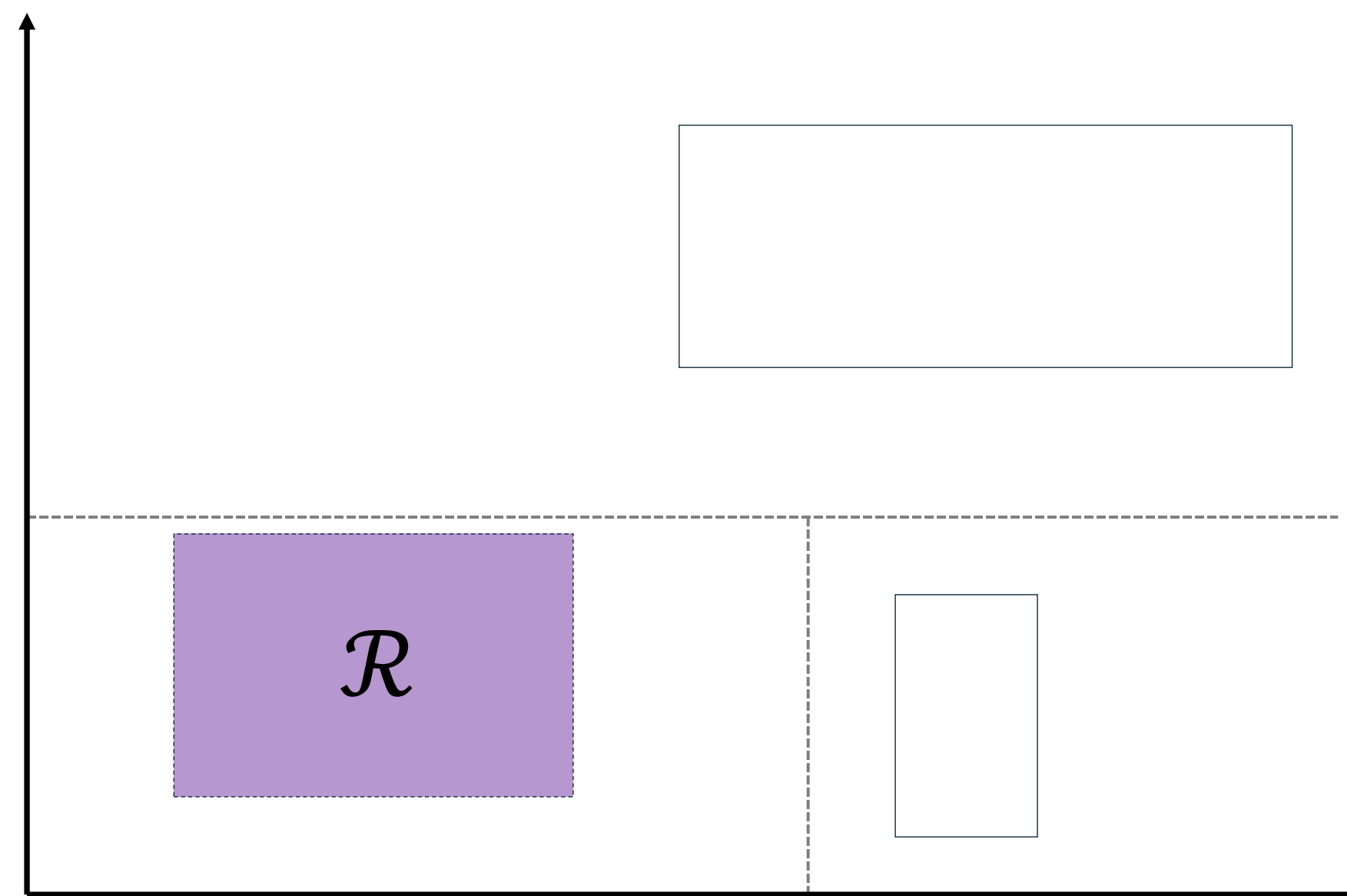
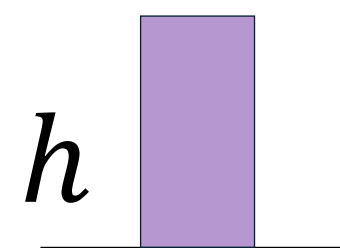
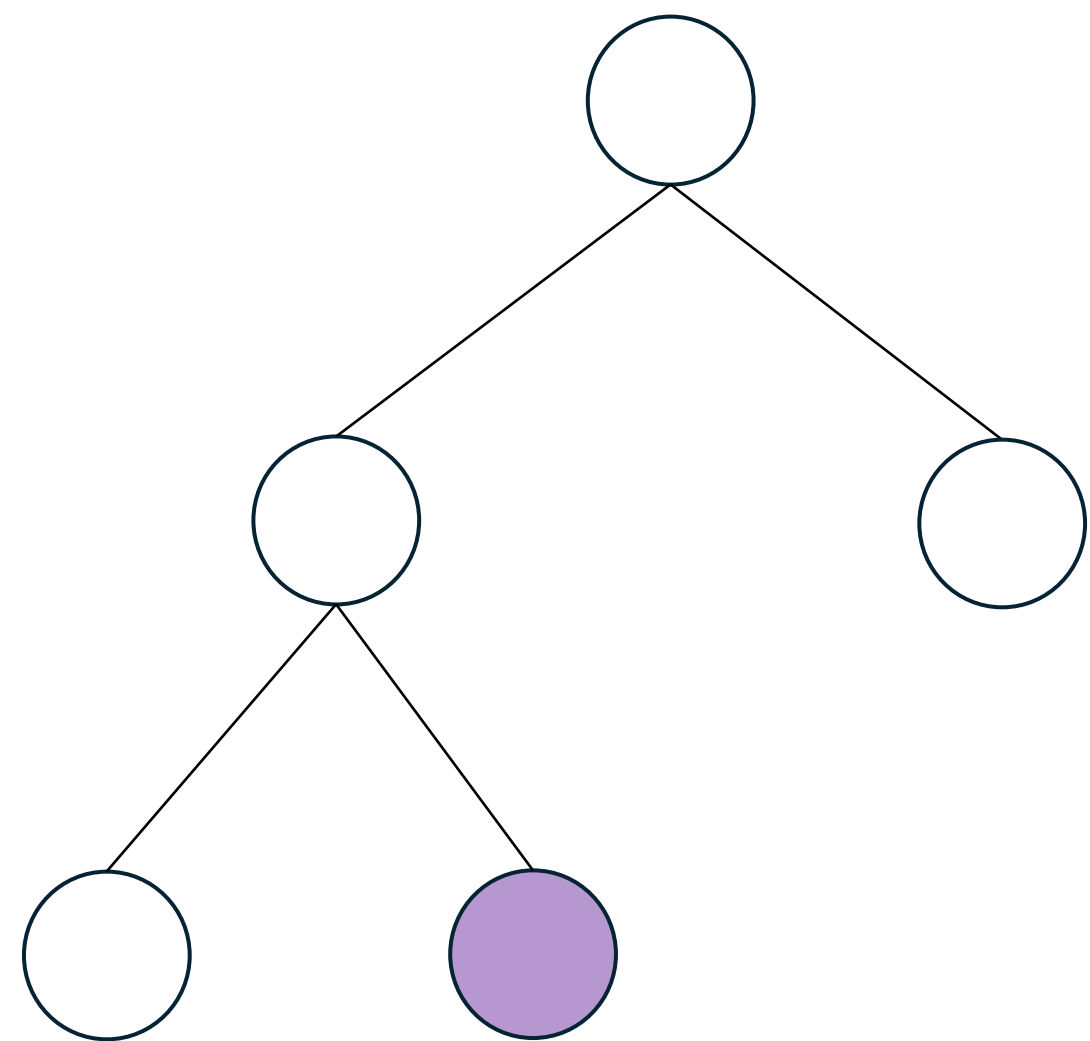
At each new point x_t

Learning procedure



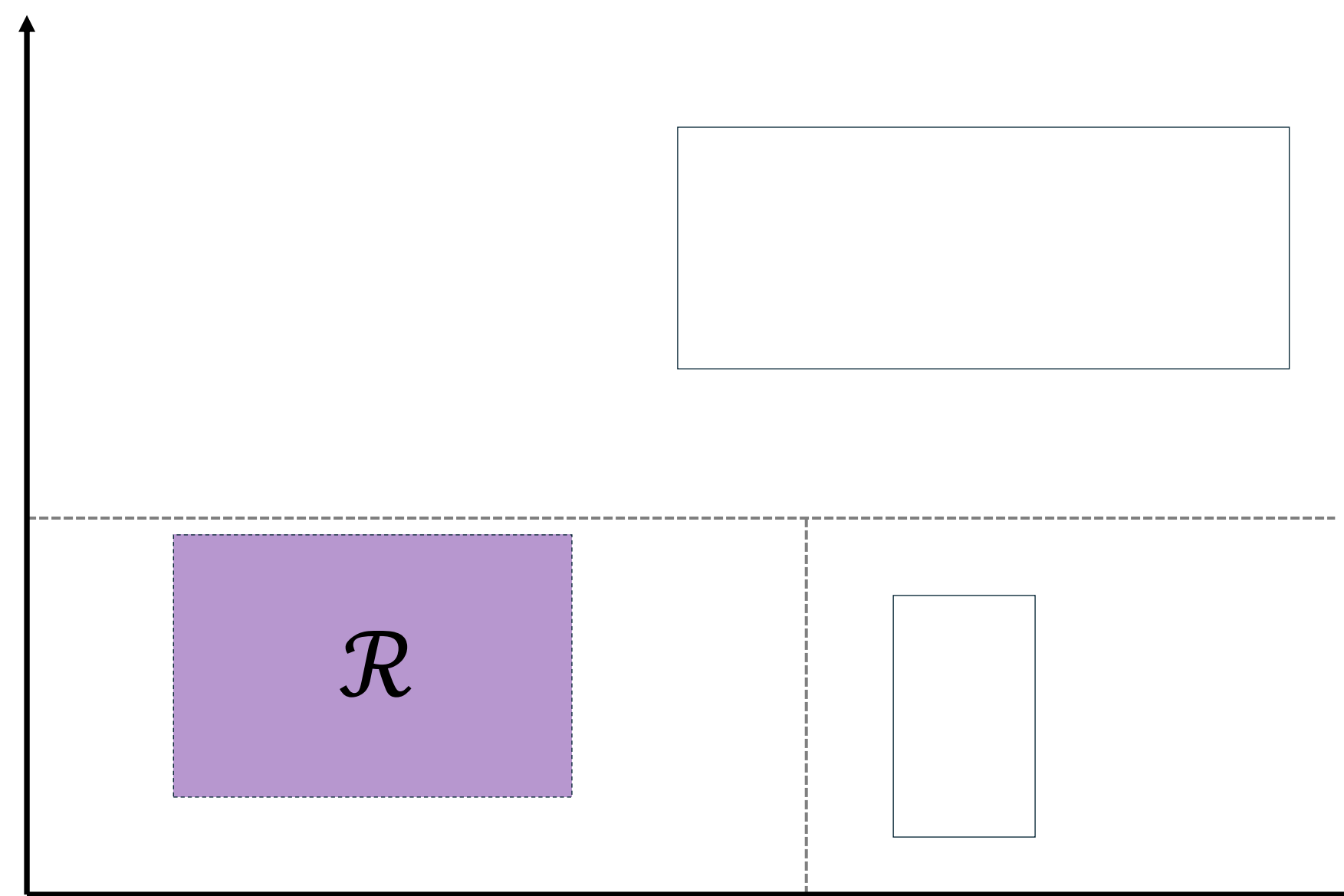
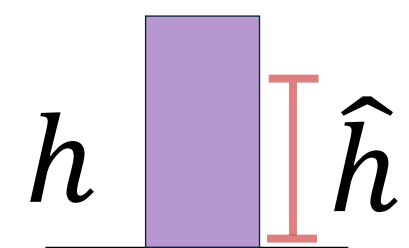
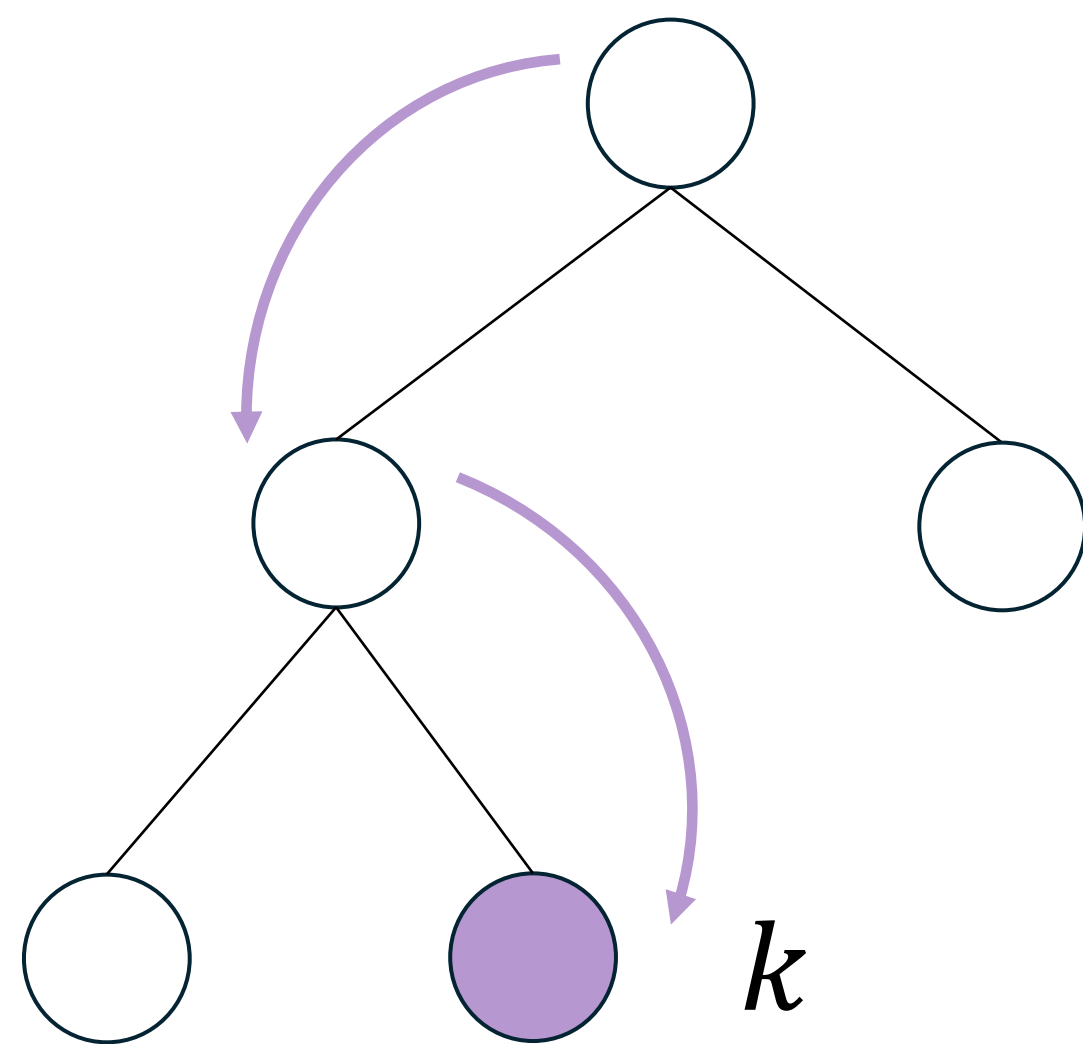
At each new point x_t , the heights h and supports $\mathcal{R}$ are updated.

Learning procedure



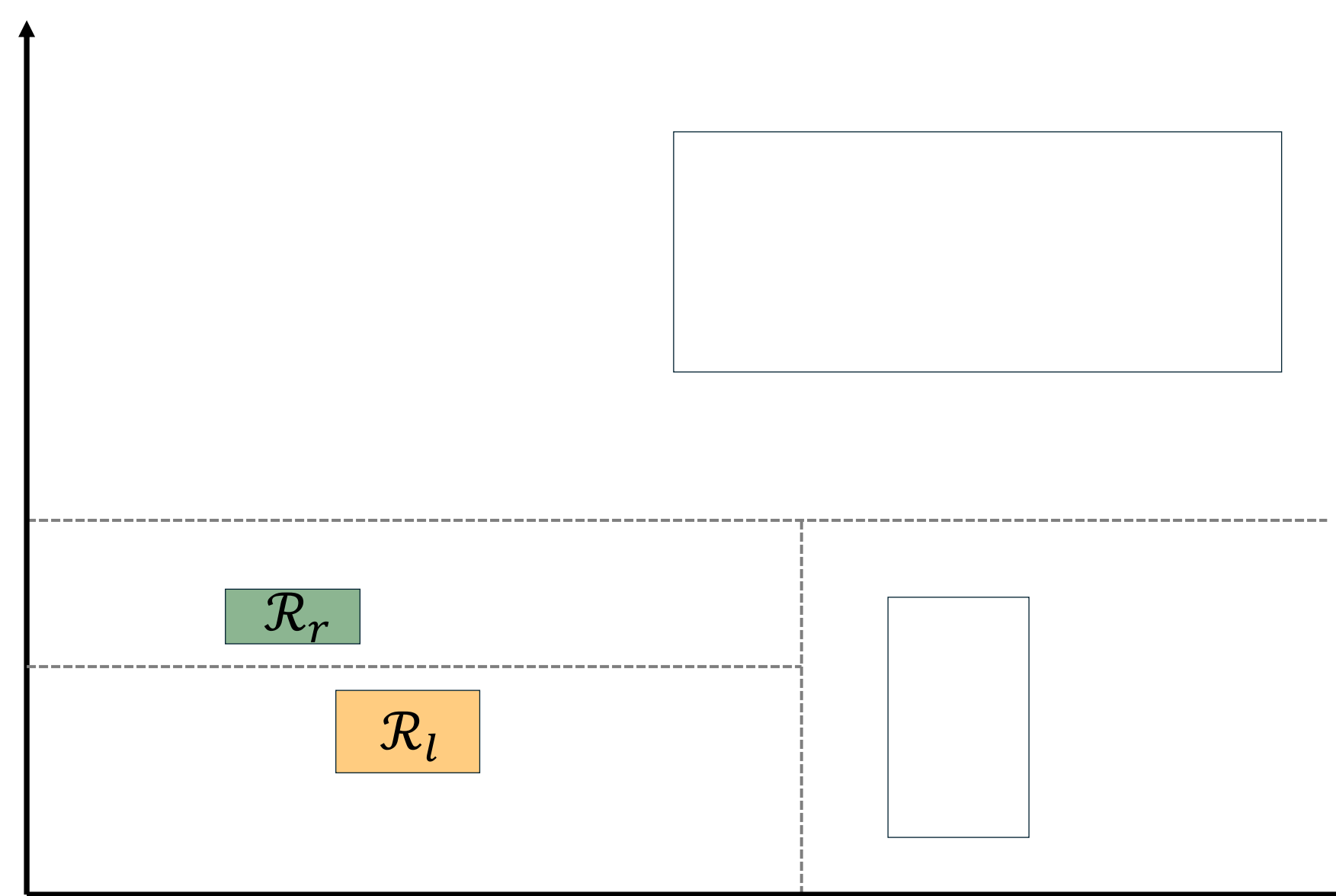
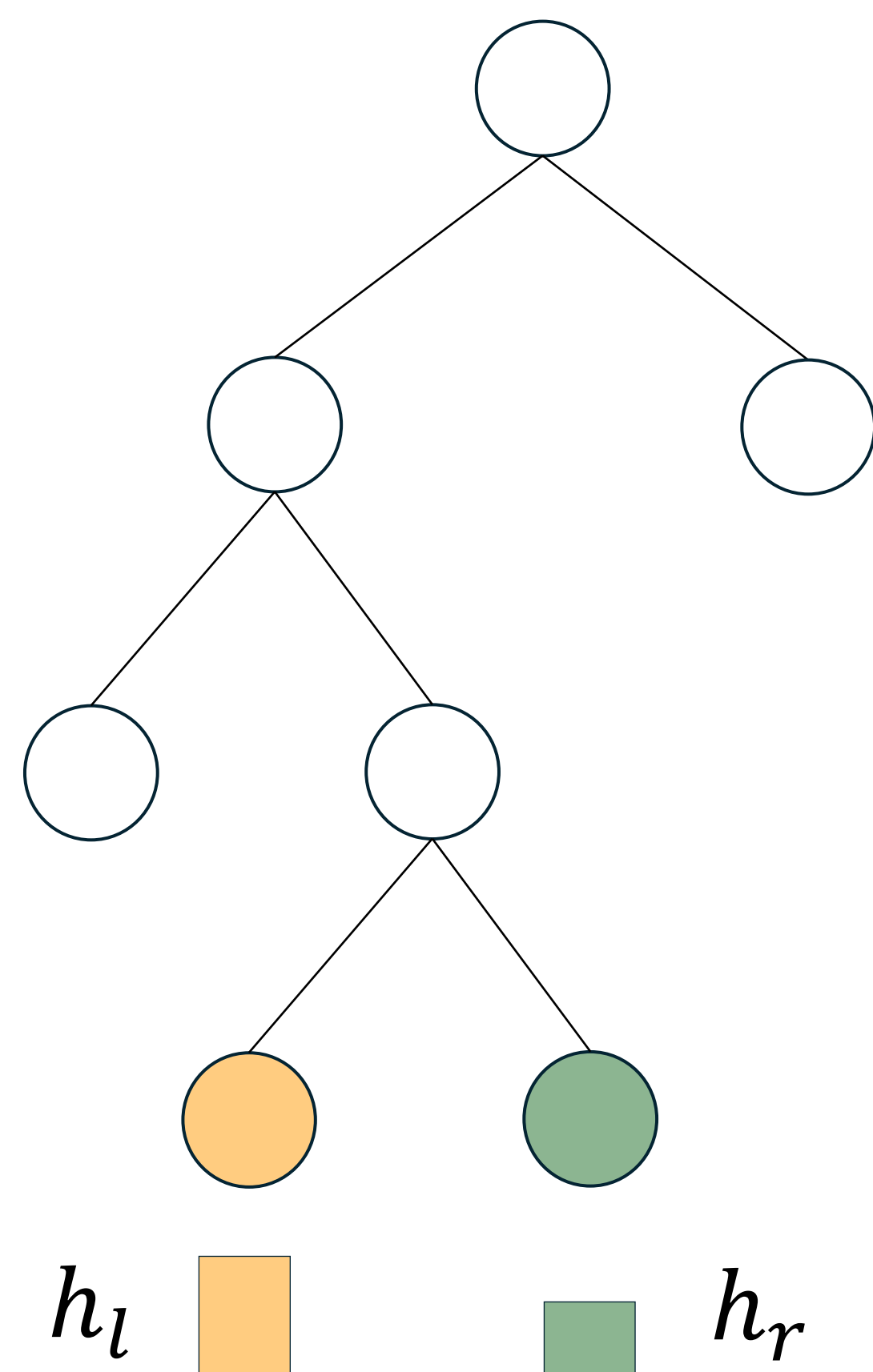
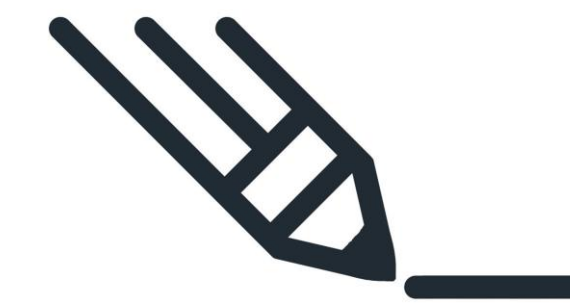
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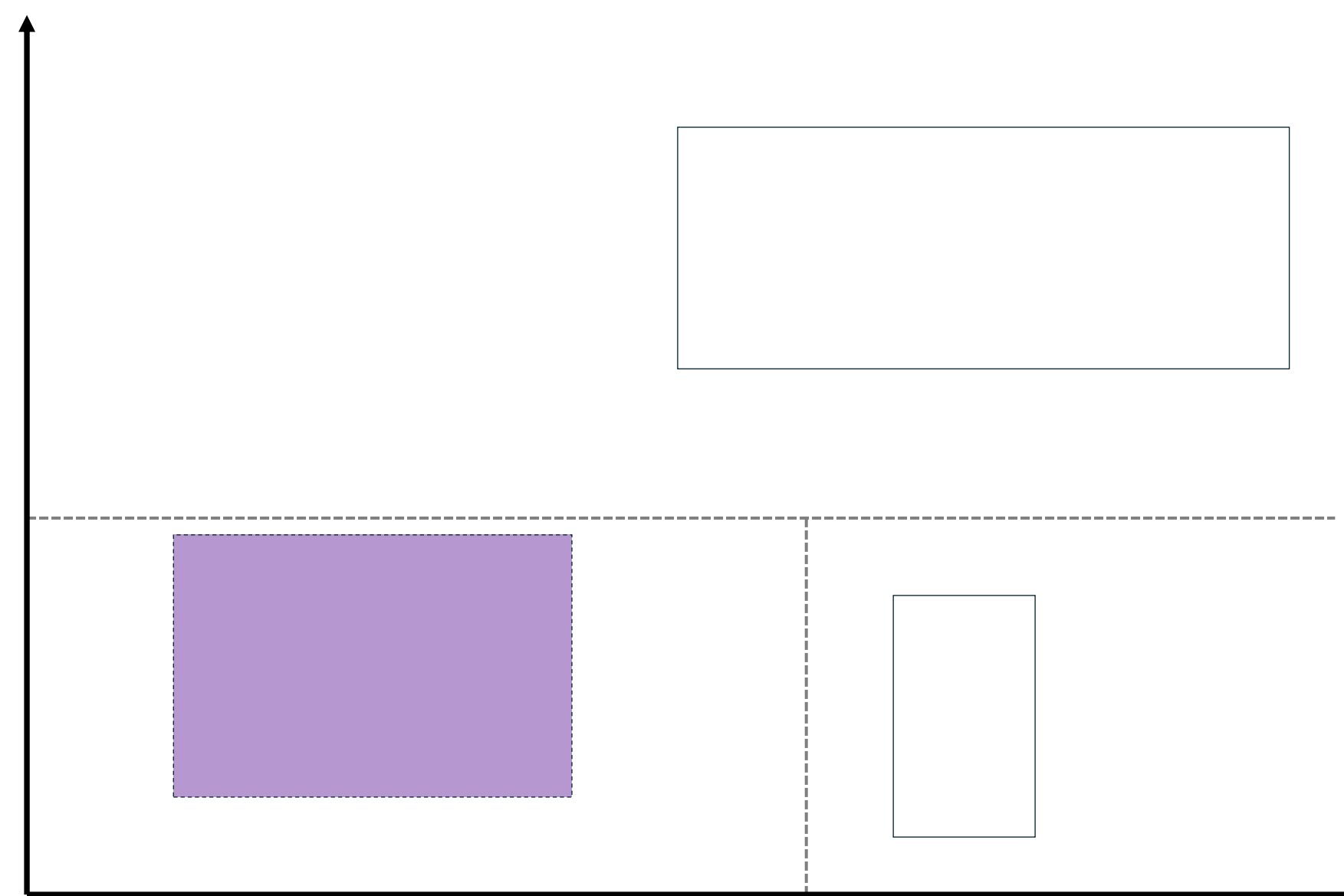
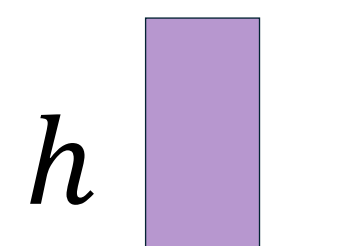
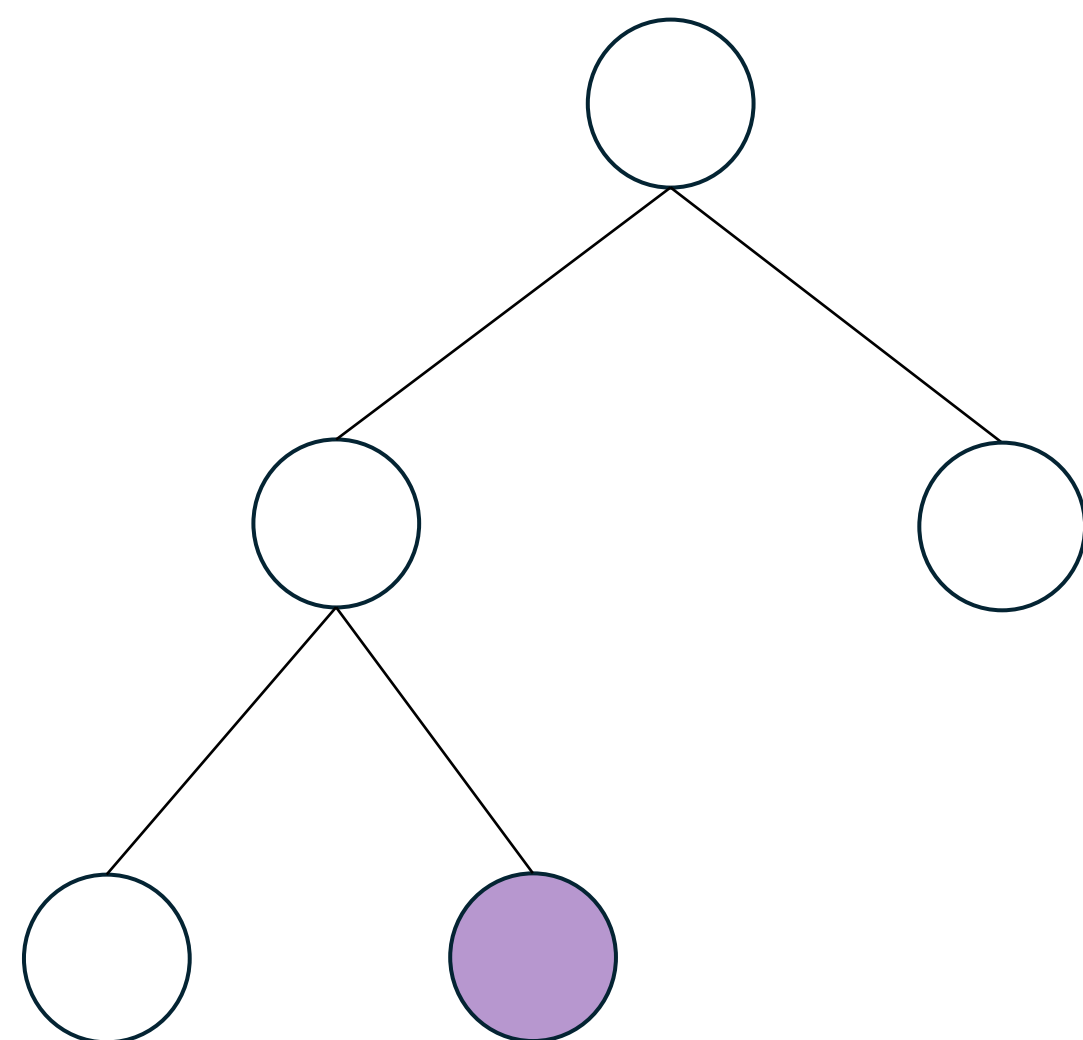
At each new point $\mathbf{x}_t$, the heights h and supports $\mathcal{R}$ are updated.
If the leaf height h reaches the maximum value $\hat{h} = \eta 2^k$

Learning procedure

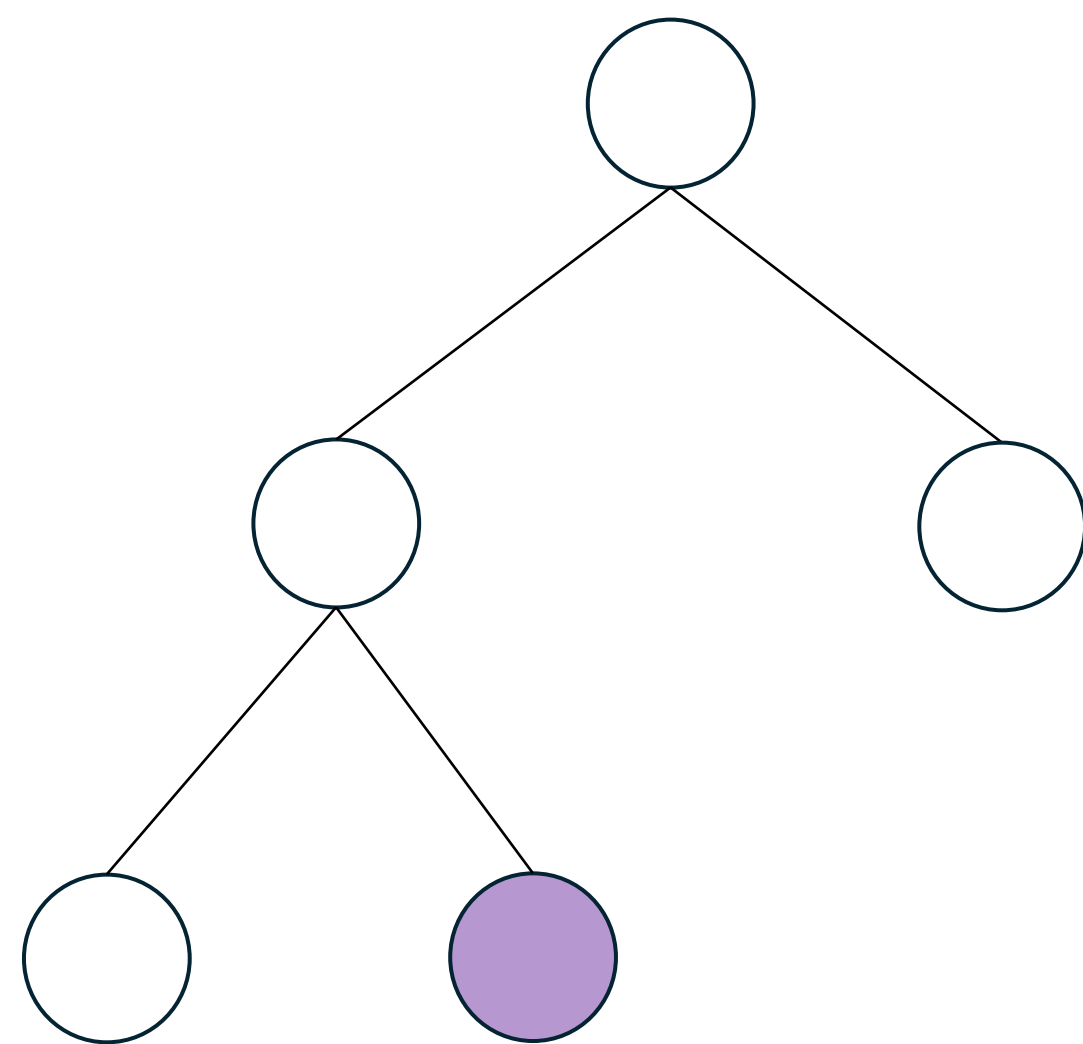


At each new point $\mathbf{x}_t$, the heights h and supports $\mathcal{R}$ are updated. If the leaf height h reaches the maximum value $\hat{h} = \eta 2^k$, we split the corresponding bin, leading to a **tree expansion**.

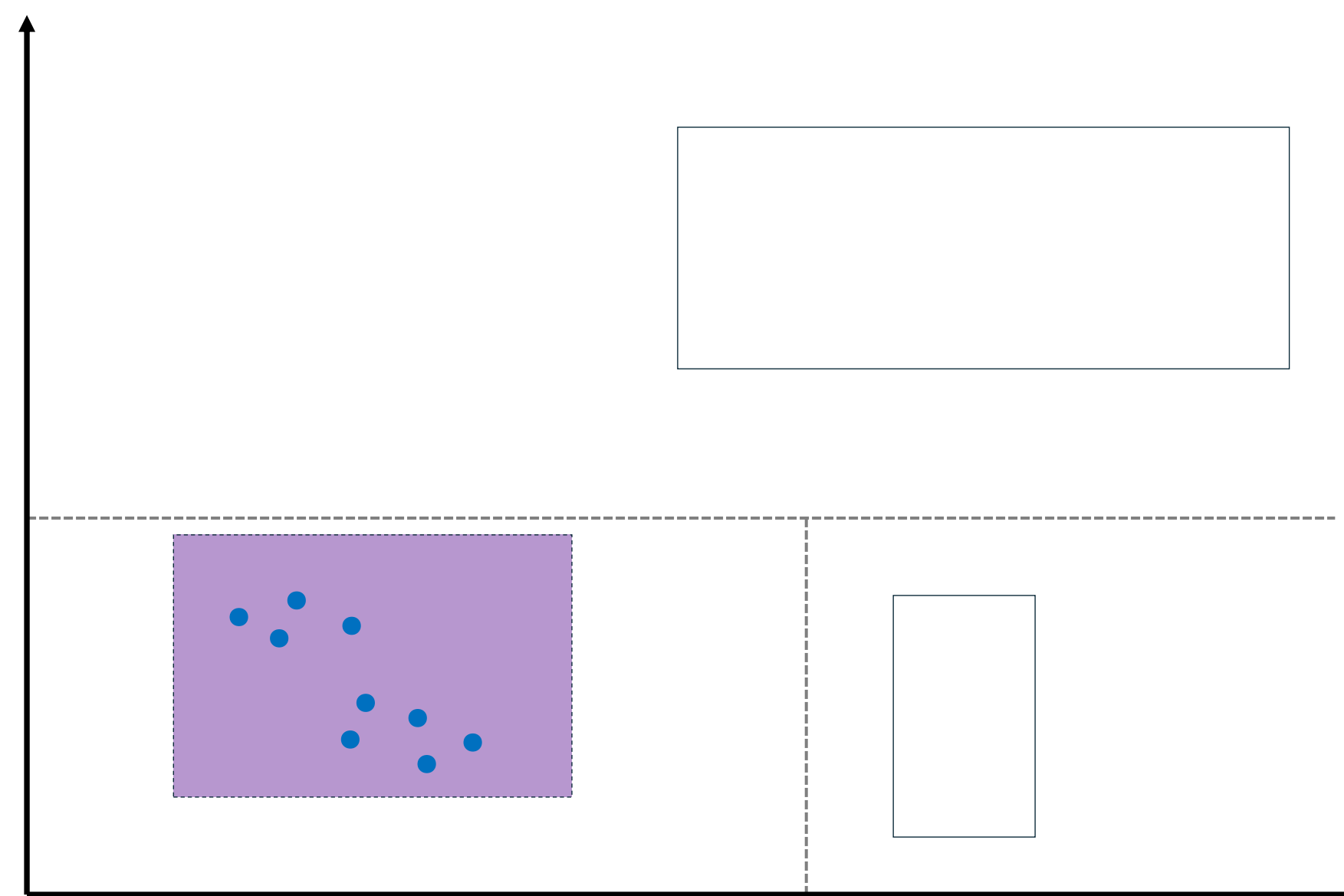
Learning procedure



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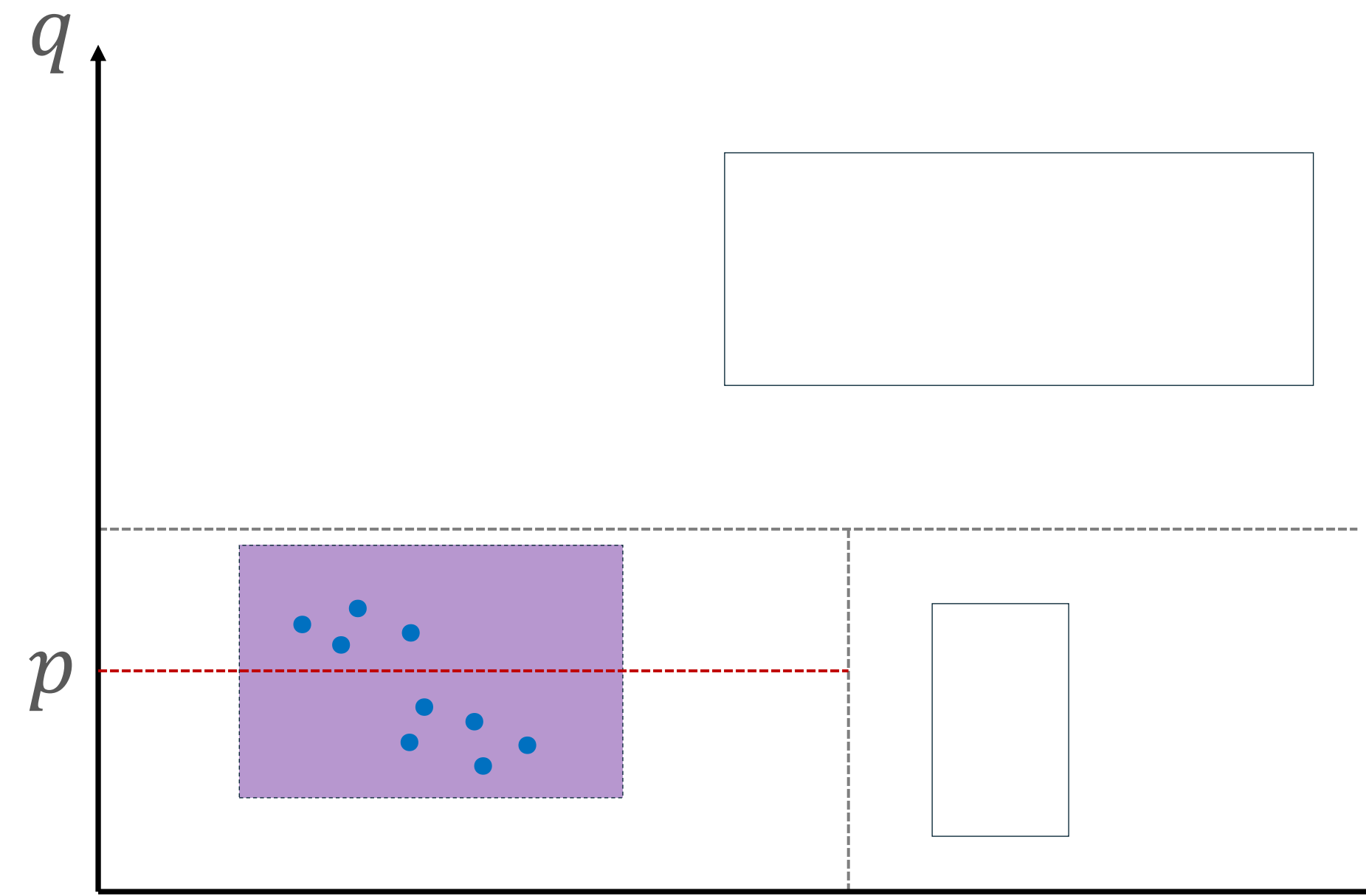
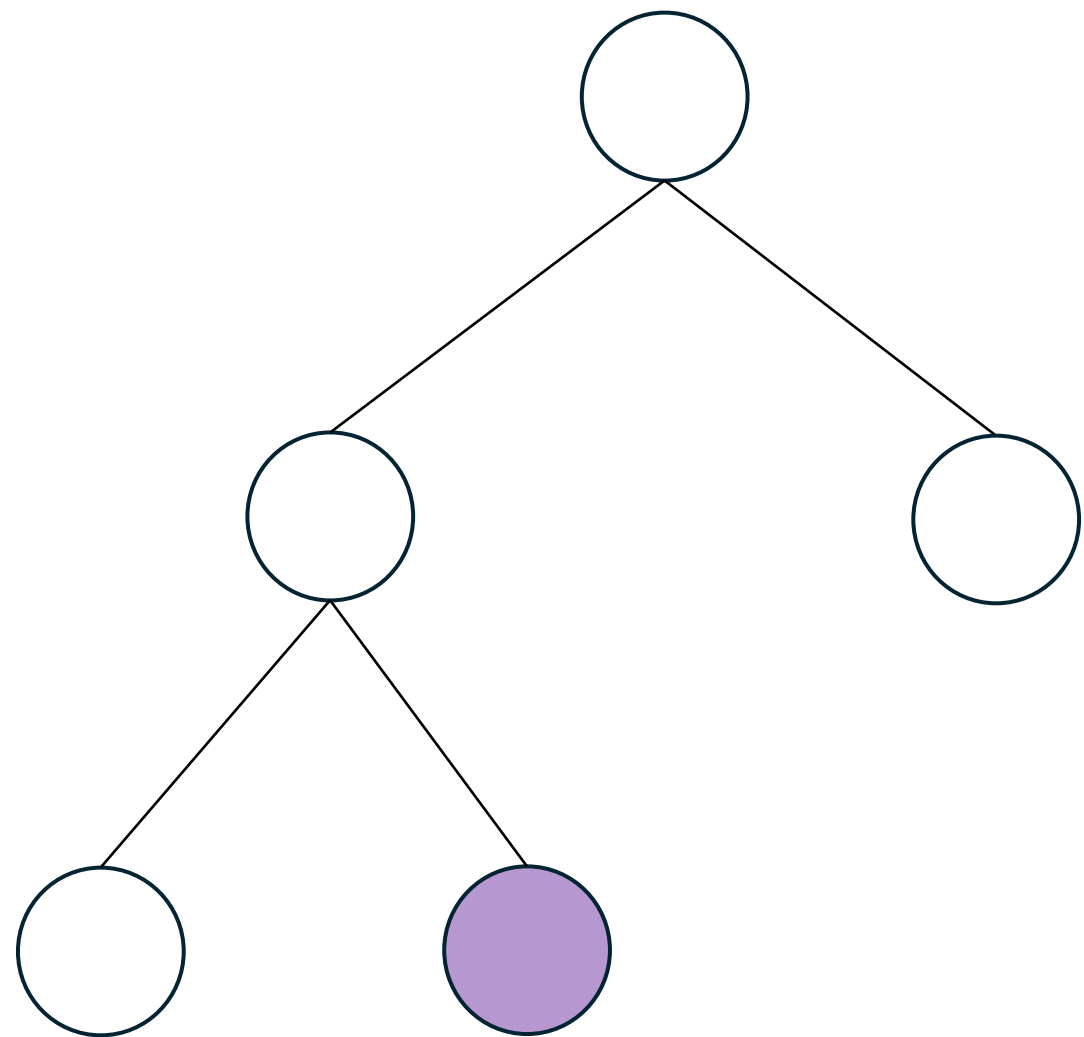


h

A solid purple rectangular bar.

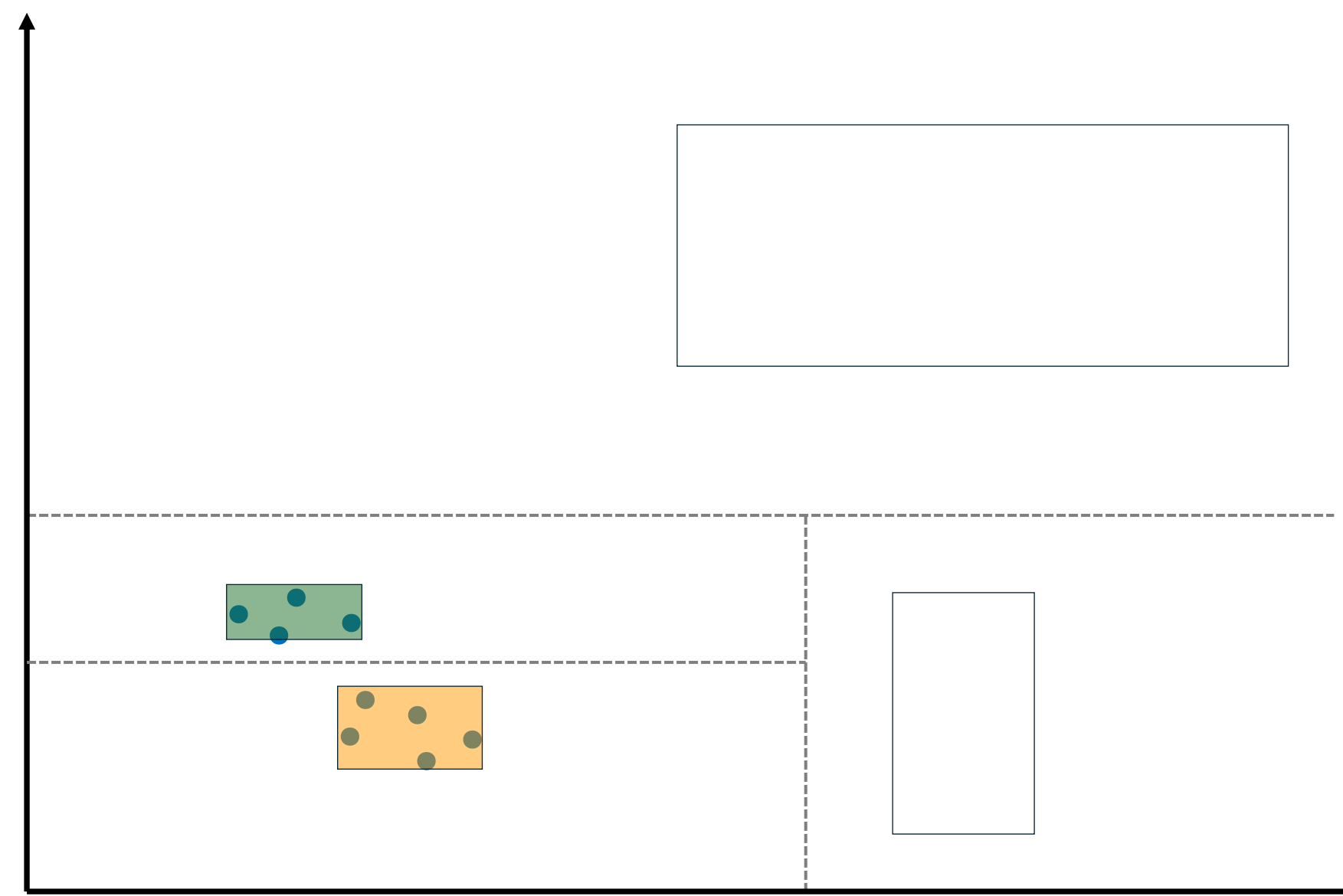
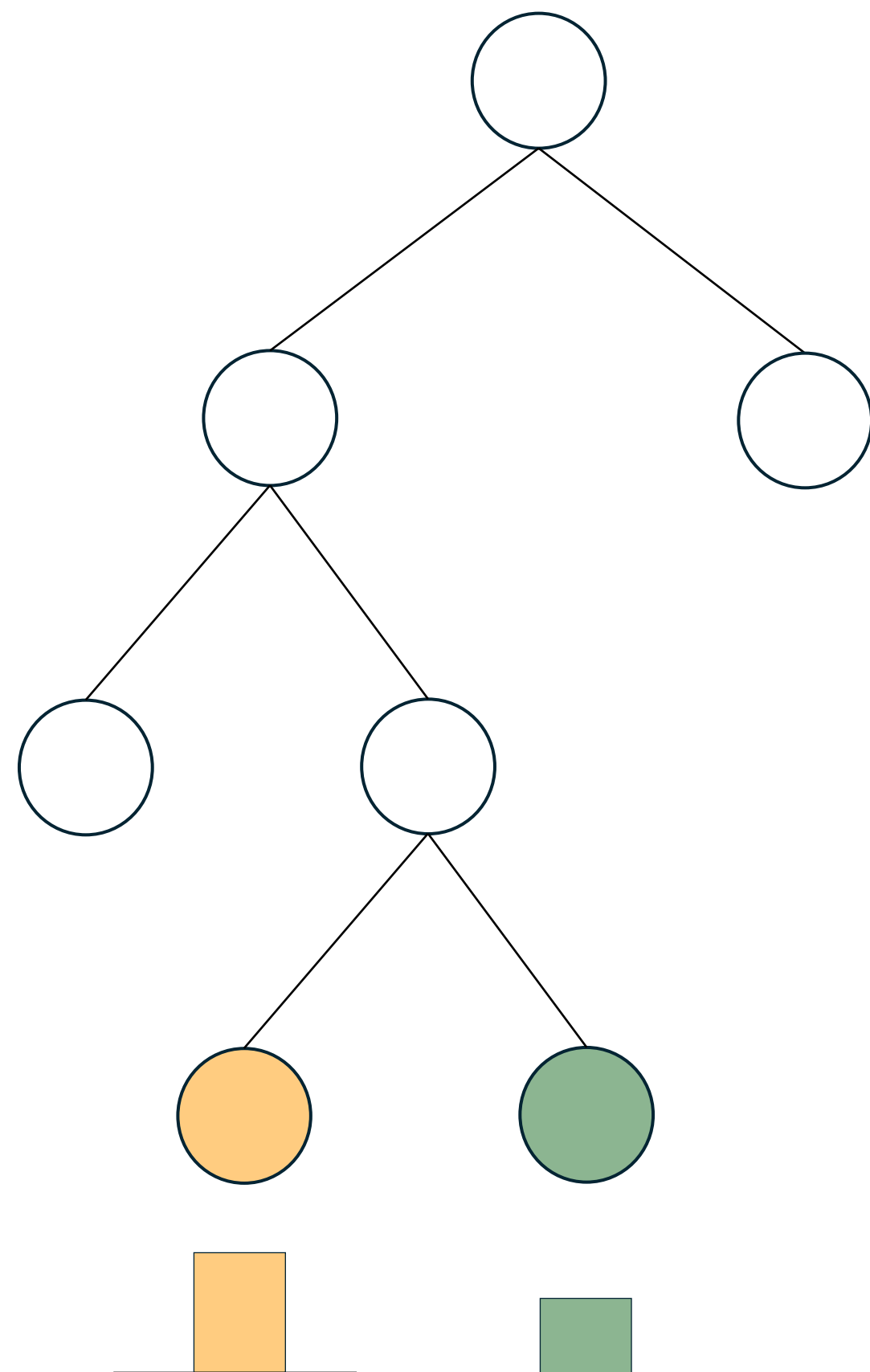
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Learning procedure



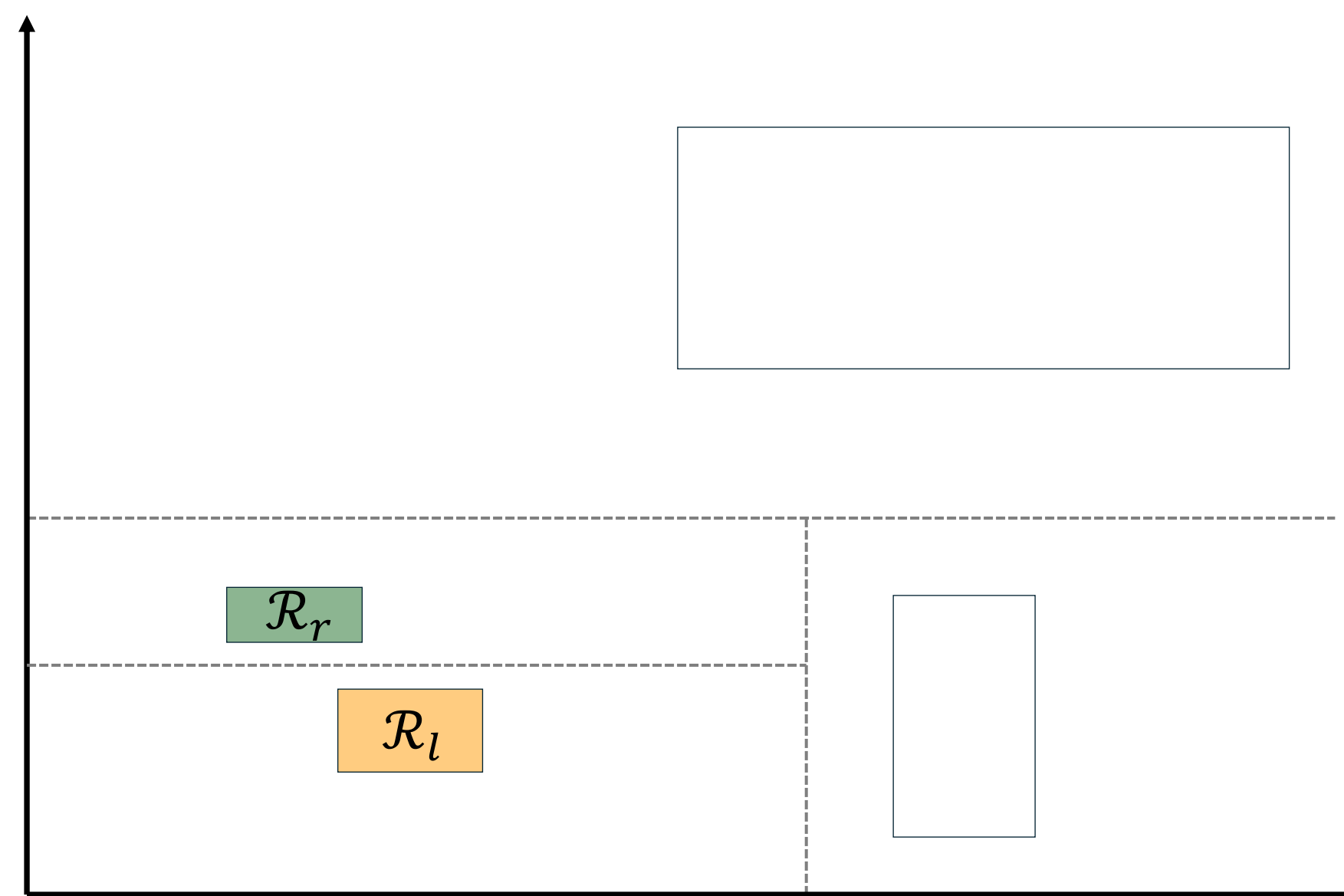
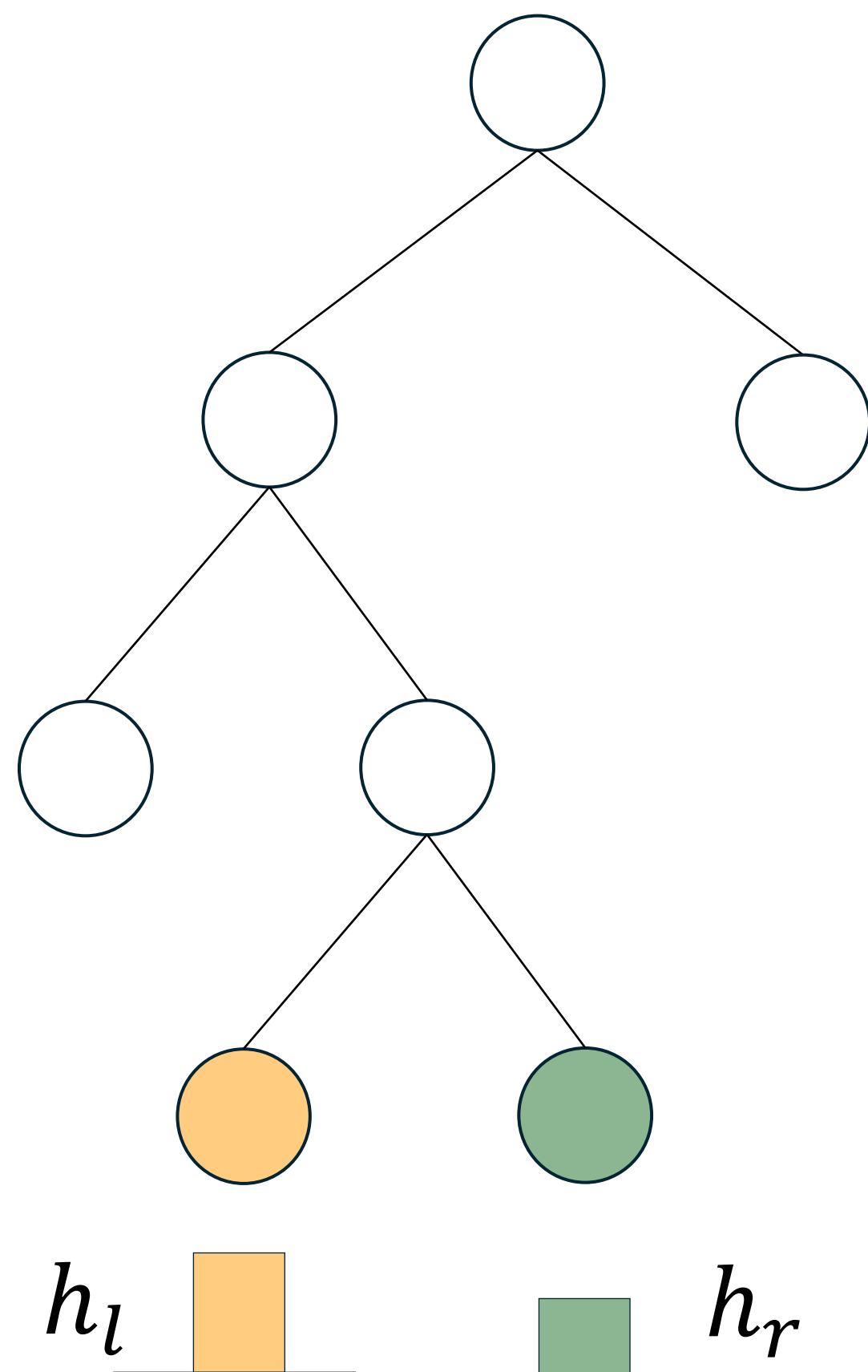
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Learning procedure



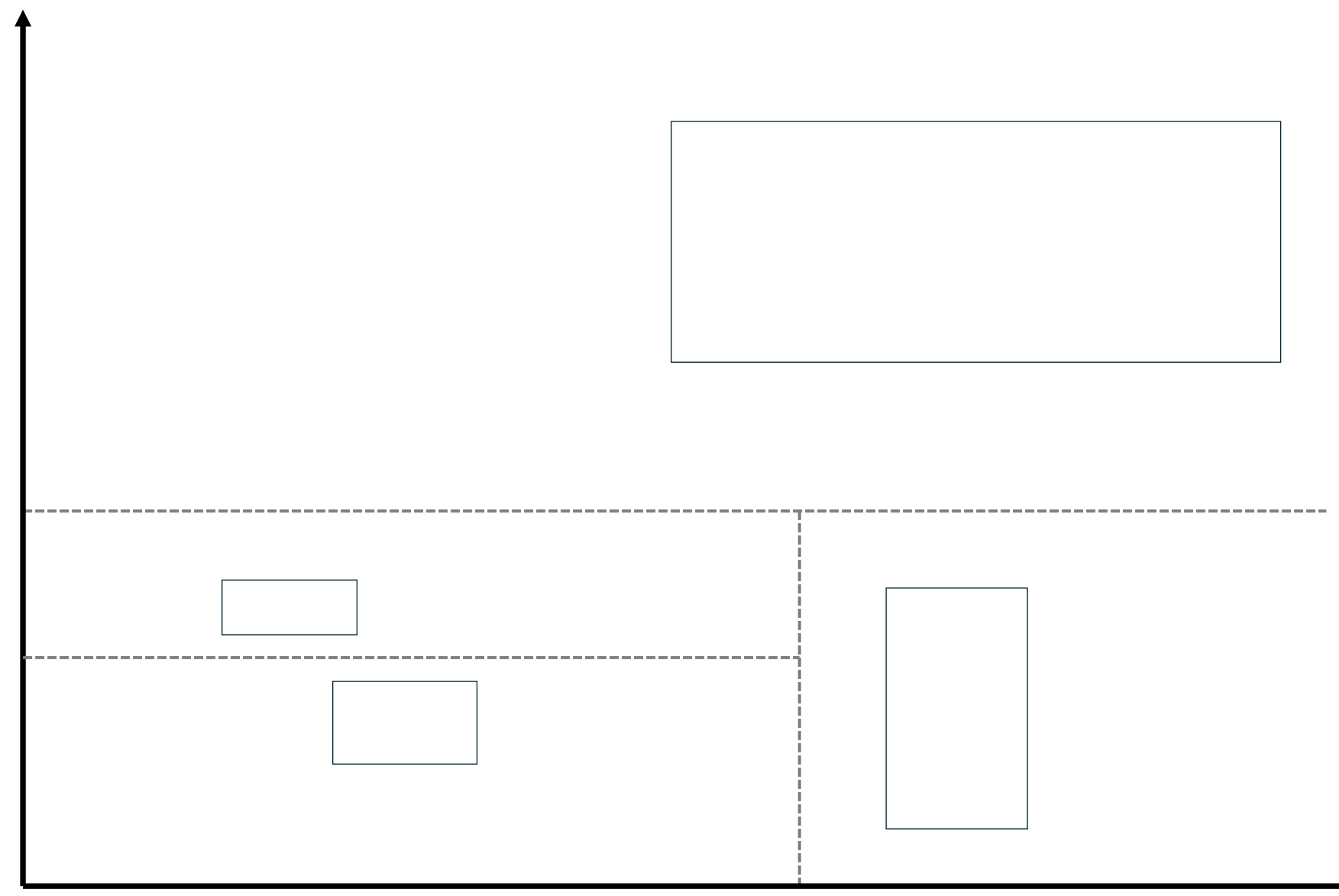
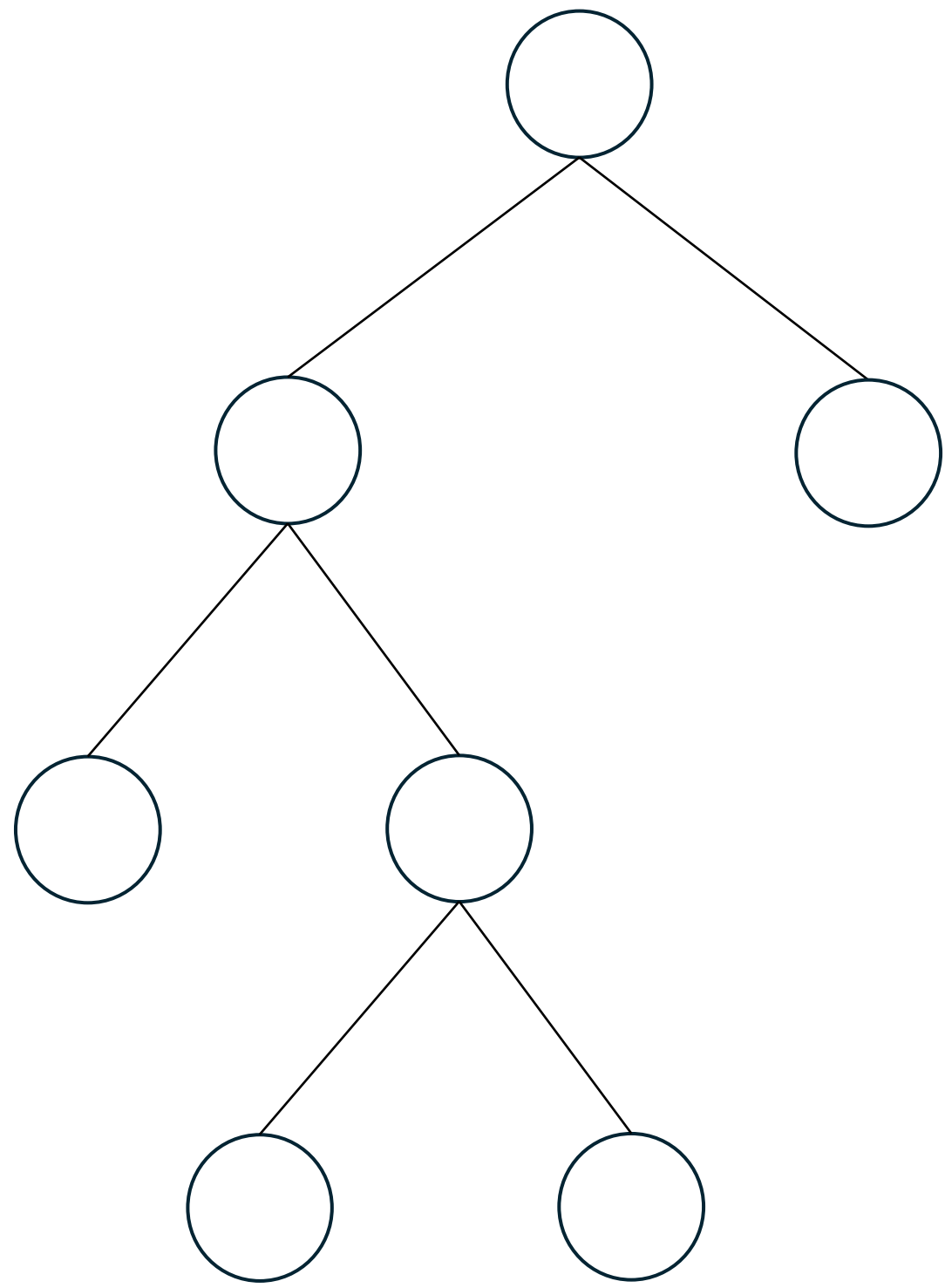
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3. Initialize heights h_l , h_r and supports $\mathcal{R}_l$, $\mathcal{R}_r$ of left and right child nodes N_l and N_r .

Learning procedure

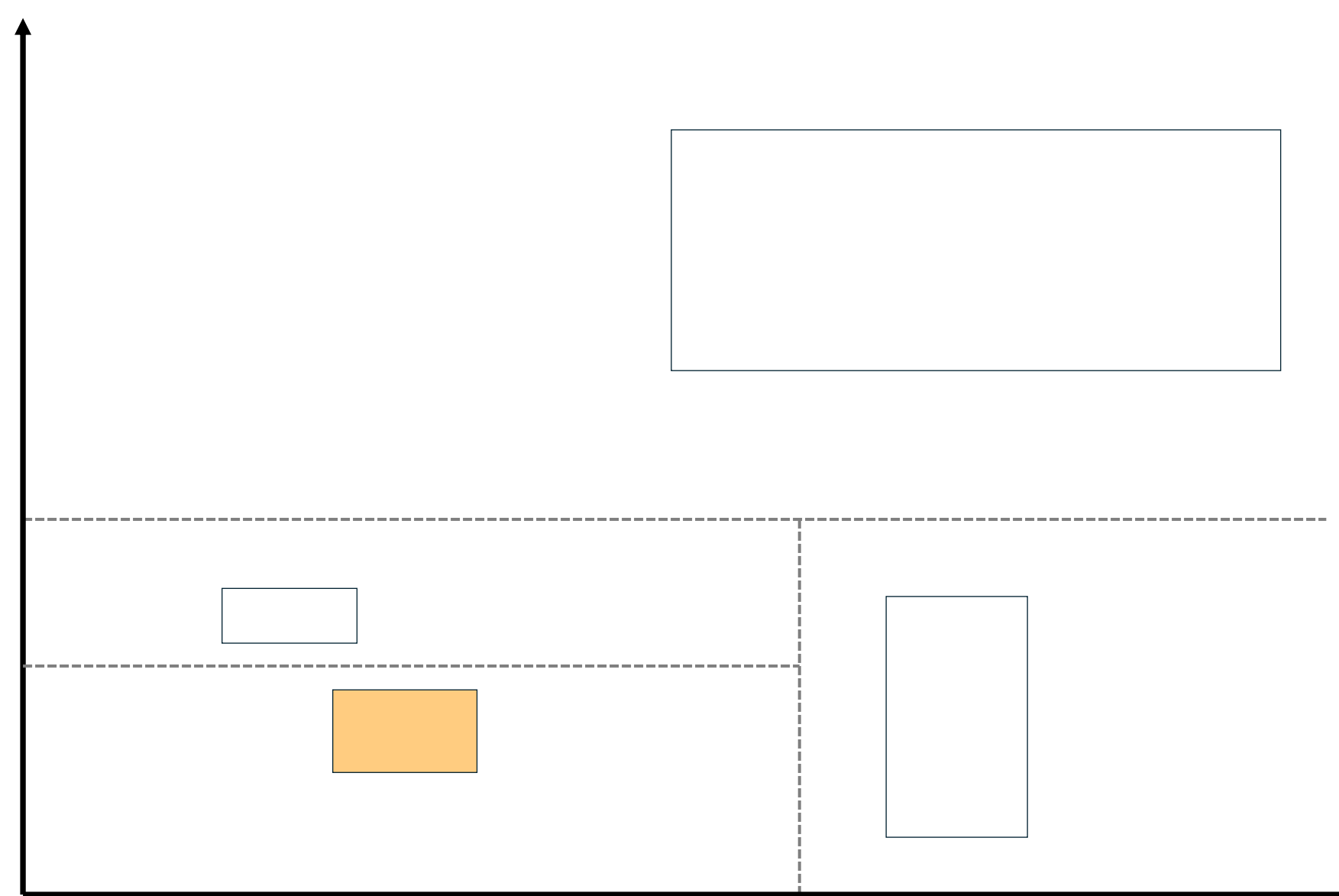
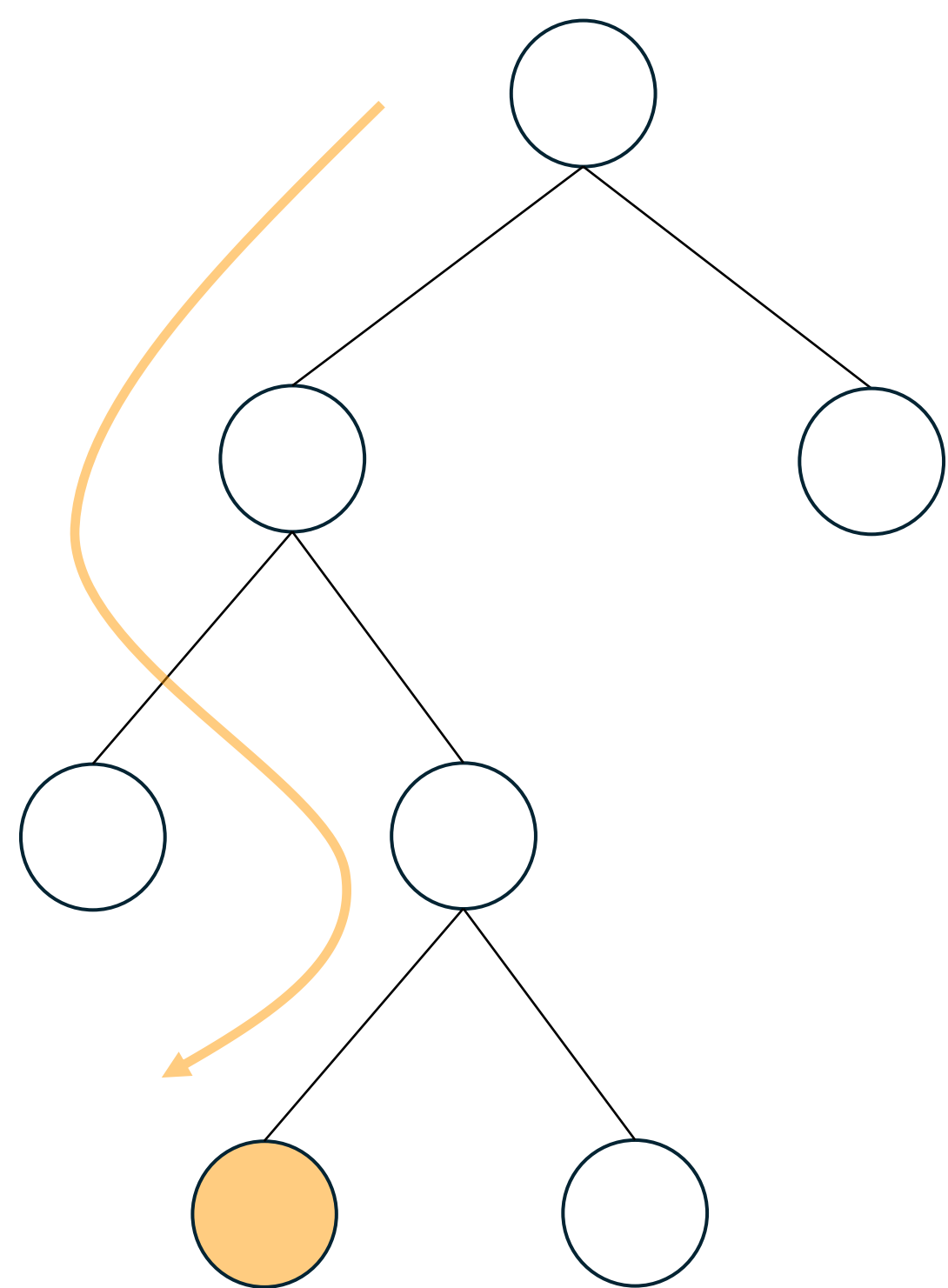


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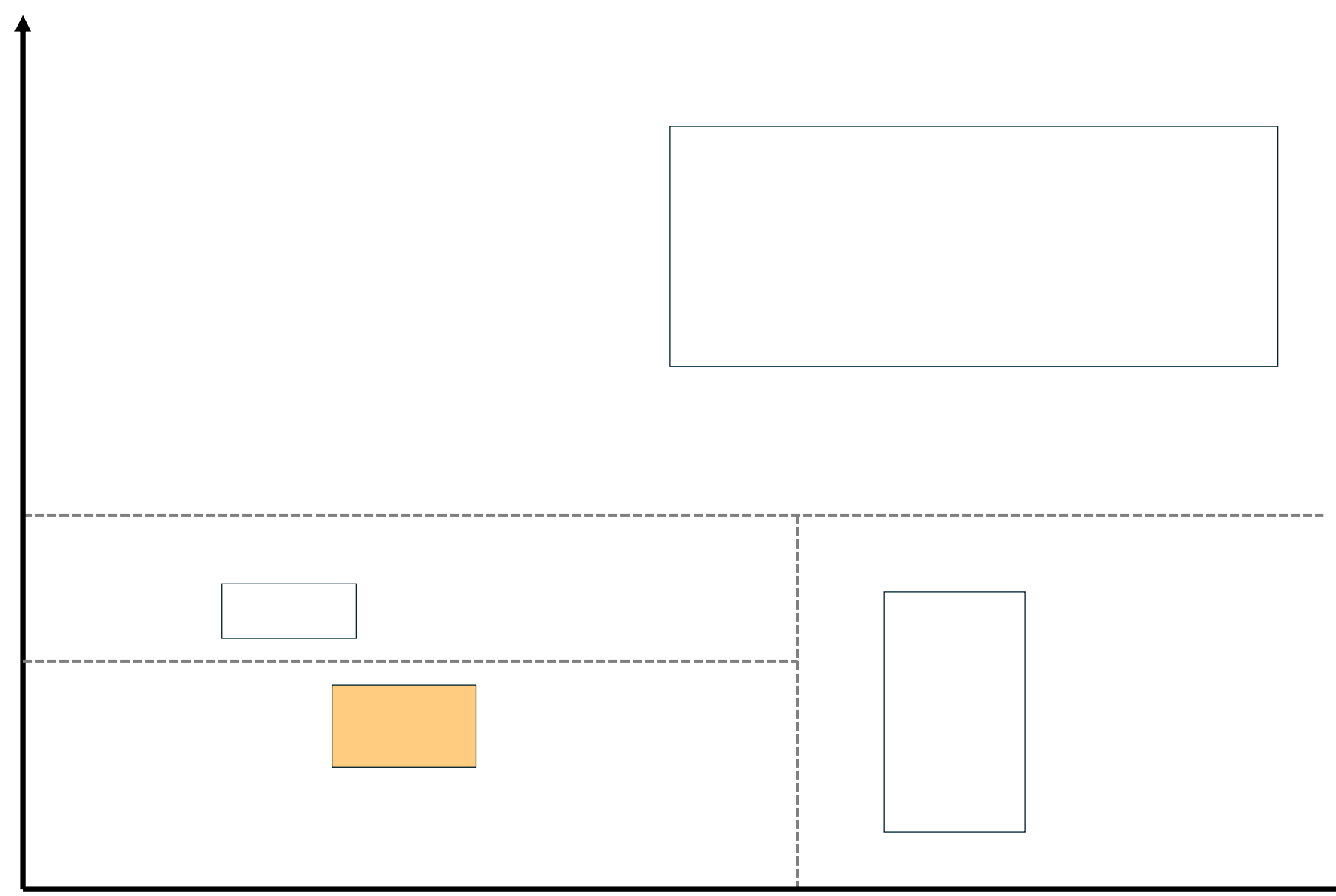
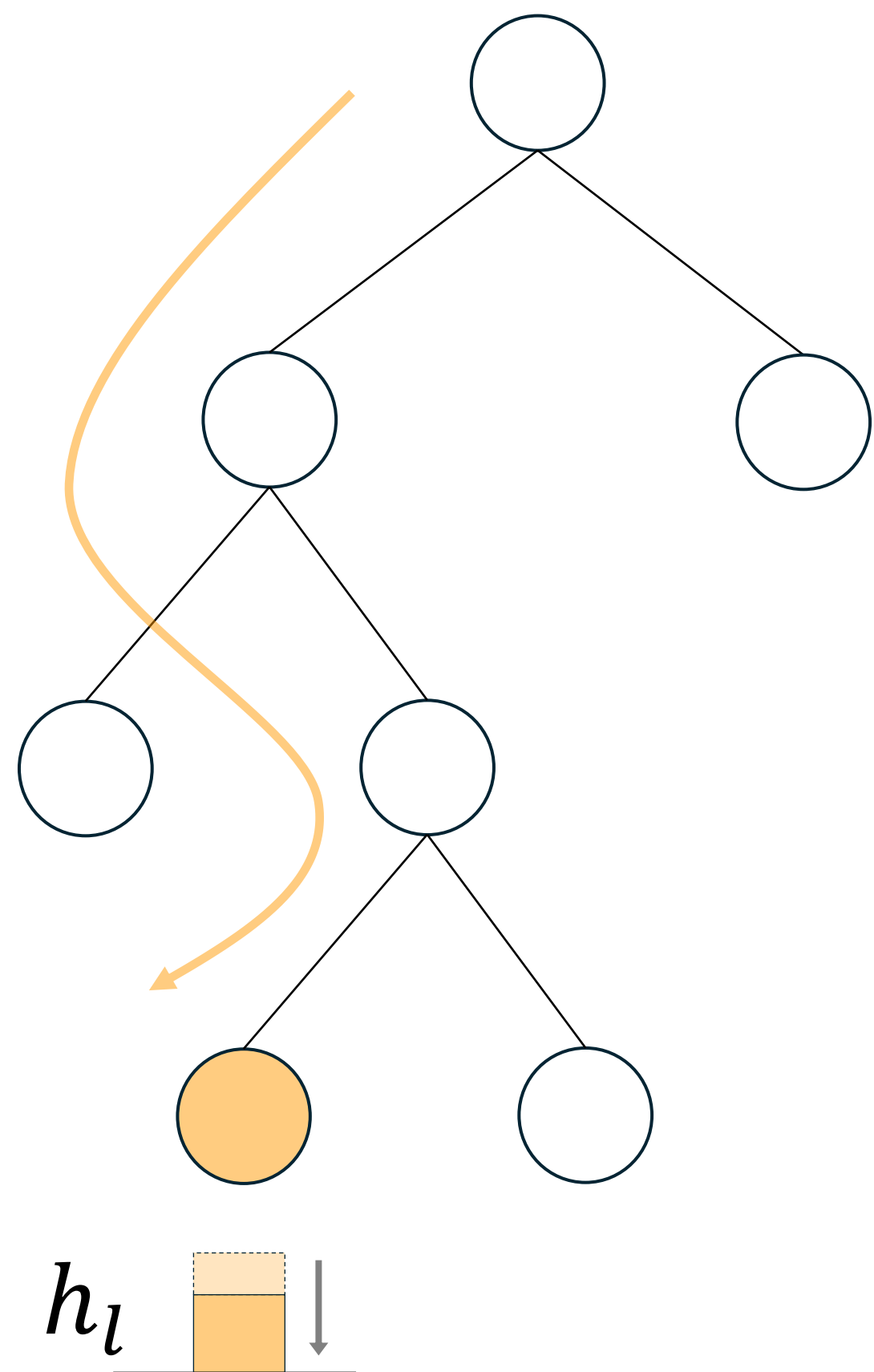
Forgetting procedure



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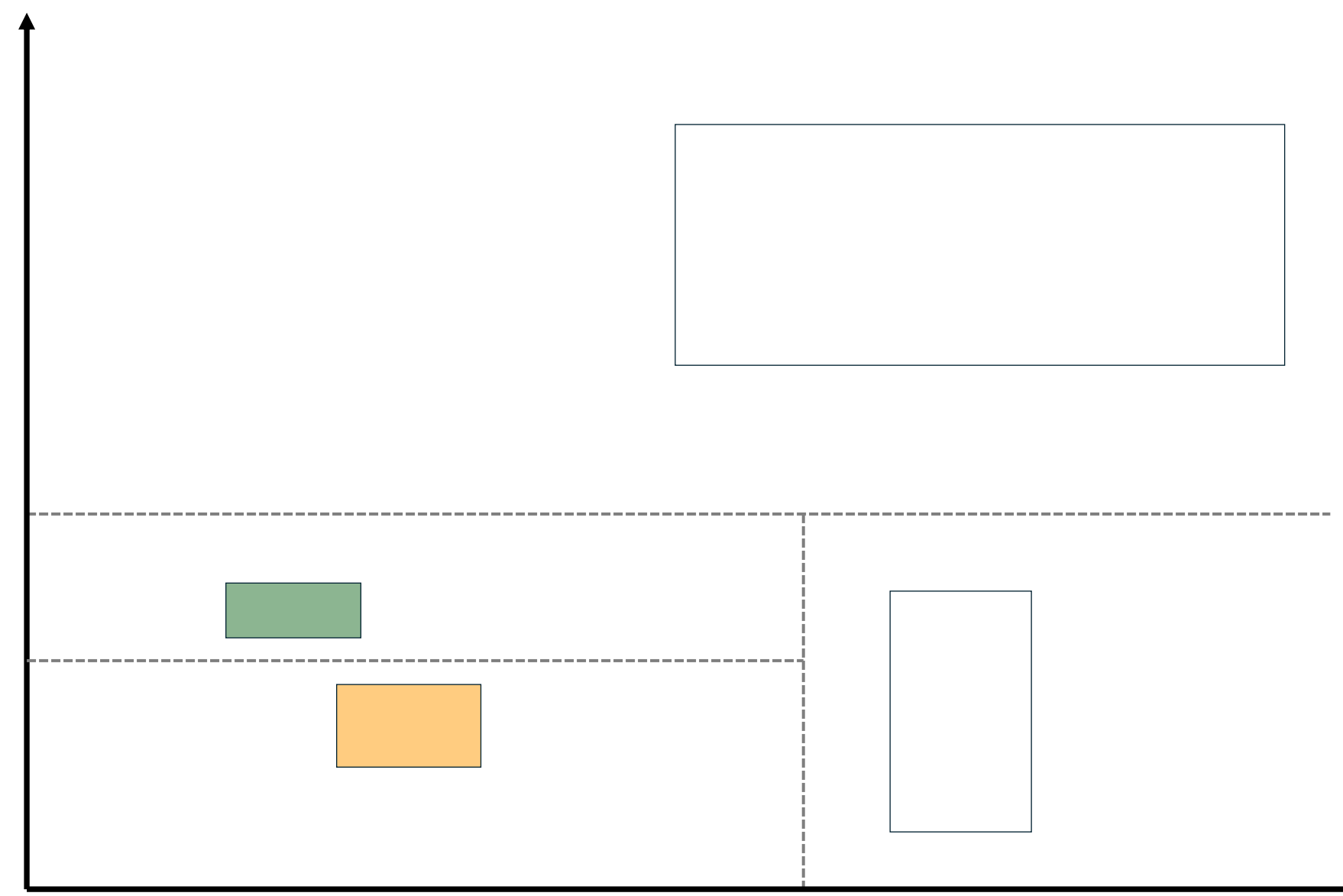
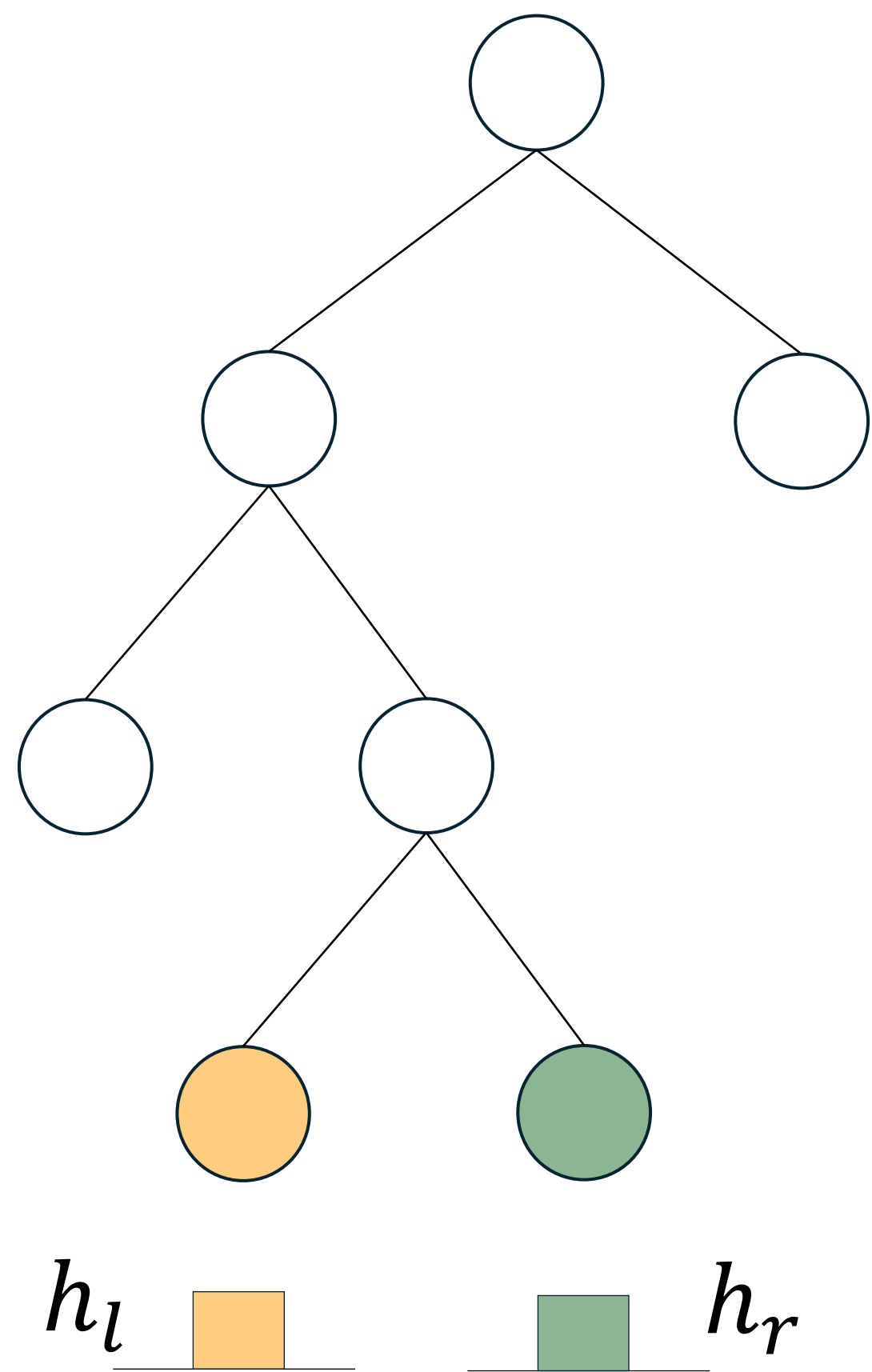


Forgetting procedure



Bin heights h are decreased along the path of $x_{t-\omega}$.

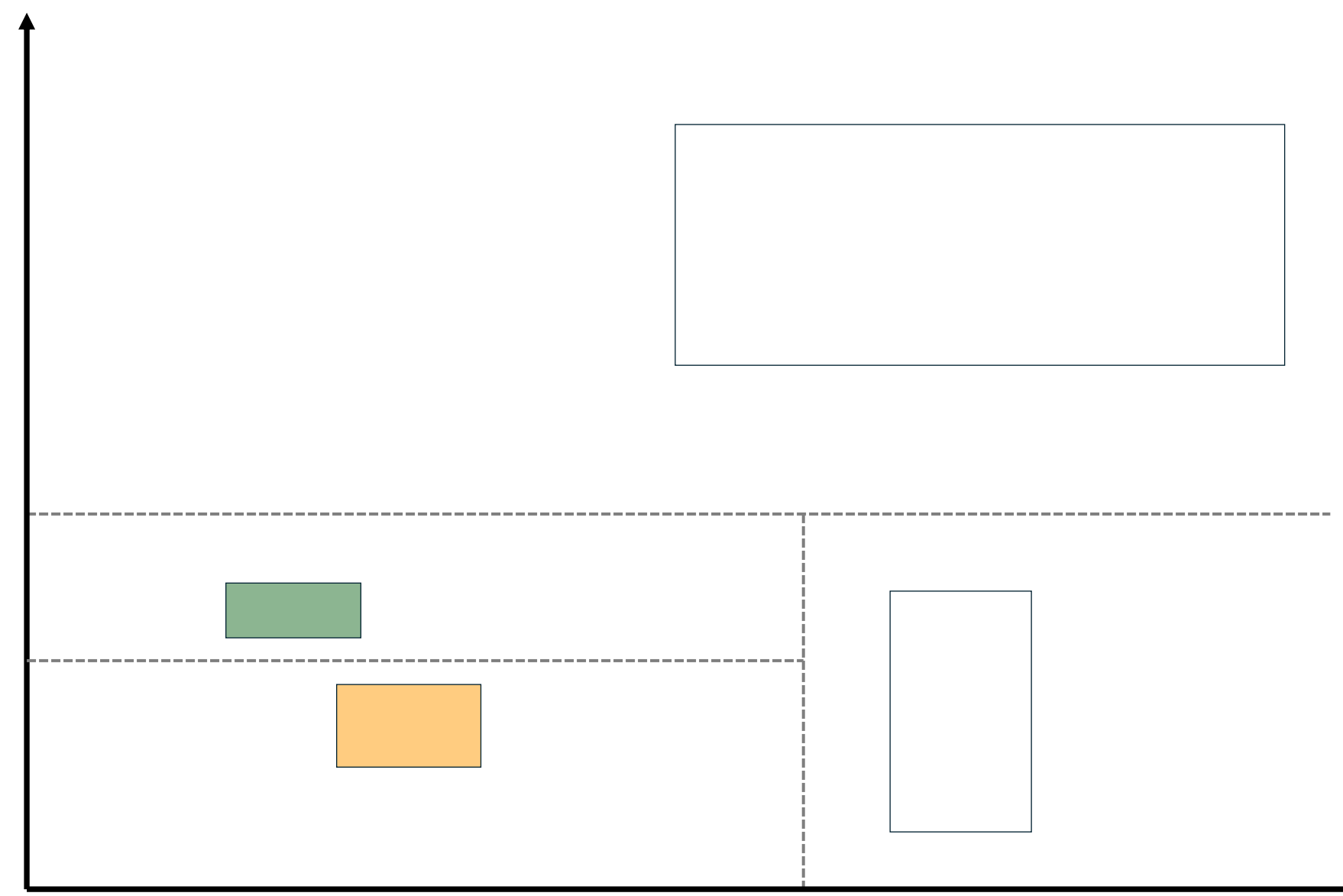
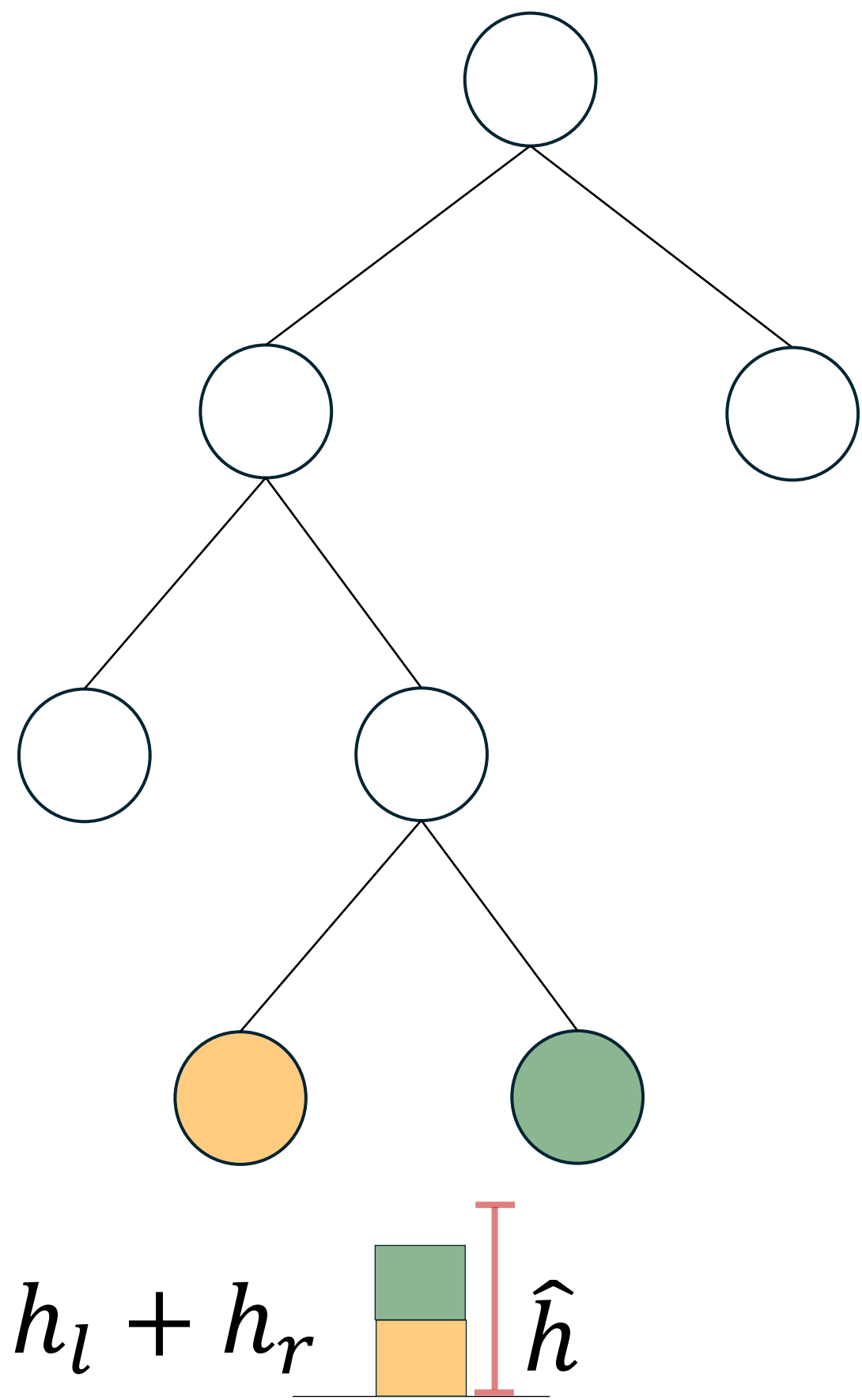
Forgetting procedure



Bin heights h are decreased along the path of $x_{t-\omega}$.

If the sum of two child nodes heights $h_l + h_r$ drops below the value $\hat{h}$, we merge the corresponding bins, leading to a **tree contraction**.

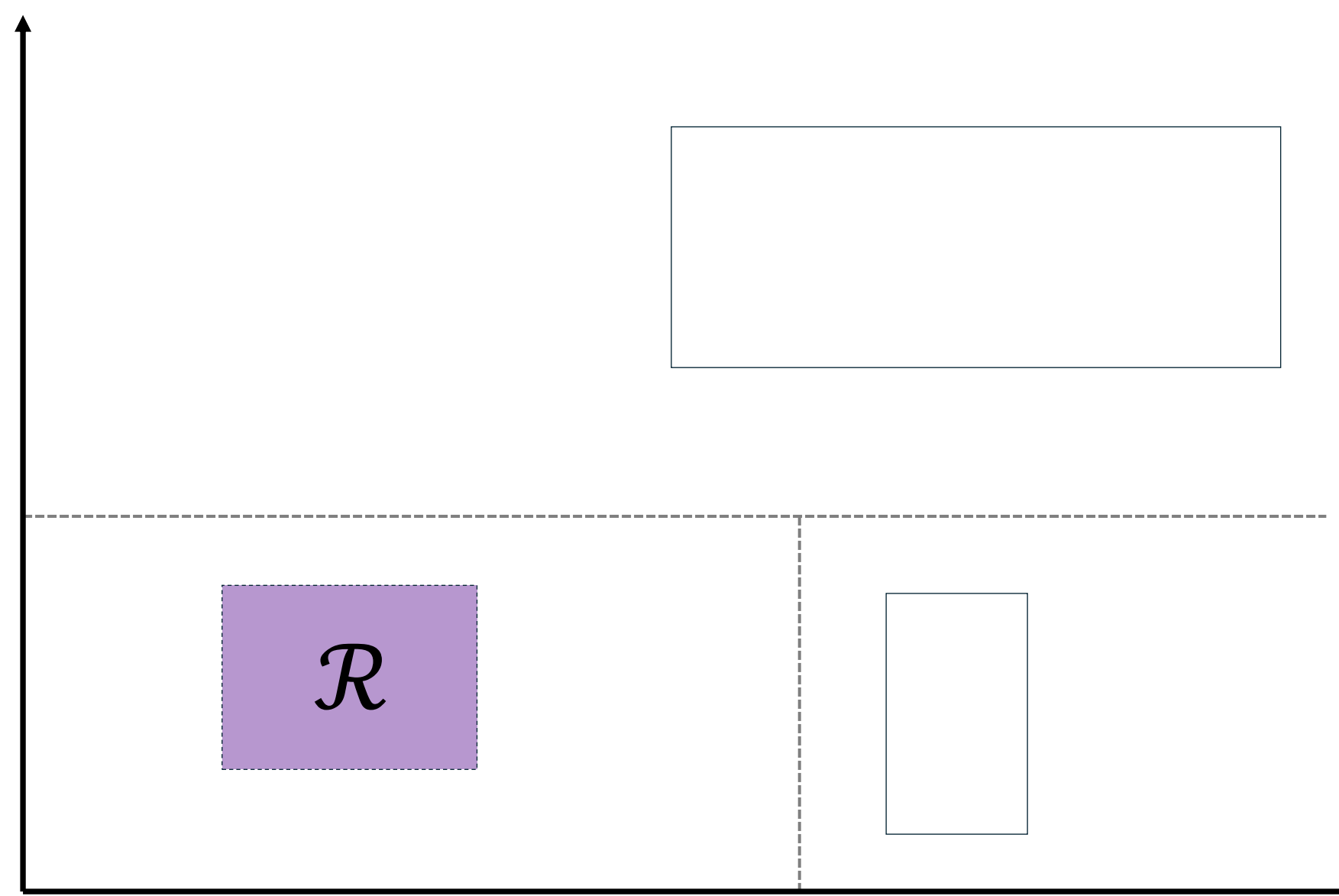
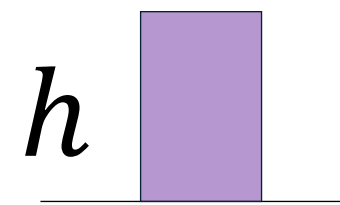
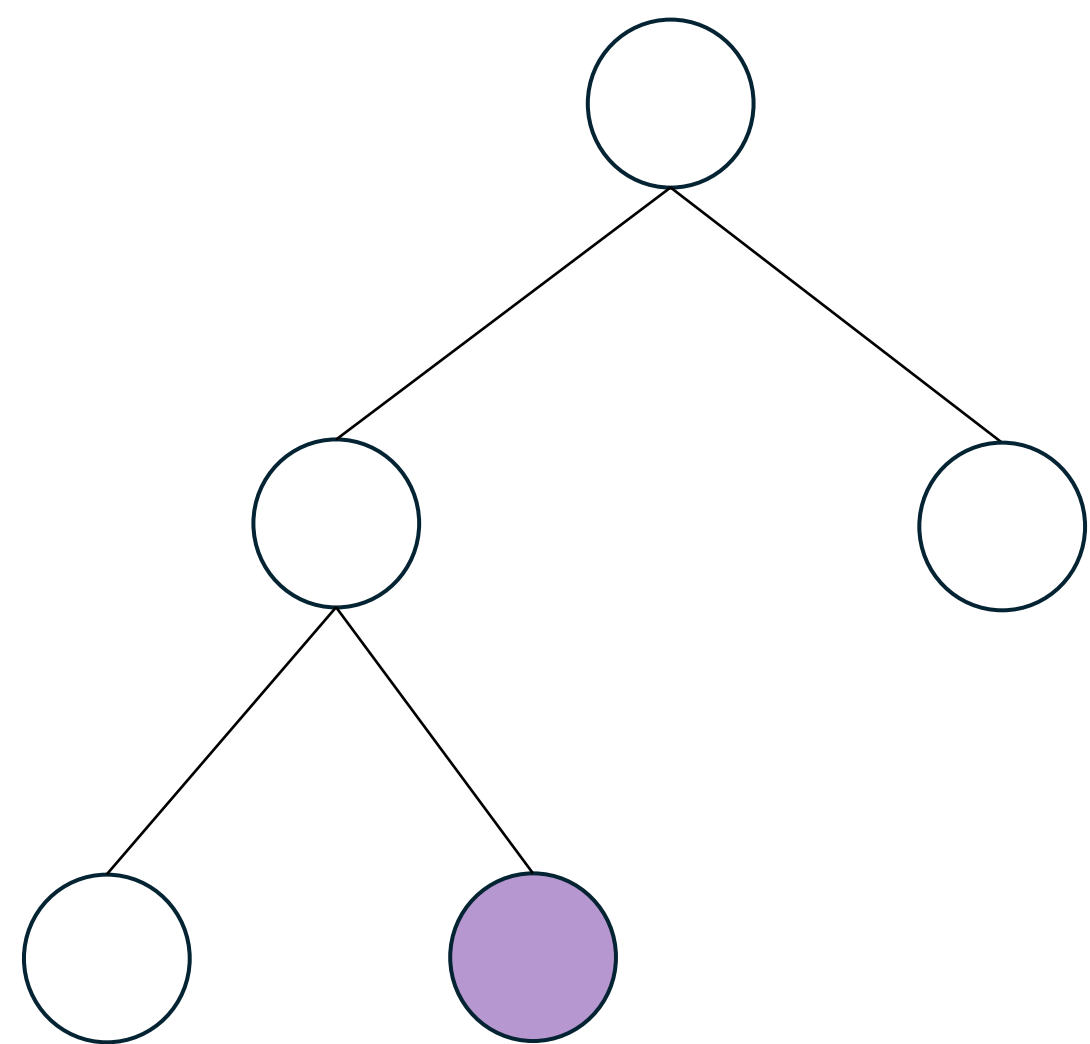
Forgetting procedure



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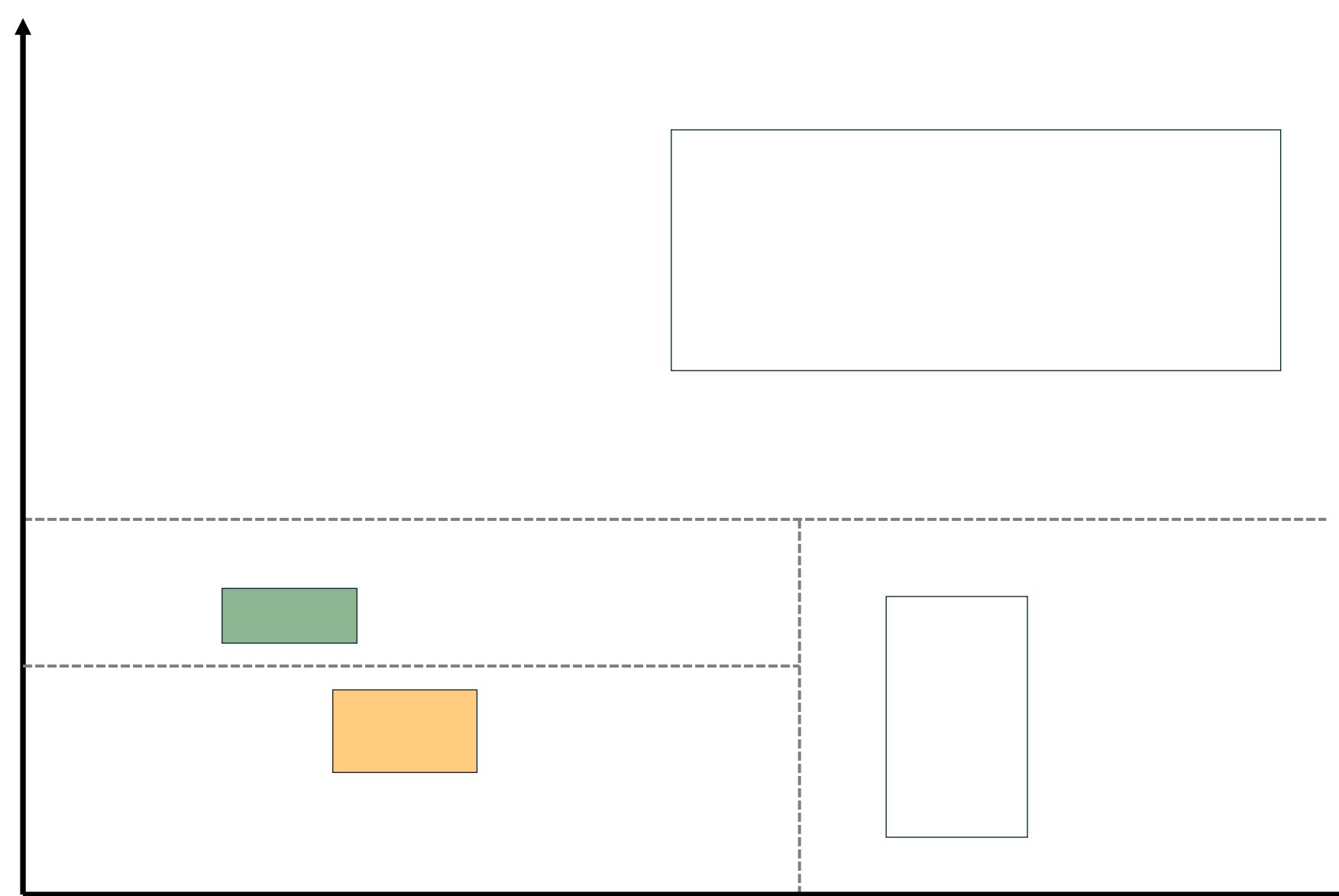
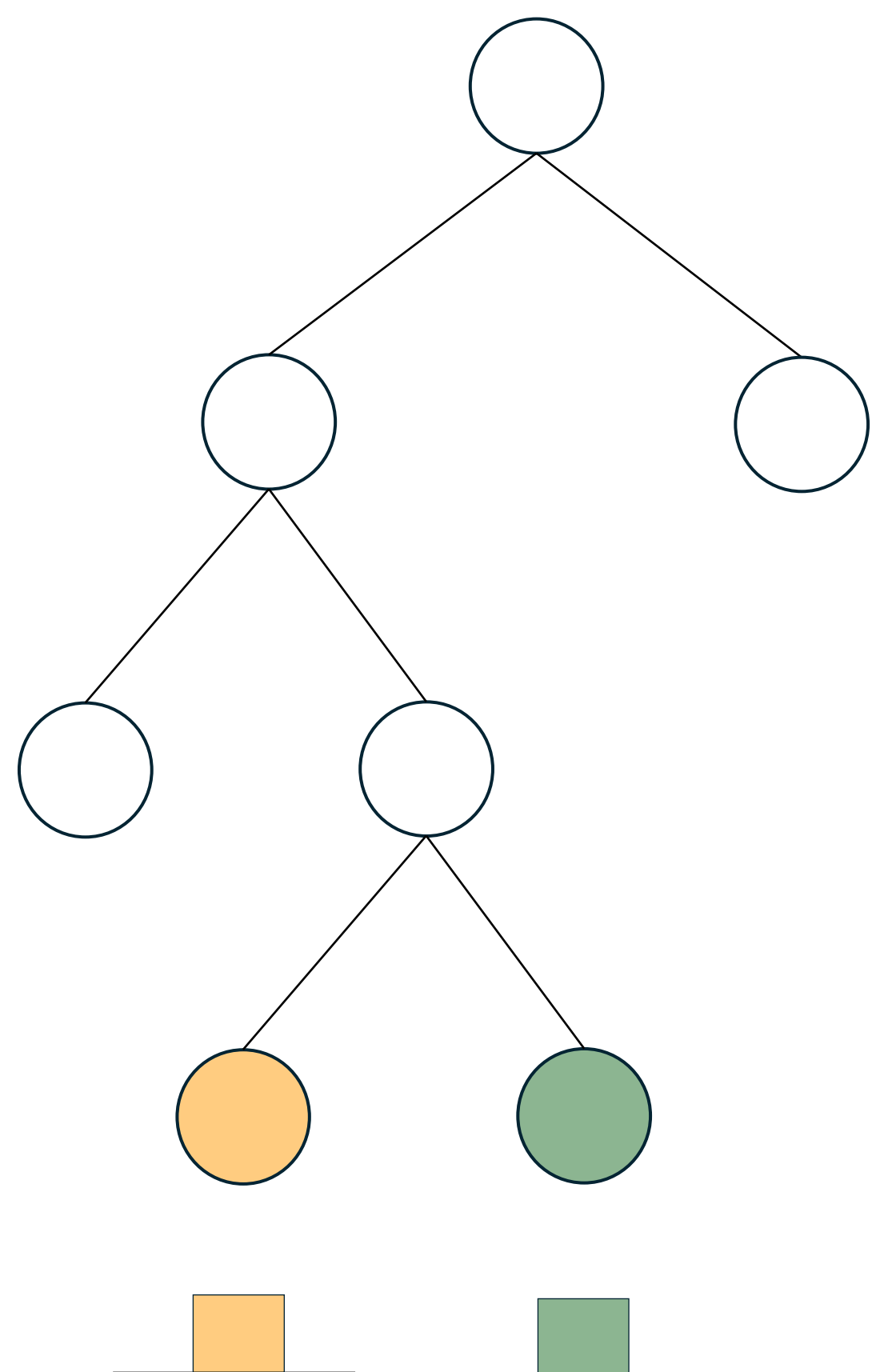
Forgetting procedure



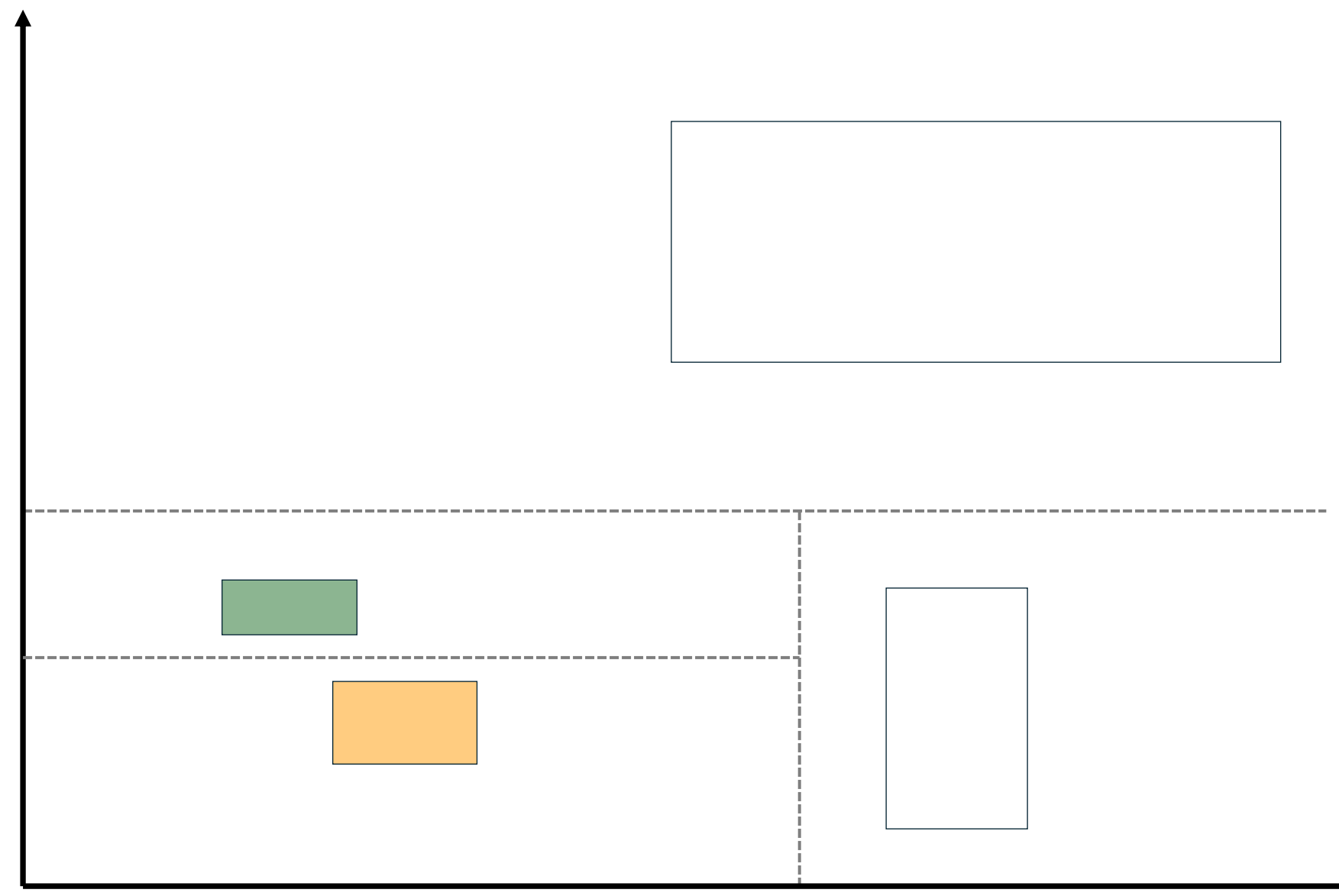
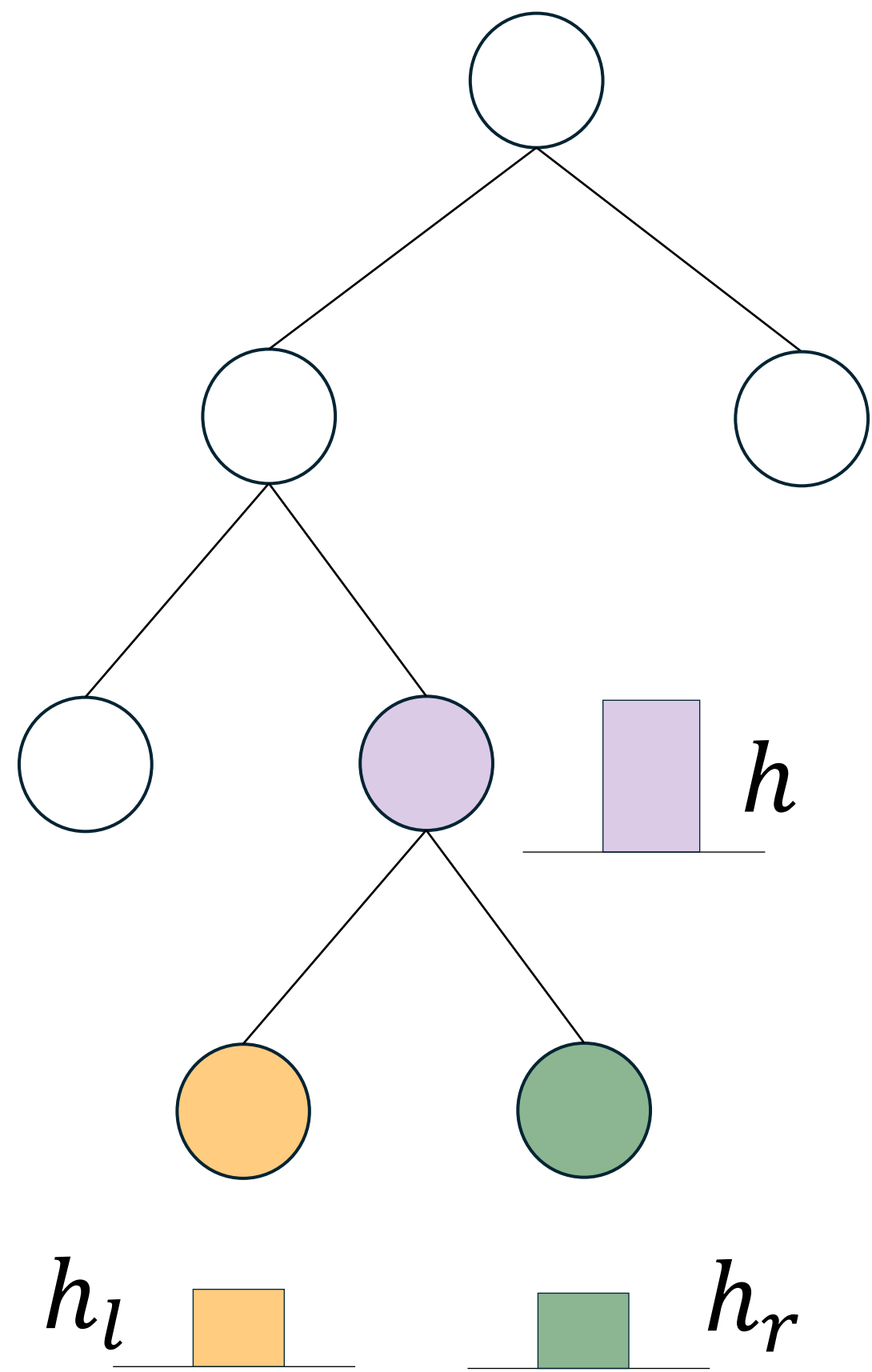
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Forgetting procedure

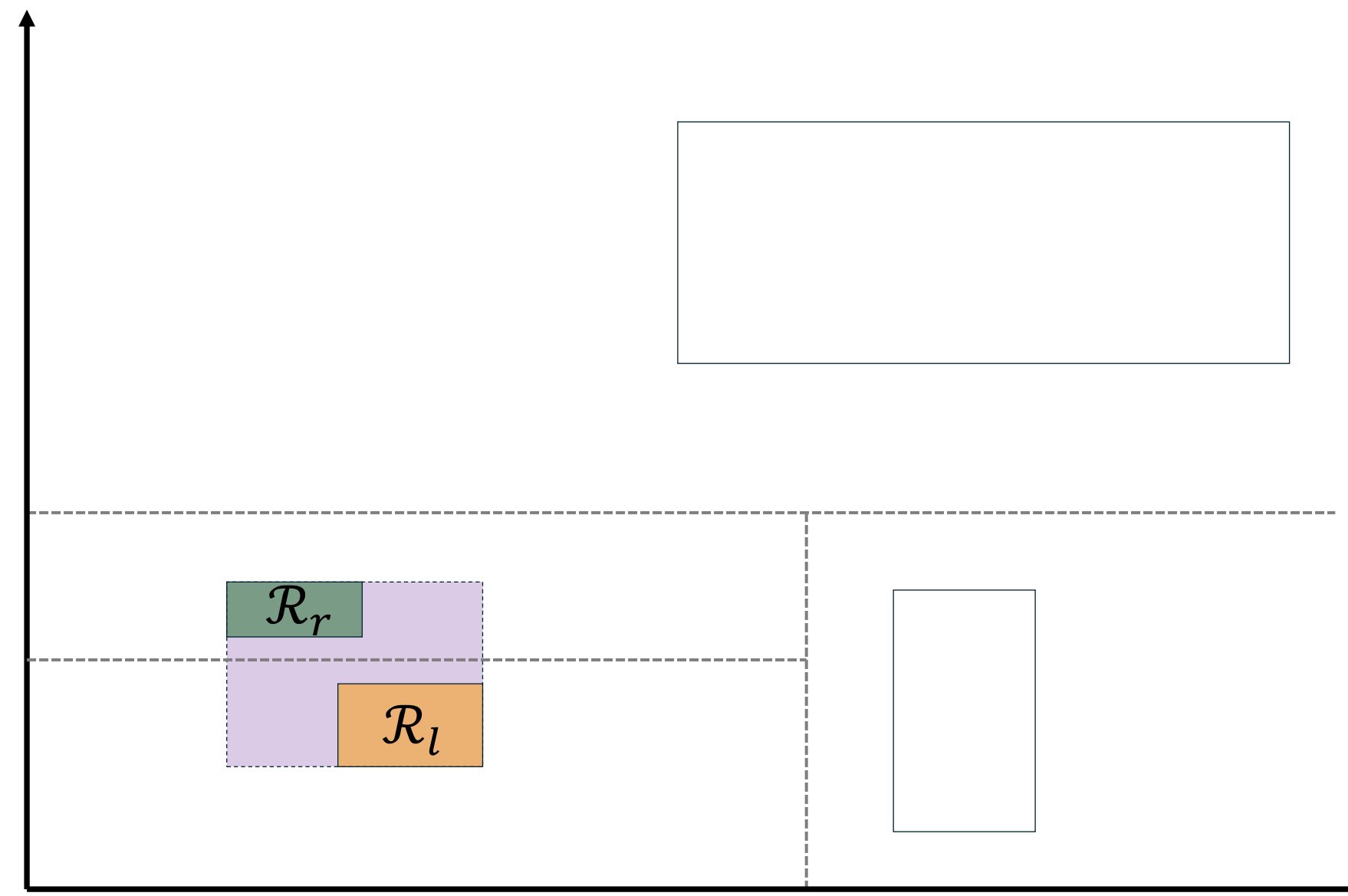
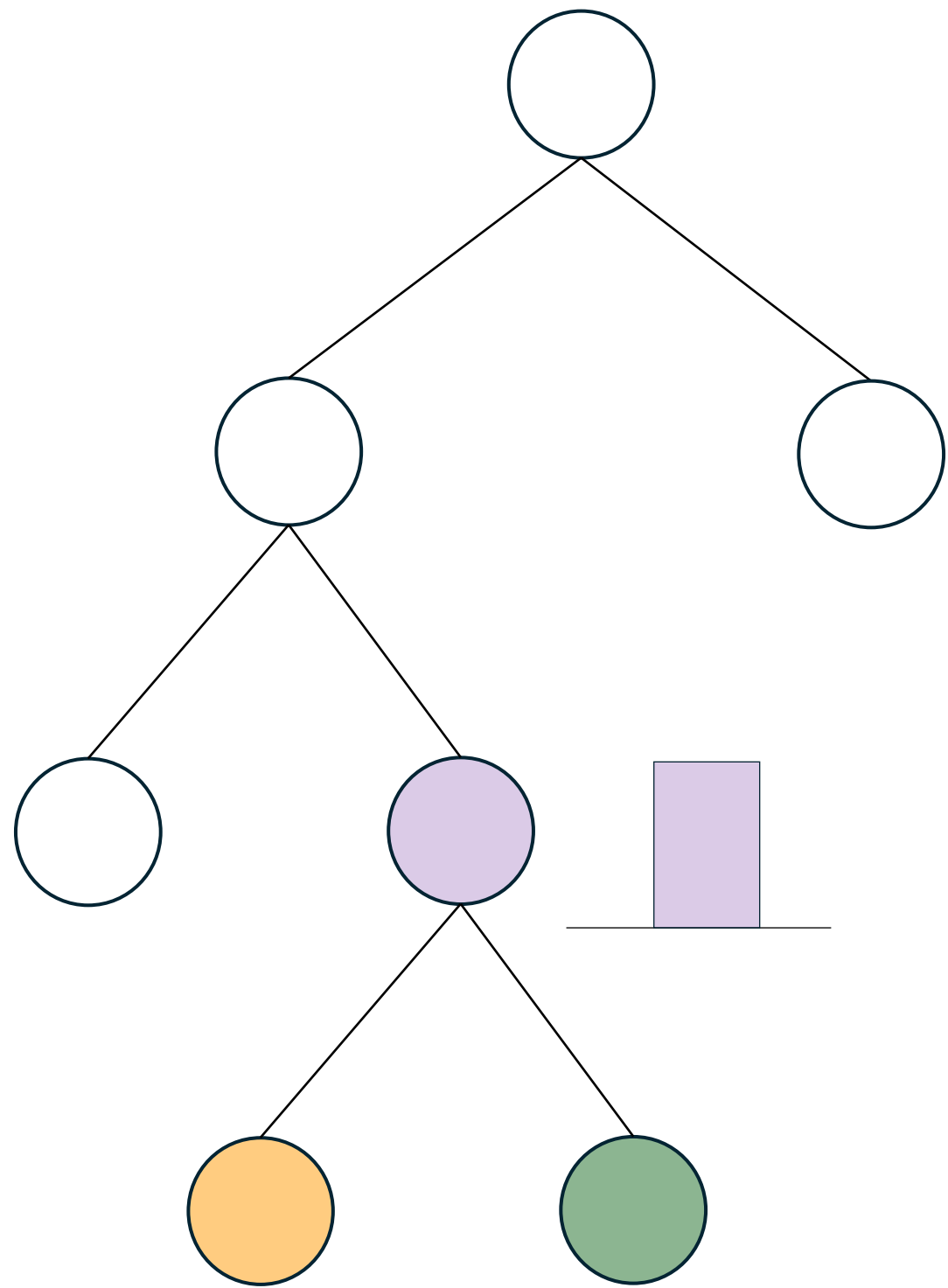


Forgetting procedure



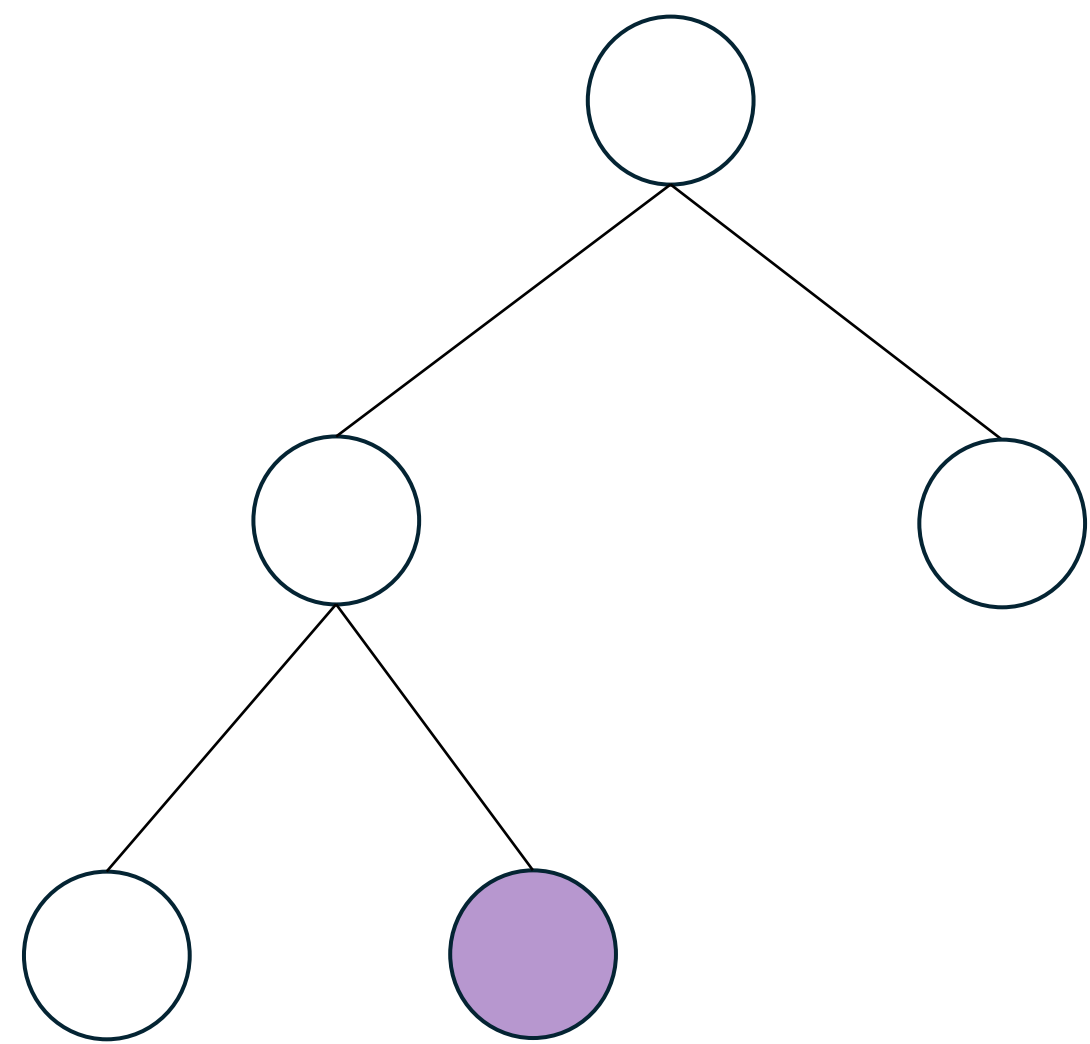
1. Set parent height h as the sum of h_l and h_r ,

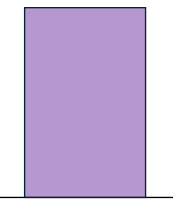
Forgetting procedure

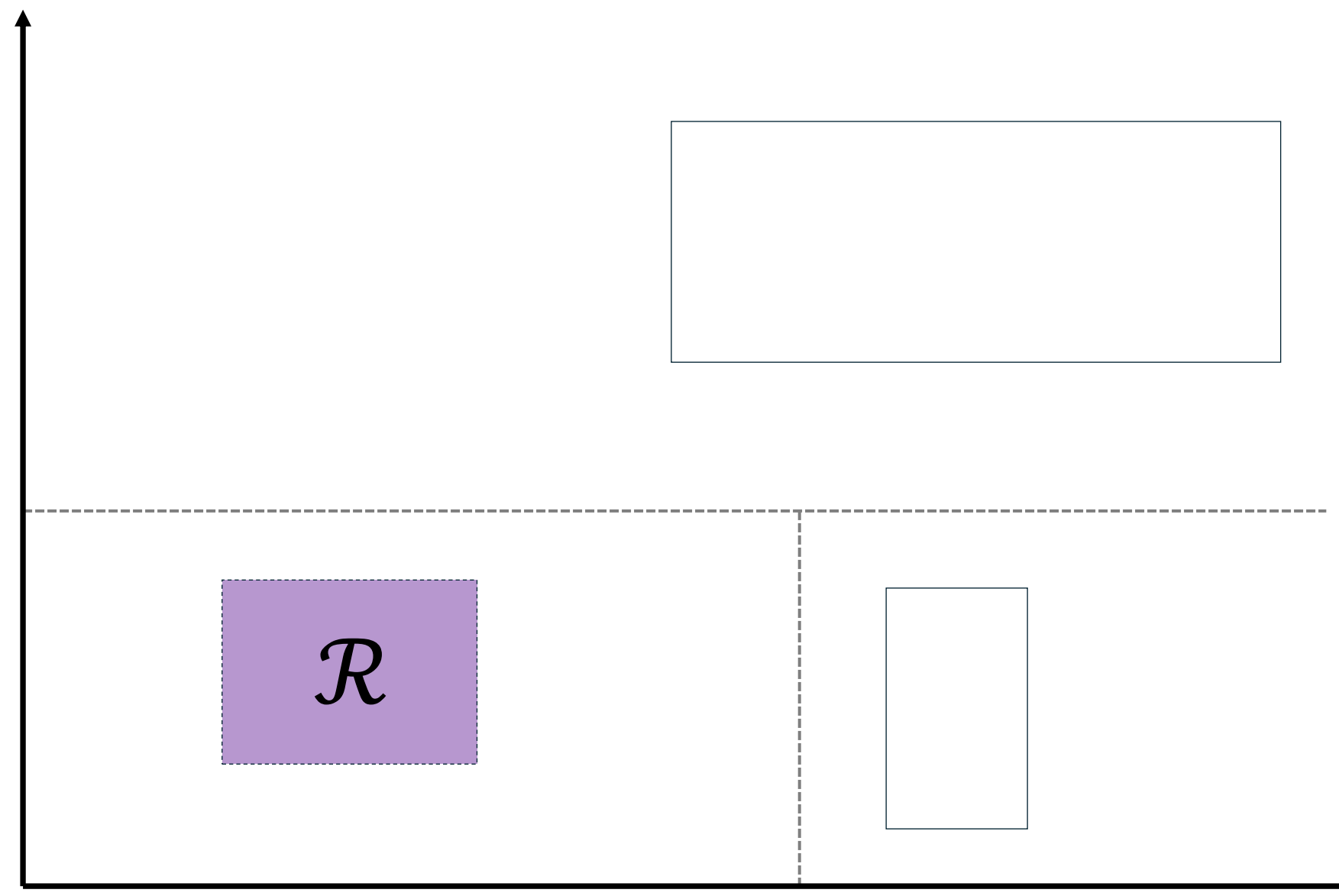


1. Set parent height h as the sum of h_l and h_r ,
2. Set parent support $\mathcal{R}$ as the hyperrectangle enclosing $\mathcal{R}_l$ and $\mathcal{R}_r$,

Forgetting procedure

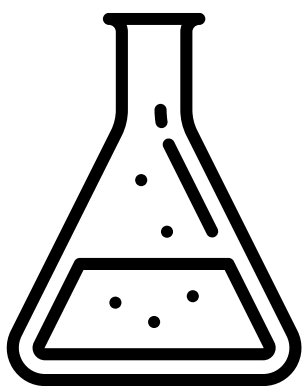


h 



1. Set parent height h as the sum of h_l and h_r ,
2. Set parent support $\mathcal{R}$ as the hyperrectangle enclosing $\mathcal{R}_l$ and $\mathcal{R}_r$,
3. Forget split information p, q and child nodes N_l, N_r .

Experimental results



DATASET	n	d	% OF ANOMALIES
DONORS	619326	10	5.90
HTTP	567497	3	0.40
FORESTCOVER	286048	10	0.90
FRAUD	284807	29	0.17
MULCROSS	262144	4	10.00
SMTP	95156	3	0.03
SHUTTLE	49097	9	7.00
MAMMOGRAPHY	11183	6	2.00
NYC_TAXI_SHINGLE	10273	48	5.20
ANNTHYROID	6832	6	7.00
SATELLITE	6435	36	32.00

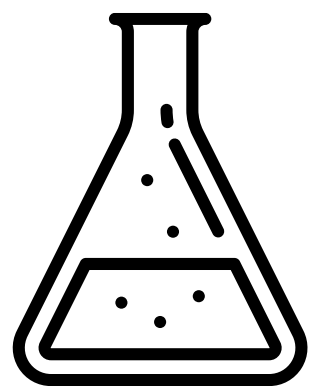
References

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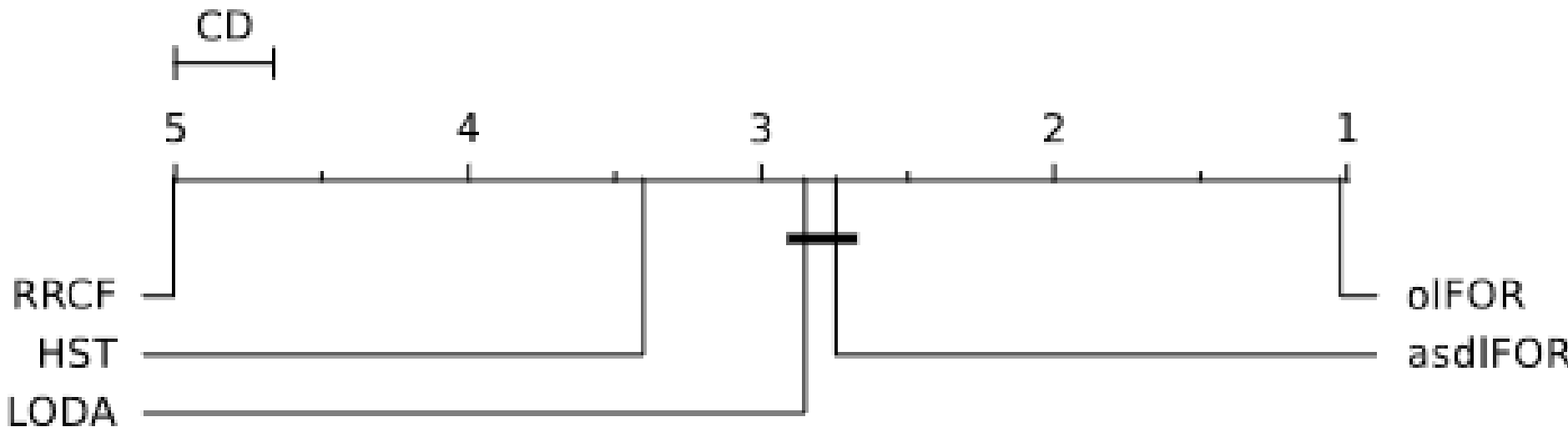
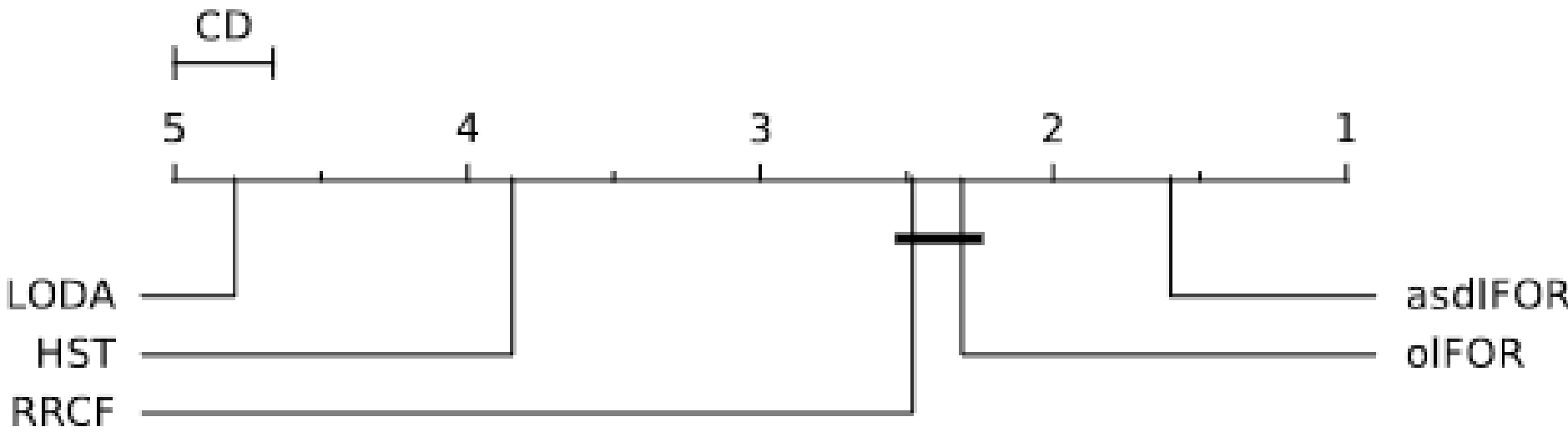
- Tan, S. C., Ting, K. M., and Liu, T. F. “Fast anomaly detection for streaming data”. JCAI 2011.
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Experimental results



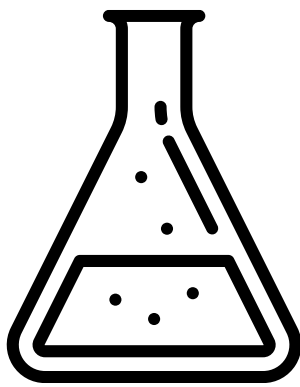
	AUC (↑)					TIME (↓)				
	oIFOR	ASDIFOR	HST	RRCF	LODA	oIFOR	ASDIFOR	HST	RRCF	LODA
DONORS	0.795	0.769	0.715	0.637	0.554	252.36	551.85	2145.85	4924.46	2111.09
HTTP	0.998	0.999	0.992	0.996	0.632	179.36	509.40	2016.00	8367.16	2017.85
FORESTCOVER	0.887	0.861	0.722	0.917	0.500	107.65	197.82	1045.39	2997.86	1009.92
FRAUD	0.936	0.946	0.910	0.951	0.722	100.09	285.69	973.91	4936.03	931.93
MULCROSS	0.995	0.952	0.011	0.800	0.506	90.33	270.79	936.01	3244.96	848.12
SMTP	0.861	0.905	0.851	0.894	0.731	29.95	142.65	325.77	1273.98	254.69
SHUTTLE	0.992	0.996	0.981	0.957	0.528	16.35	108.28	167.48	770.61	130.61
MAMMOGRAPHY	0.854	0.855	0.831	0.824	0.622	3.32	80.01	37.92	118.22	29.55
NYC_TAXI_SHINGLE	0.572	0.709	0.342	0.725	0.499	8.03	83.15	36.70	151.67	36.82
ANNTHYROID	0.685	0.810	0.636	0.740	0.589	2.00	77.06	24.26	93.40	19.79
SATELLITE	0.651	0.709	0.531	0.662	0.501	3.74	78.90	21.78	93.77	17.55
MEDIAN	0.866	0.863	0.739	0.832	0.541	29.95	142.57	323.29	1274.45	254.64
MEAN RANK	2.167	1.583	3.917	2.500	4.833	1.000	2.667	3.500	5.000	2.833



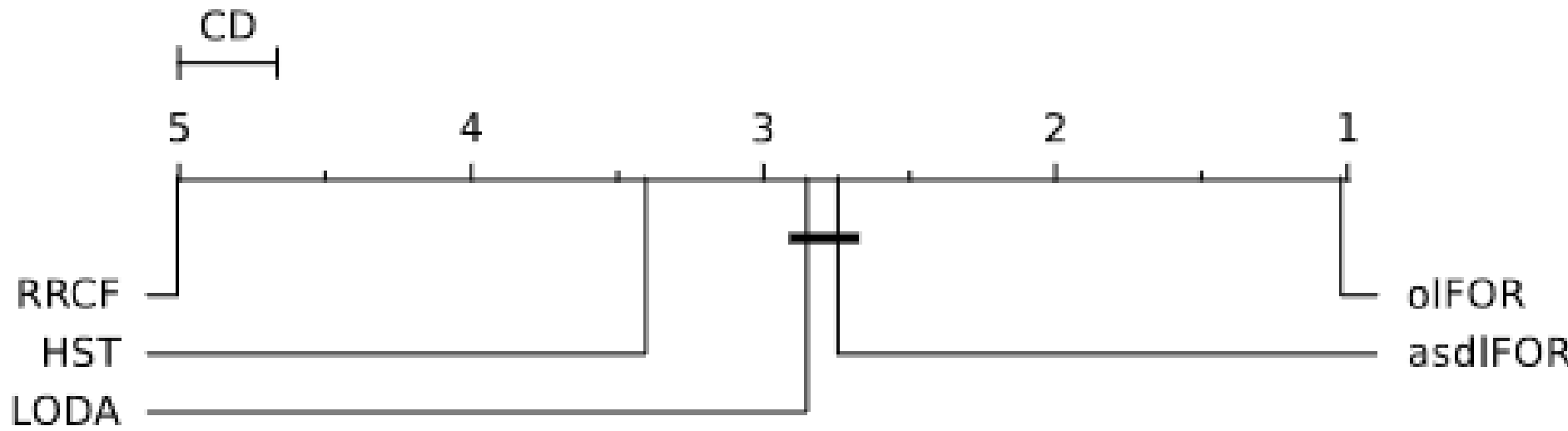
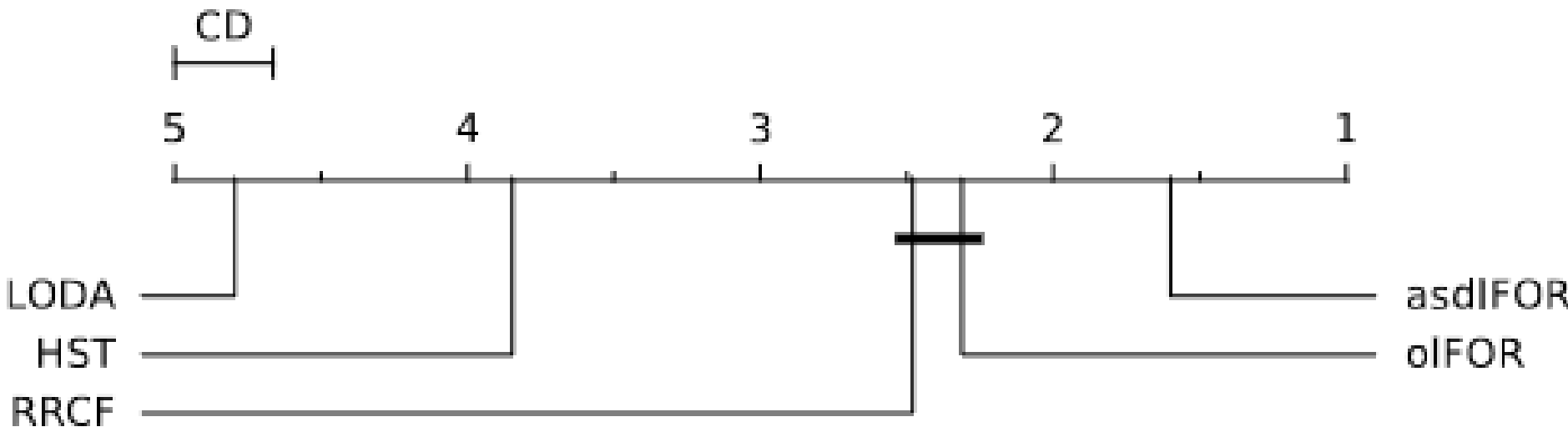
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Experimental results



	AUC (↑)					TIME (↓)				
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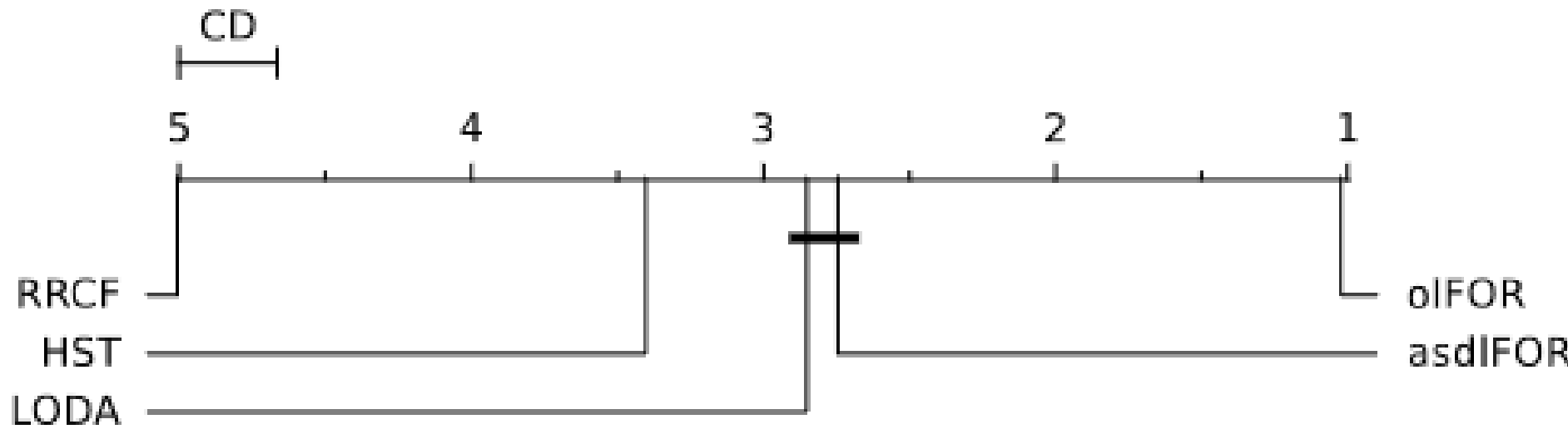
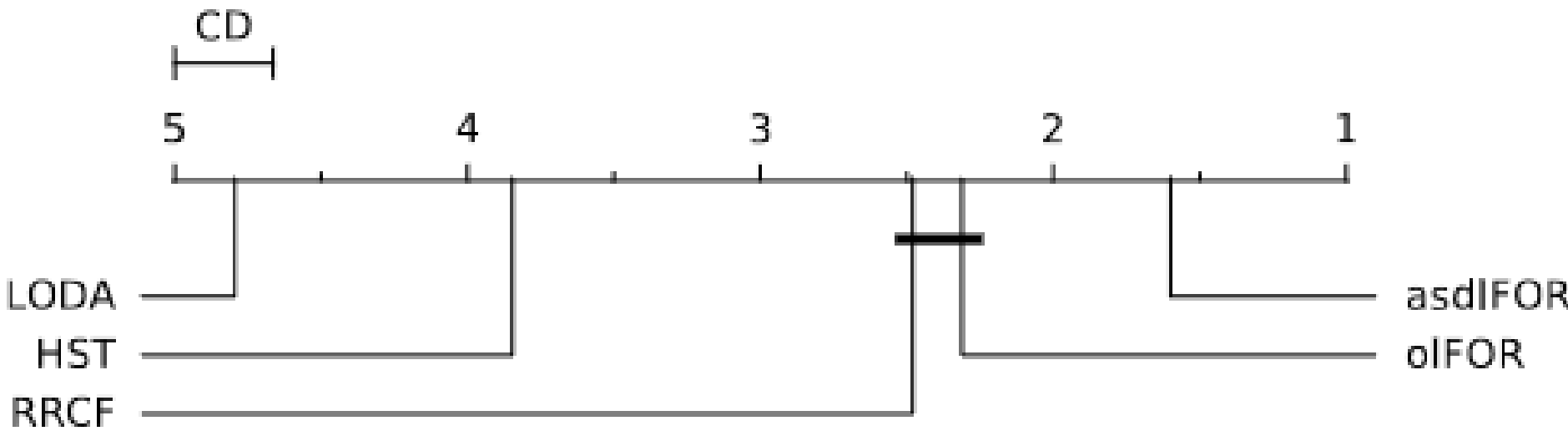


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Experimental results

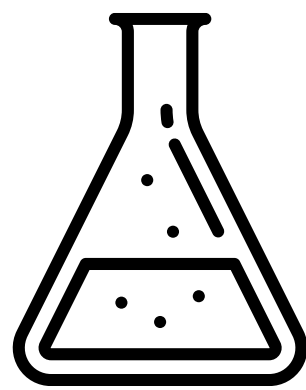
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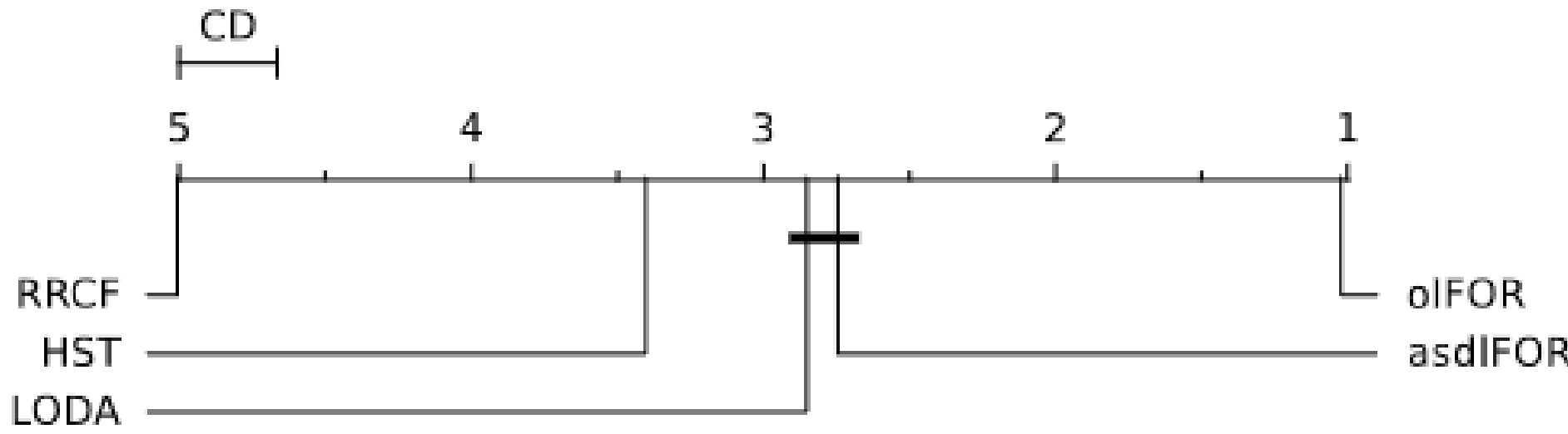
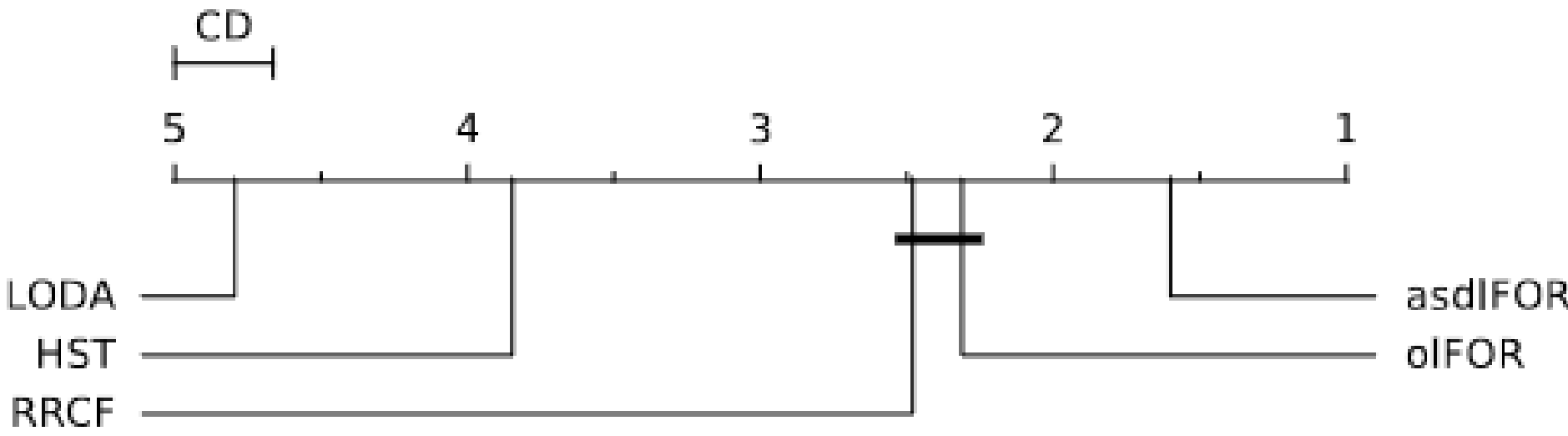
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Experimental results



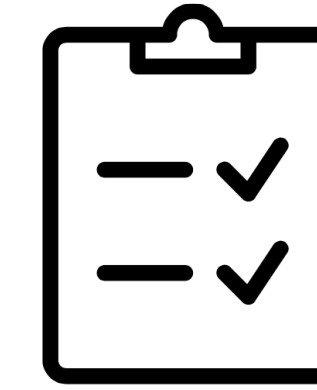
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Conclusions

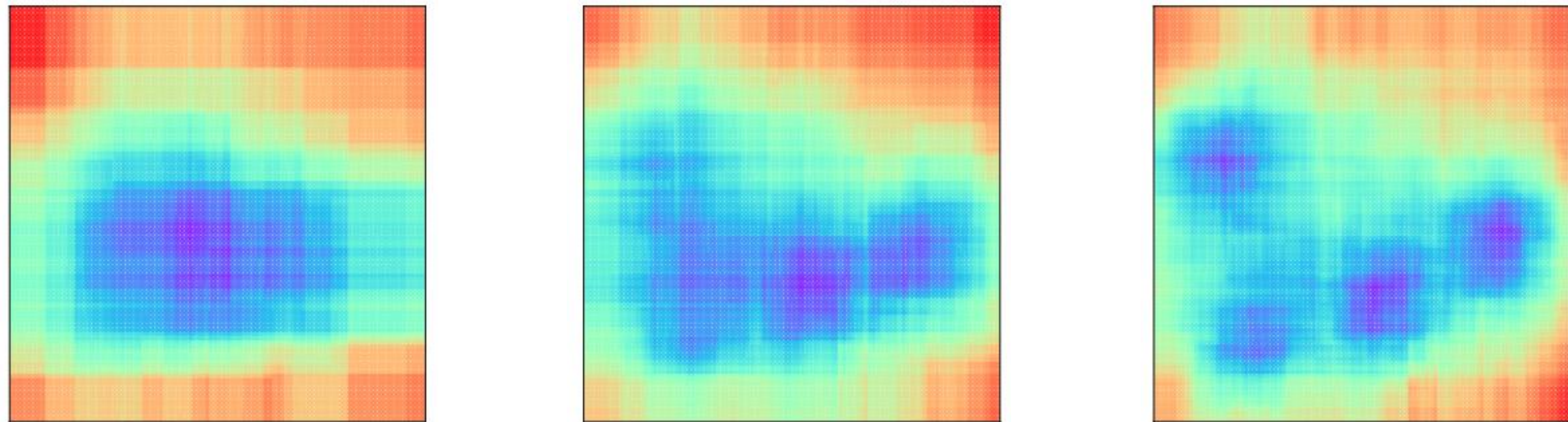


Online Isolation Forest is a truly online anomaly detection algorithm that **tracks the data generating process** as it evolves over time.

Experimental validation demonstrated that Online Isolation Forest:

- is **on par** with state-of-the art techniques in terms of **effectiveness**,
- consistently **outperforms** all competitors in terms of **efficiency**.

Thank you



filippo.leveni@polimi.it