Accelerating PDE Data Generation via Differential Operator Action in Solution Space

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Overlook

- Introduction
- Methods
- Experiments

Introduction

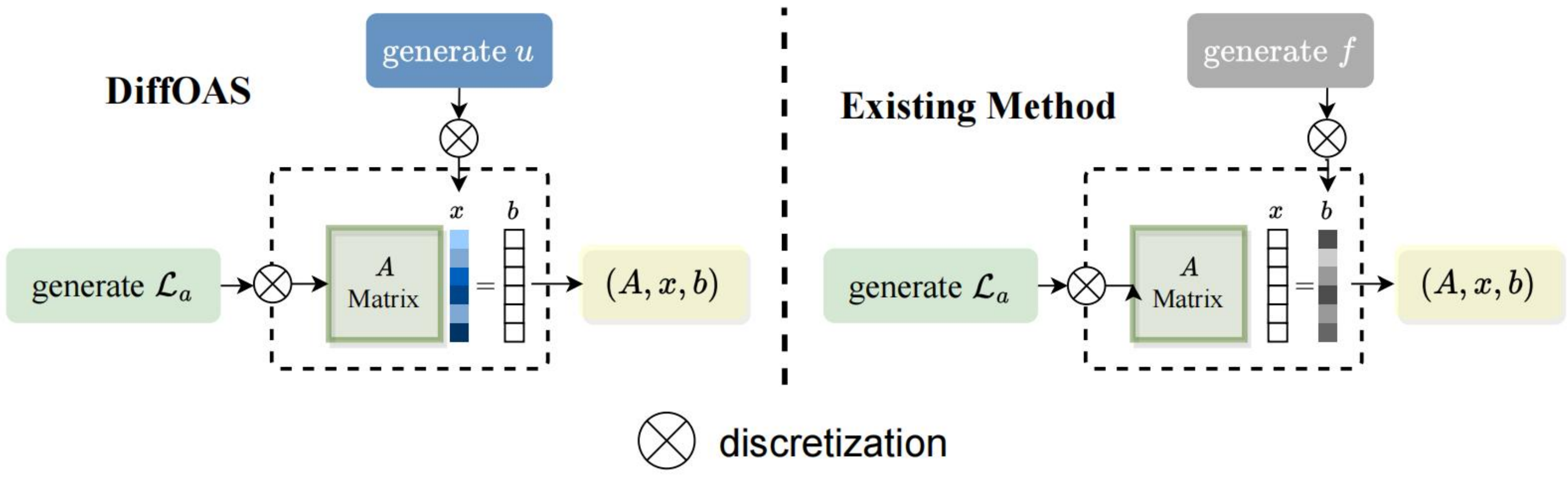
- Objective: Accelerating the Training Data Generation for PDE Neural Operators

- Challenge:The method for generating data by solving PDE results in significant time overhead.

- Idea: The method of operator action in solution space is used to generate data, so as to avoid solving PDEs.

Method

- Considering that solving the linear equation system $Ax = b$ requires significantly more time compared to computing the matrix-vector multiplication Ax, we aim to develop a PDE data generation method based on computing Ax rather than solving $Ax = b$.
- In the continuous case, the process of computing Ax can be represented as calculating $L_\alpha u$. Therefore, we have devised a method to generate data using the differential operator L_α and the solution function u.



Method

Solution Functions Generation

- Since data from different distributions significantly impact neural operator training, our solution functions need to match the actual physical background.

- We generate basis functions by solving a small number of PDEs numerically. These basis functions are fewer than the required training data. They form fundamental elements in the solution space under a specific physical distribution. We then use a Gaussian distribution to randomly weight and normalize these basis functions, creating new solution functions $u_{new}(x)$.

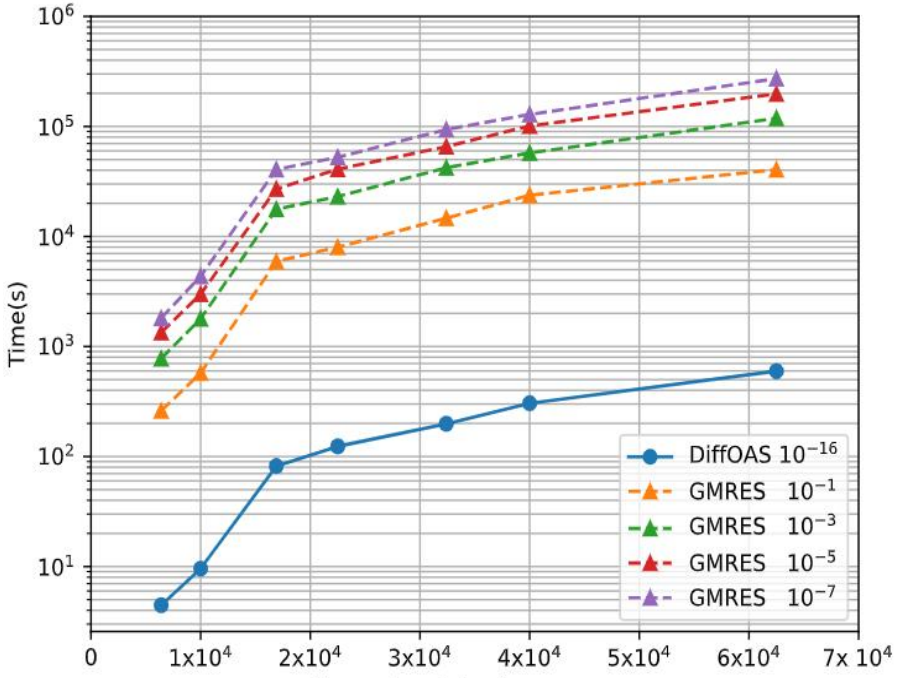
$$u_{\text{new}} = \sum_{i=1}^{N_{basis}} \alpha_i u_i \qquad \alpha_i = \frac{\mu_i}{\sum_{j=1}^{N_{basis}} \mu_j}$$

$$\mu_i \sim N(0,1) \quad i = 1, 2, \ldots, N_{basis}.$$

Experiments

- Speed

size	DiffOAS		GMRES			
	time1	time2	1e-1	1e-3	1e-5	1e-7
2500	9.732e-1	2.687e-1	4.530e1	1.394e2	2.349e2	3.265e2
10000	9.582e0	5.931e-1	5.750e2	1.790e3	2.996e3	4.350e3
16900	8.192e1	8.066e-1	5.933e3	1.771e4	2.704e4	4.066e4
22500	1.236e2	1.004e0	7.994e3	2.305e4	4.086e4	5.251e4
32400	1.983e2	1.462e0	1.466e4	4.216e4	6.561e4	9.384e4
40000	3.051e2	1.943e0	2.371e4	5.739e4	1.010e5	1.285e5
62500	5.971e2	3.815e0	4.052e4	1.186e5	1.977e5	2.722e5



Experiments

Data Quality

Dataset	Darcy Flow 10000			Wave 22500			Diffusion 62500		
	TIME(s)	FNO	DeepONet	TIME(s)	FNO	DeepONet	TIME(s)	FNO	DeepONet
GMRES 1000	2.99E2	4.56E-3	6.82E-2	3.27E3	4.05E-4	6.12E-2	1.99E4	4.29E-3	7.70E-2
DiffOAS 1000	8.97E0	2.15E-2	**6.82E-2**	1.60E2	1.71E-3	**4.57E-2**	5.99E2	1.76E-2	**6.40E-2**
DiffOAS 5000	9.20E0	6.86E-3	**5.58E-2**	1.60E2	7.25E-4	**4.74E-2**	6.01E2	5.66E-3	**5.80E-2**
DiffOAS 10000	9.49E0	**4.26E-3**	**5.62E-2**	1.60E2	6.52E-4	**4.72E-2**	6.03E2	4.79E-3	**6.04E-2**

Ablation Experiments on Basis Functions

Basis	FNO	DeepONet
DiffOAS	4.260%	5.629%
GRF	31.44%	88.08%
Fourier	93.56%	96.98%
Chebyshev	63.11%	68.23%

Thank You!