





tn GPS

Discovering Unknown Tensor Network Structure Search Algorithms via Large Language Models (LLMs)

Presenter: Junhua Zeng {jh.zenggdut@gmail.com}

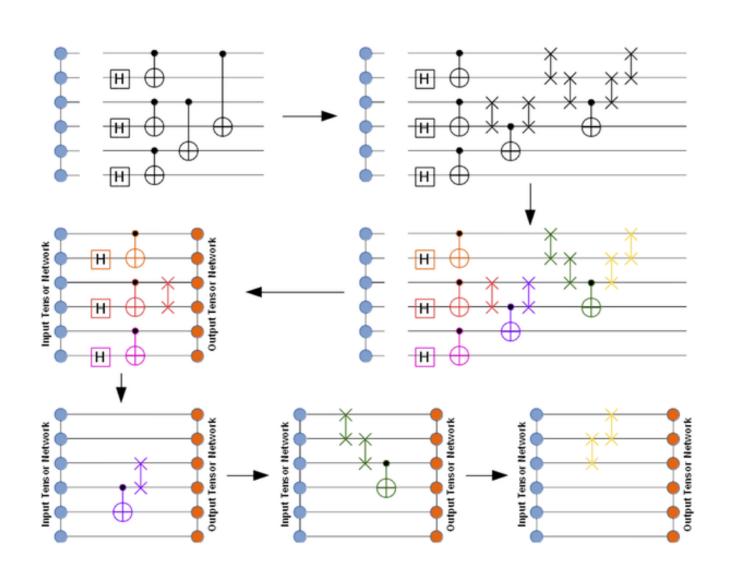
Junhua Zeng^{1*}, Chao Li^{2*}, Zhun Sun³, Qibin Zhao² and Guoxu Zhou^{1,4}

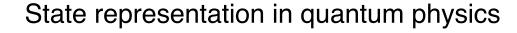
¹Guangdong University of Technology, China ²RIKEN-AIP, Japan ³Tencent Inc., China

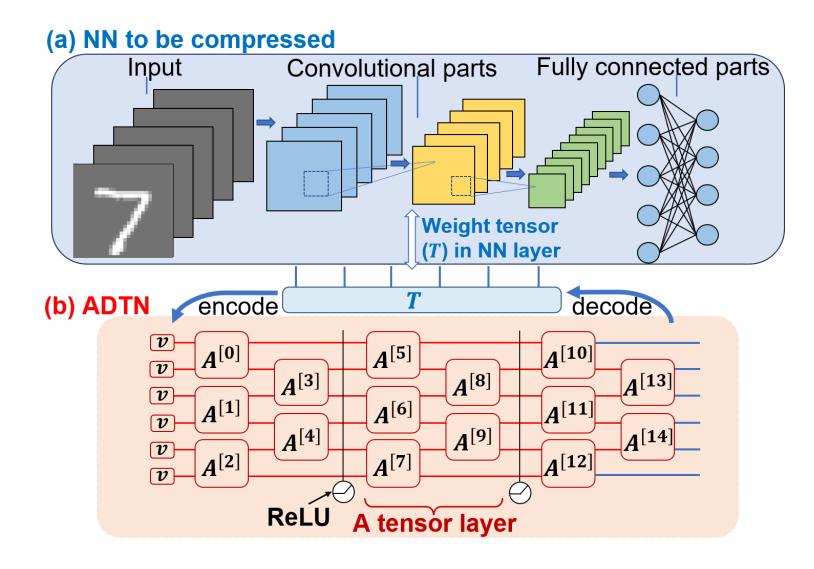
⁴Key Laboratory of Intelligent Detection and the Internet of Things in Manufacturing, China

Applications of Tensor Networks (TNs)

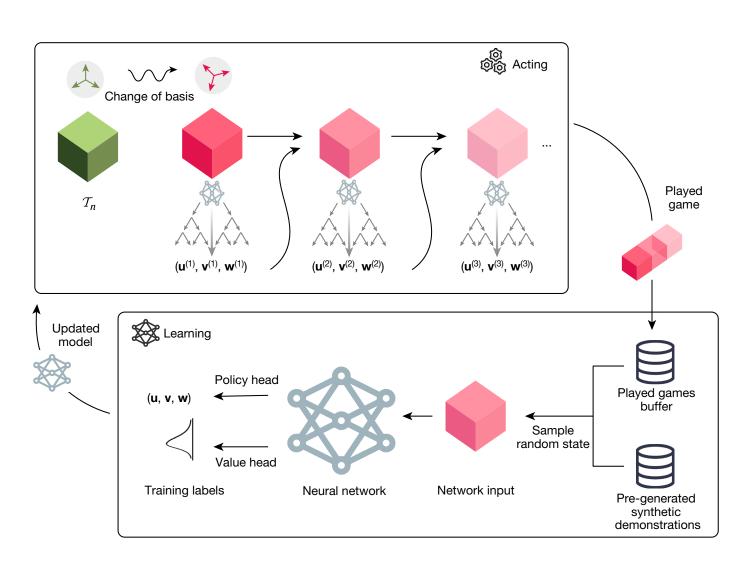
TN is an efficient framework for modeling complex systems by decomposing it into simpler, interconnected parts.







Model representation in machine learning



Discovering faster *matrix multiplication* (AlphaTensor, Fawzi et al., Nature'22)

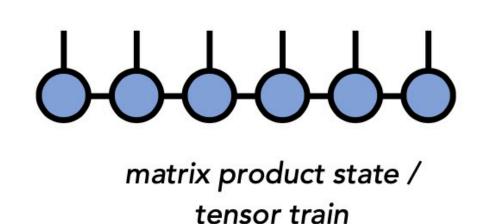
Why TN so Powerful?

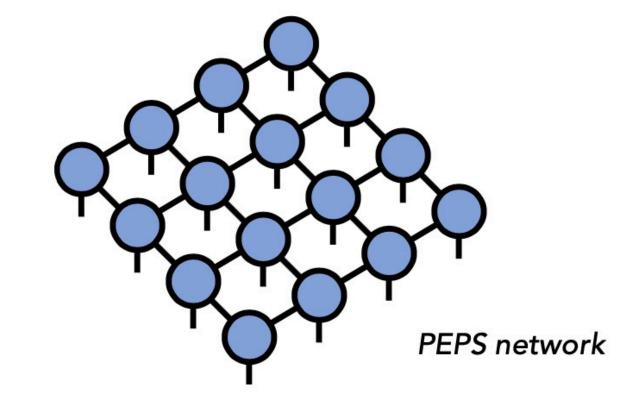
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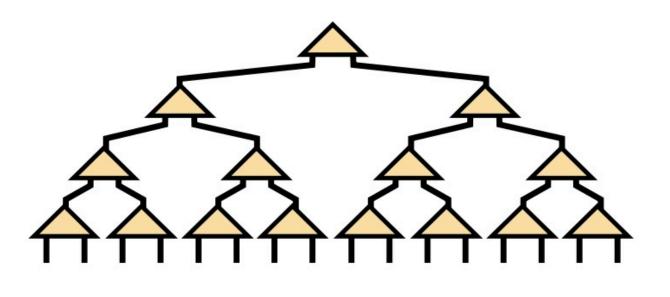
Tensor networks can efficiently represent high-dimensional space.

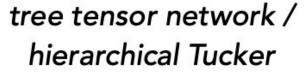
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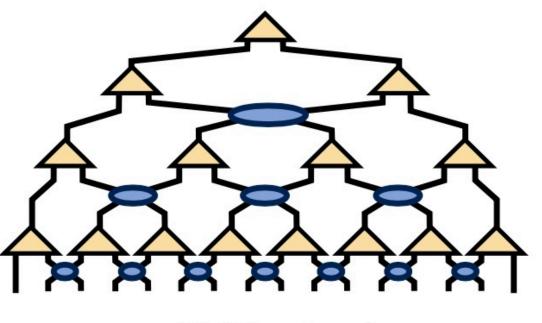
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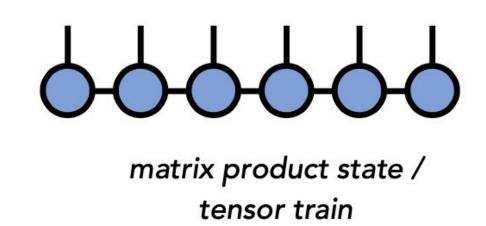


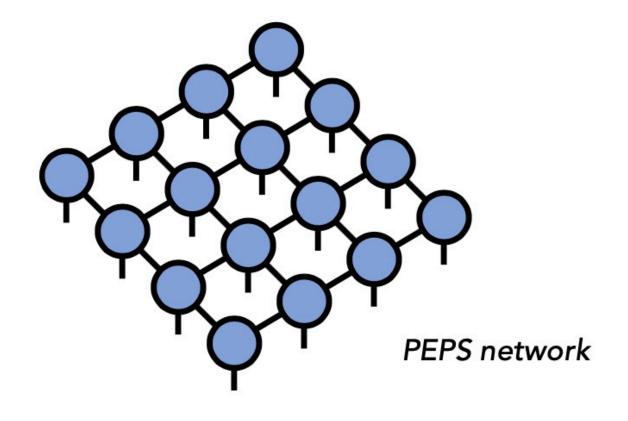
MERA network

R. Orus, Ann. of Phys. 349, 117-158 (2014)

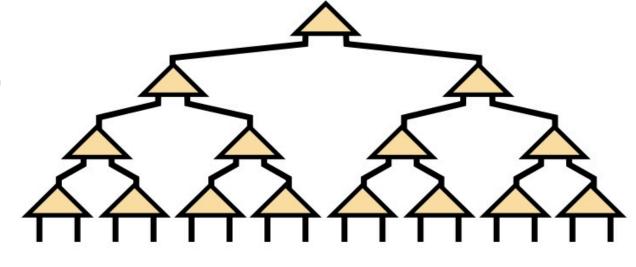
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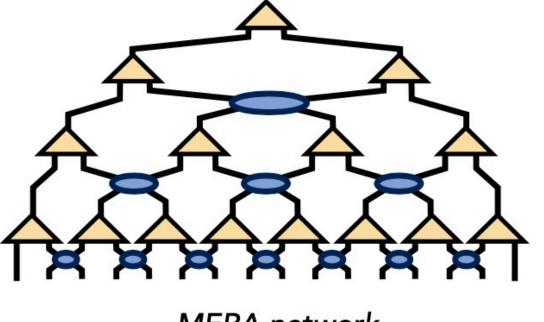




How to select the promising tensor network models?



tree tensor network / hierarchical Tucker

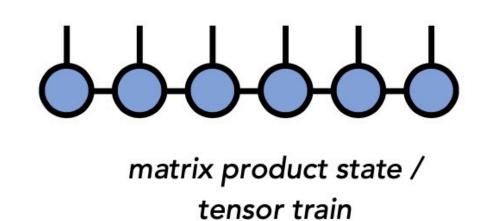


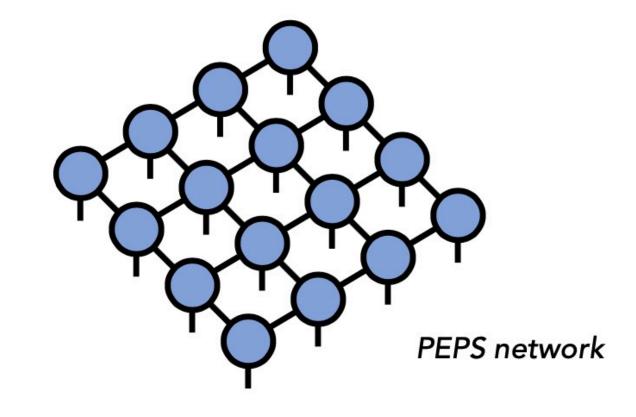
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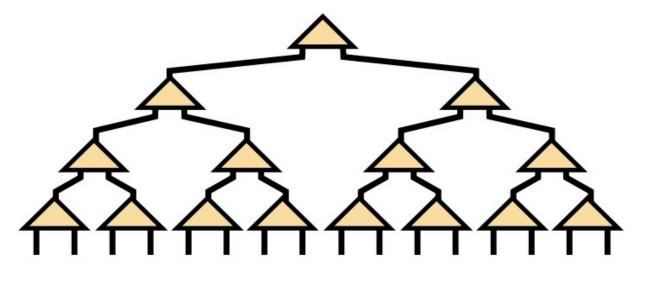
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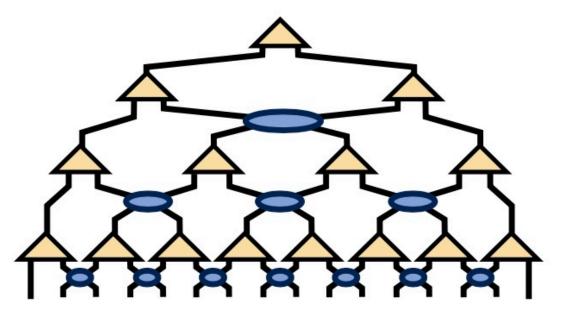


How to select the promising tensor network models?

Tensor Network Structure Search (TN-SS)



tree tensor network / hierarchical Tucker



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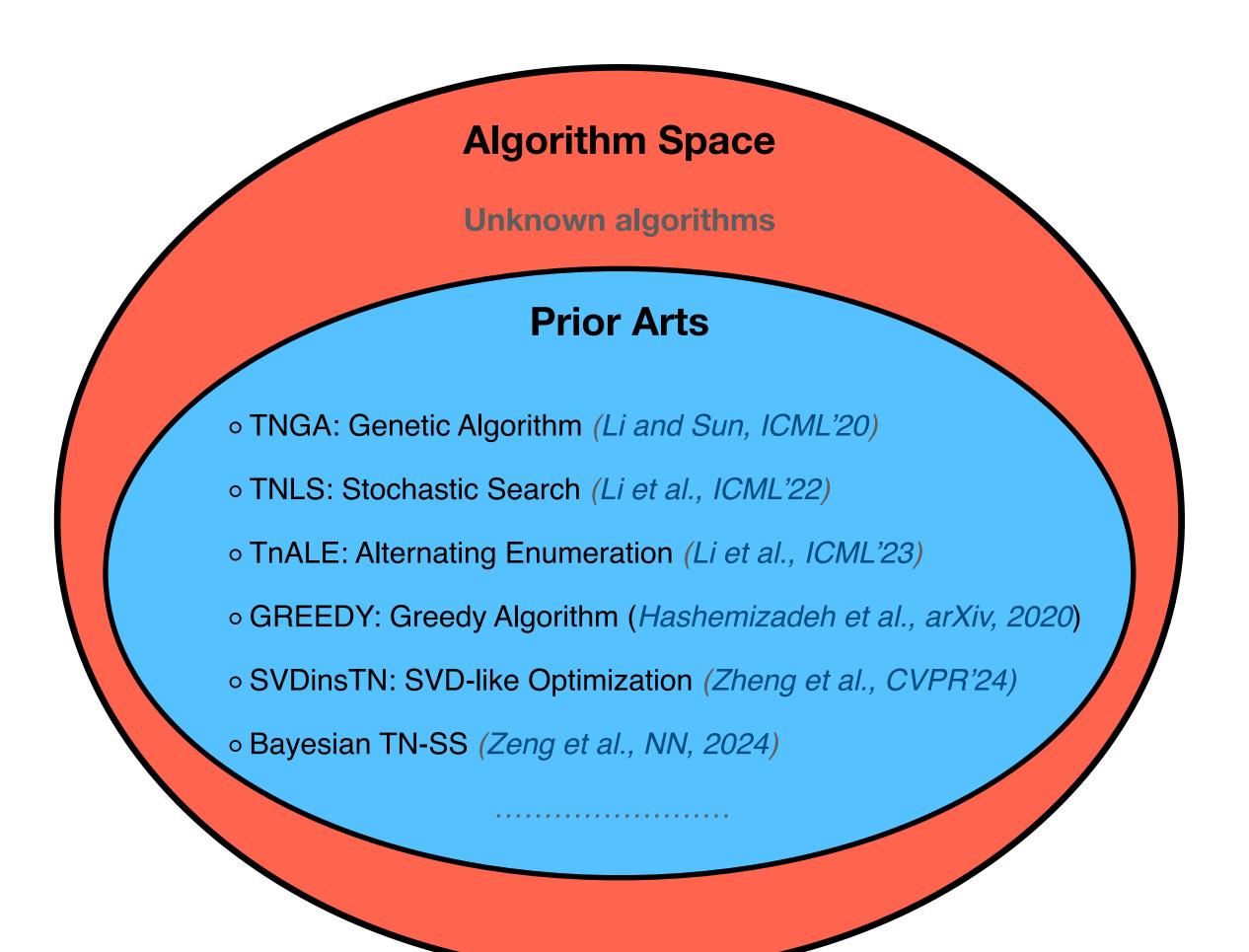
Motivations

Prior Arts

- TNGA: Genetic Algorithm (Li and Sun, ICML'20)
- TNLS: Stochastic Search (Li et al., ICML'22)
- TnALE: Alternating Enumeration (Li et al., ICML'23)
- GREEDY: Greedy Algorithm (Hashemizadeh et al., arXiv, 2020)
- o SVDinsTN: SVD-like Optimization (Zheng et al., CVPR'24)
- o Bayesian TN-SS (Zeng et al., NN, 2024)

.....

Motivations



Motivations





Language Space

Algorithm Space

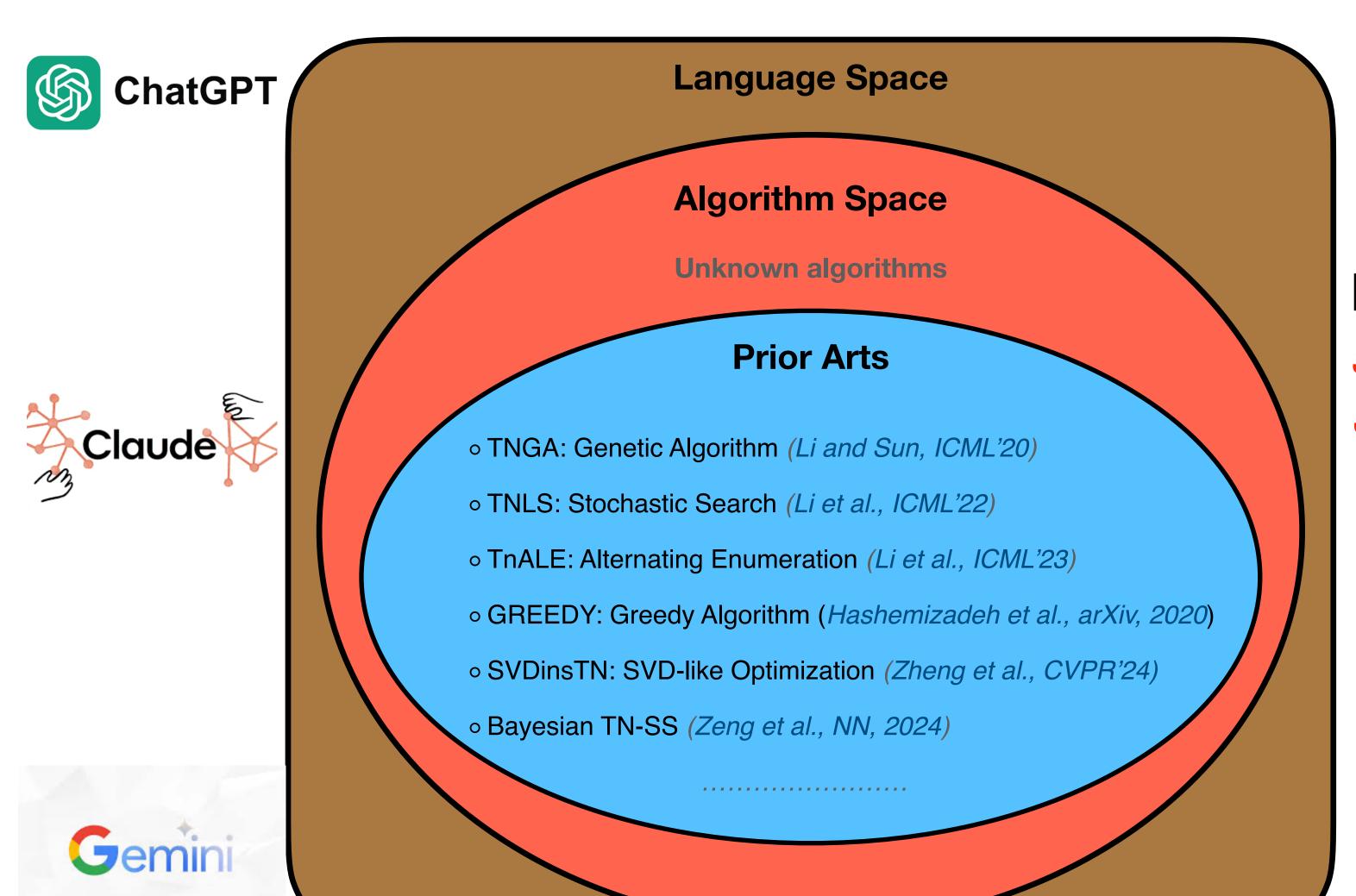
Unknown algorithms

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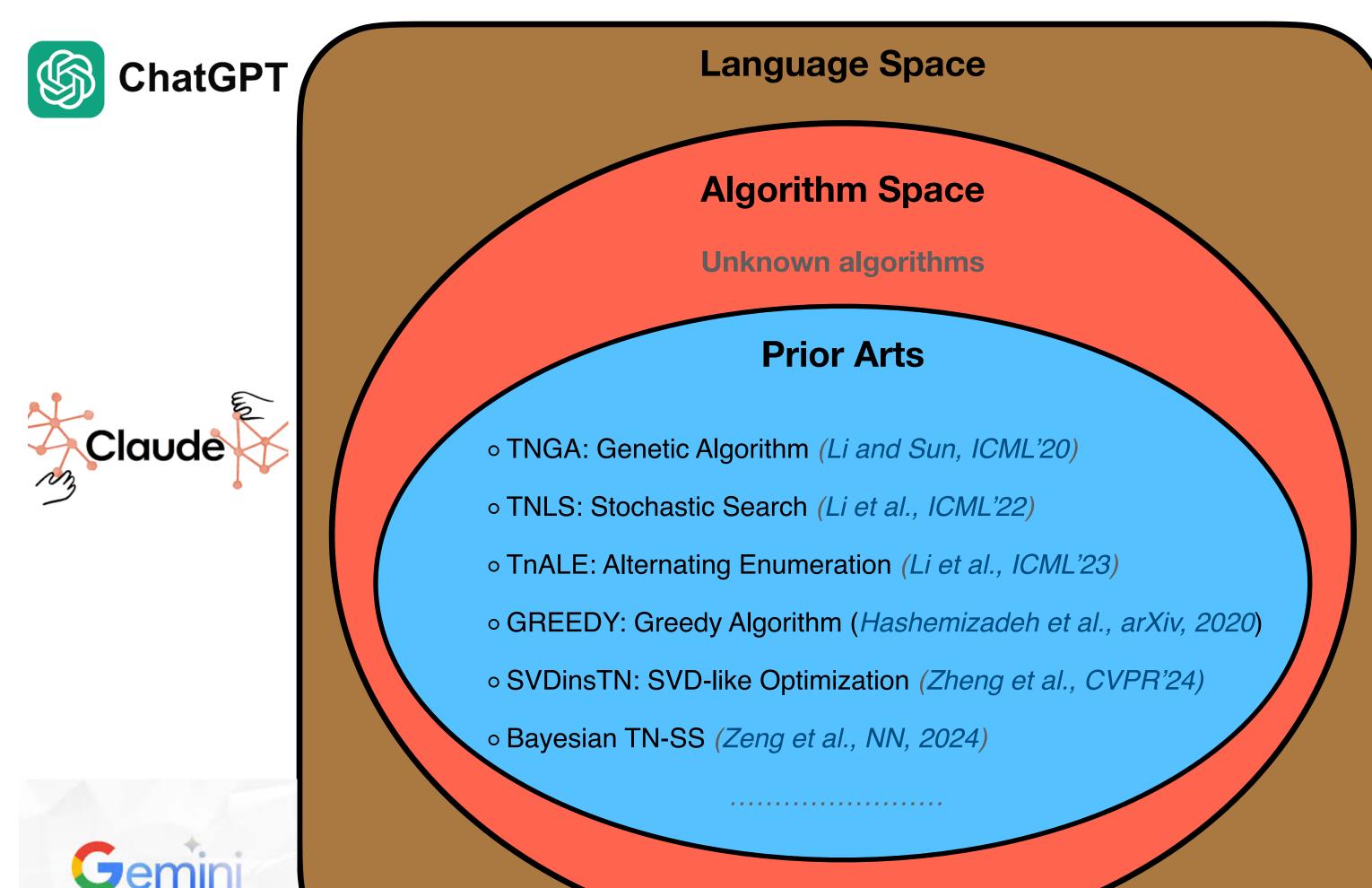


Motivations



Exploiting the *enormous language* space of LLMs for *autonomous TN-SS algorithm discovery*.

Motivations



Exploiting the *enormous language* space of LLMs for autonomous TN-SS algorithm discovery.

Saving human experts from the labor-intensive algorithm design process and letting them focus on more challenging problems.



Background Contributions

Contributions

1. We propose tensor-network-purposed GPT-driven structure search (tnGPS), a LLM-driven automation framework designed to automatically generate novel and effective TN-SS algorithms tailored to specific downstream tasks;

Contributions

- 1. We propose tensor-network-purposed GPT-driven structure search (tnGPS), a LLM-driven automation framework designed to automatically generate novel and effective TN-SS algorithms tailored to specific downstream tasks;
- 2. Experimental results demonstrate that the algorithms discovered by tnGPS outperform existing TN-SS algorithms on benchmark data.

How Human Experts Conduct Innovative Research



How Human Experts Conduct Innovative Research

Idea Pool

, SCORE, CONTEXT)

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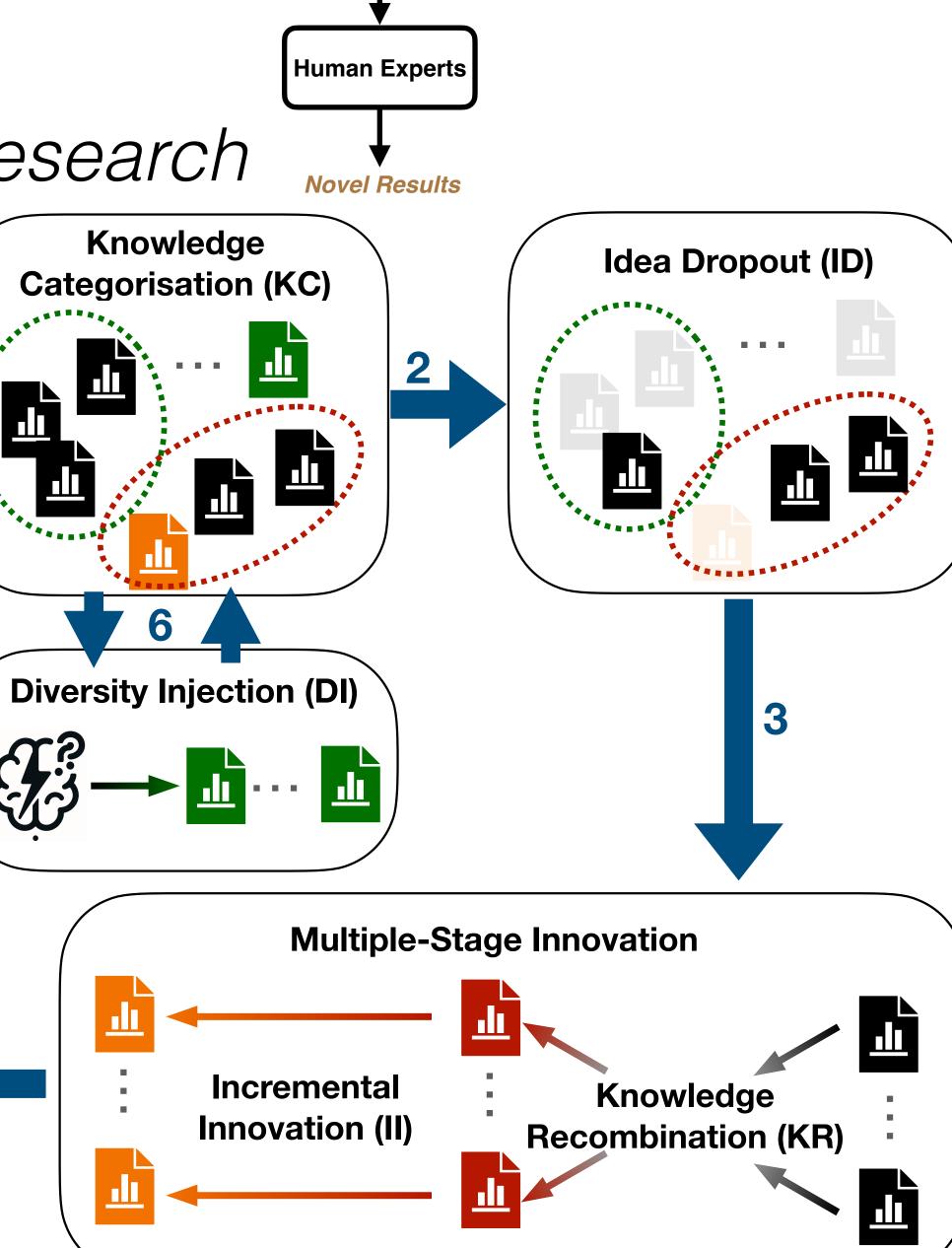
, SCORE, CONTEXT)

Experiments

(Evaluation)

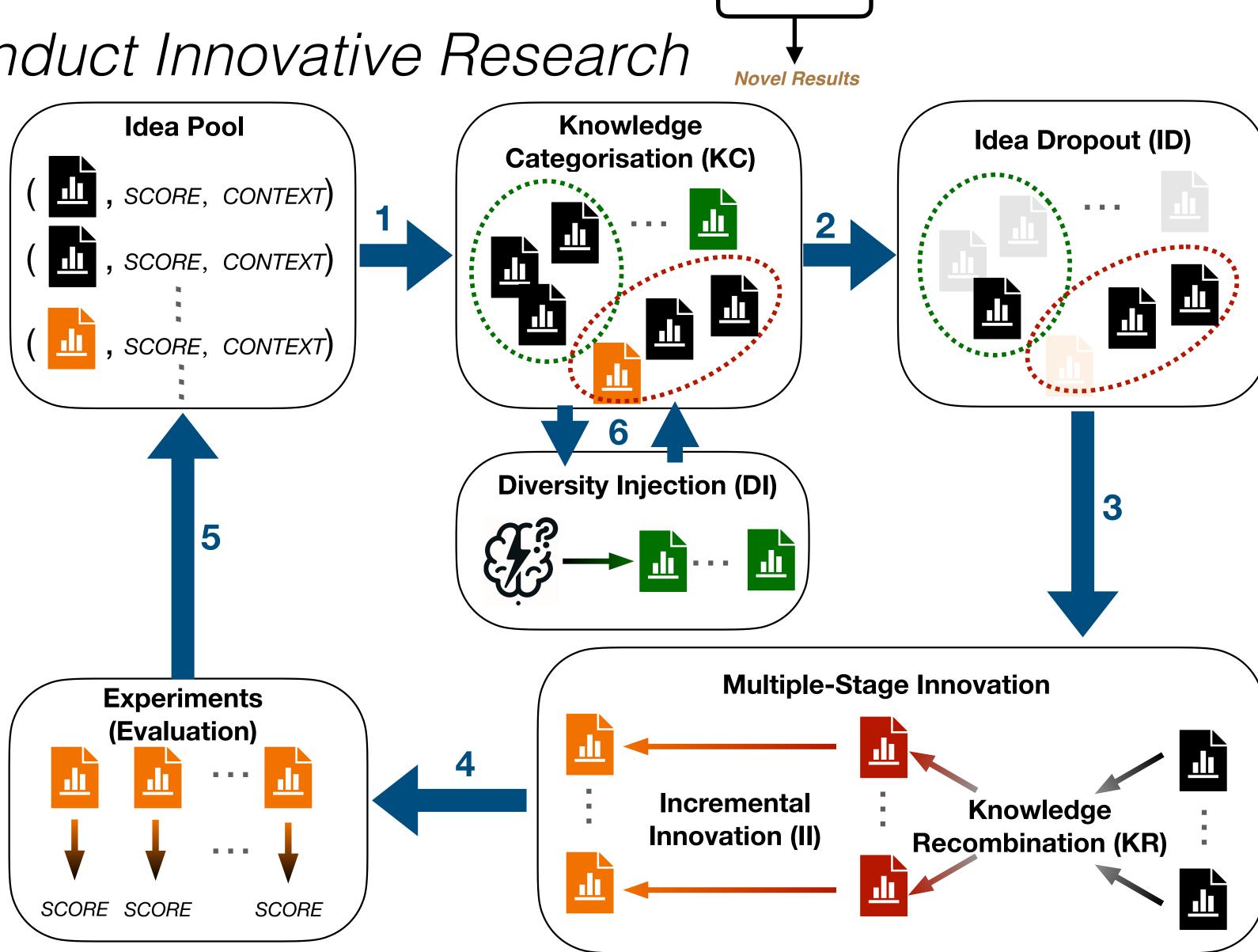
SCORE

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How Human Experts Conduct Innovative Research

Idea Pool: Gather information through literature reviews and paper retrieval



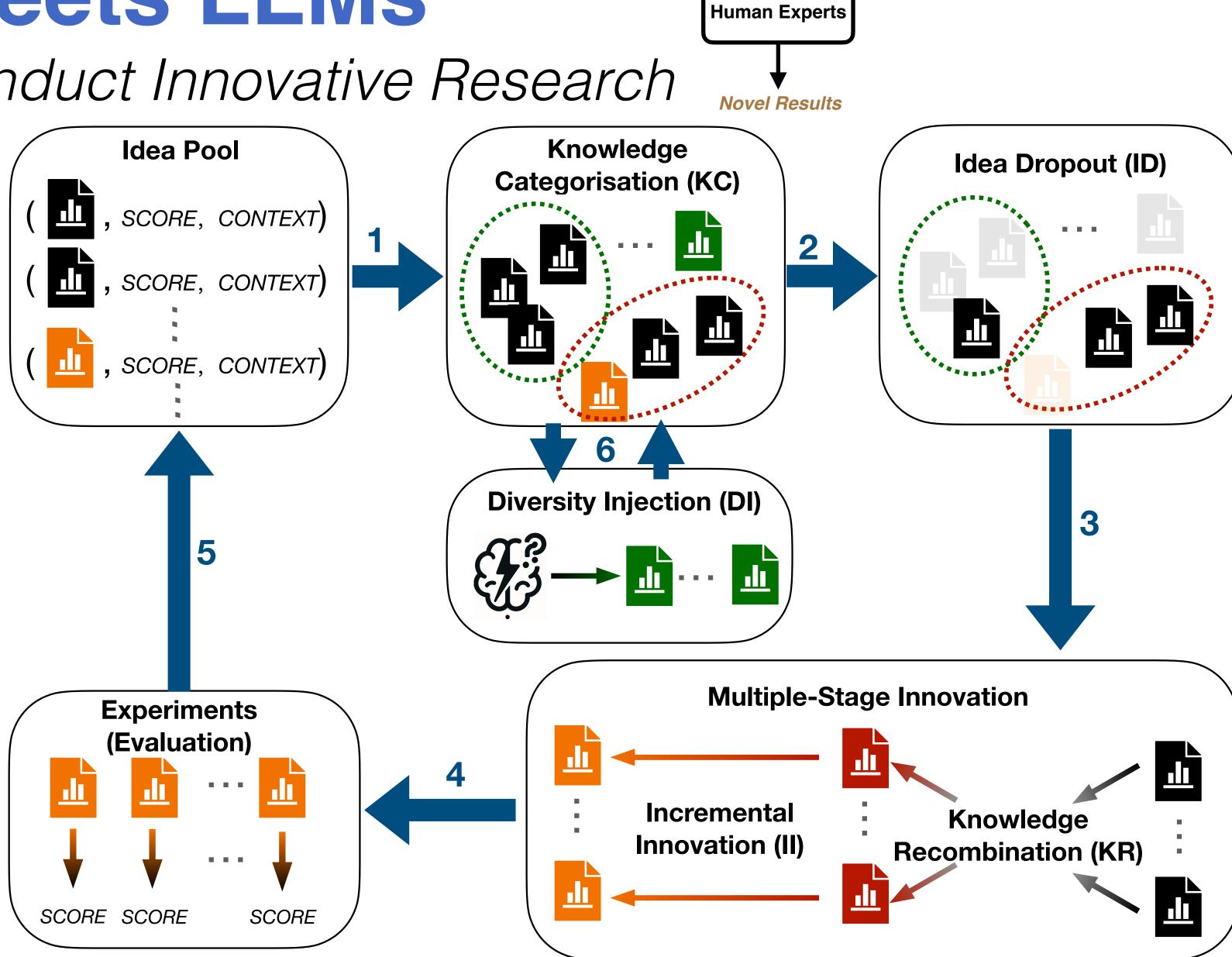
Existing Studies

Human Experts

How Human Experts Conduct Innovative Research

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Knowledge Categorization (KC): Refine ideas into knowledge clusters.

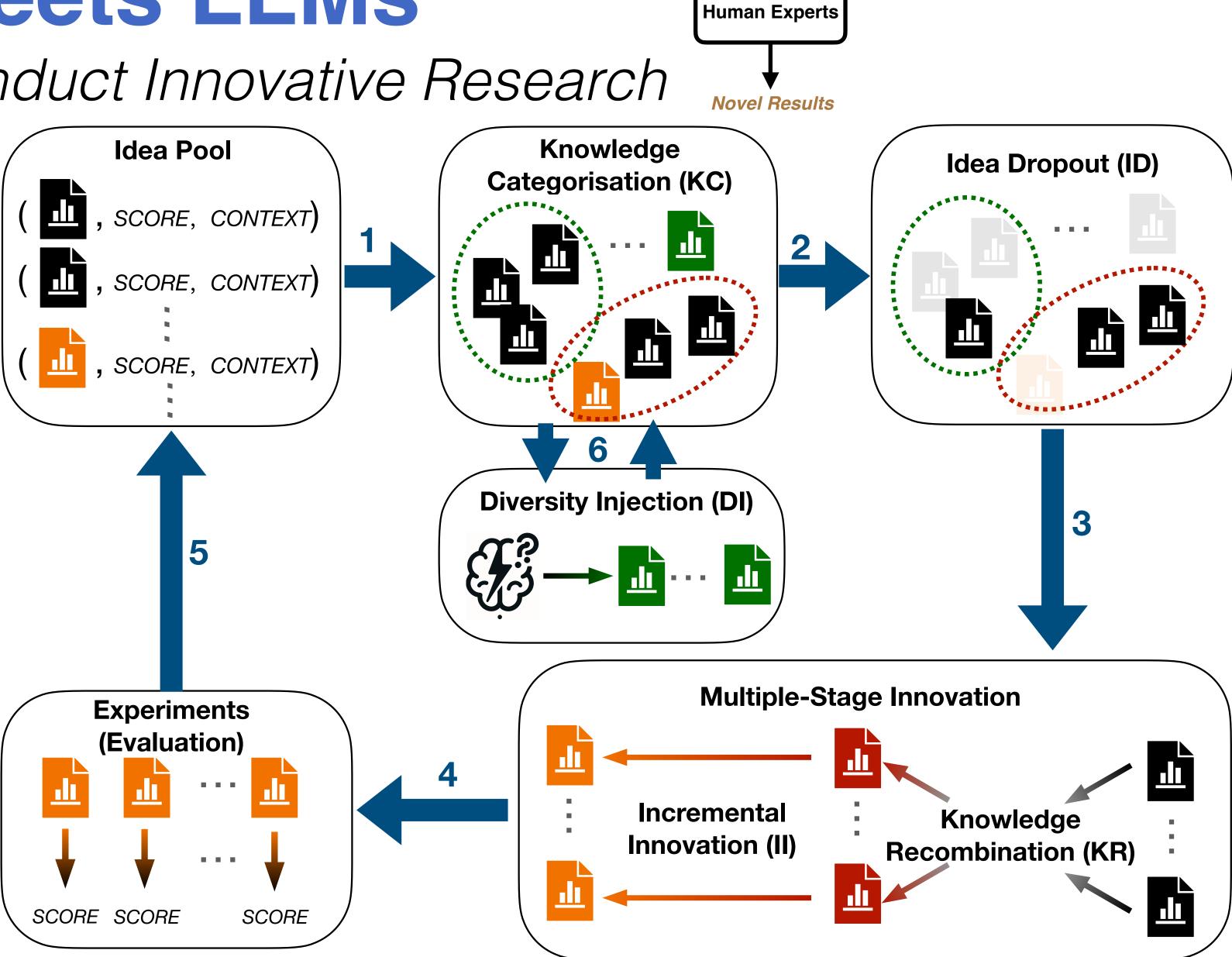


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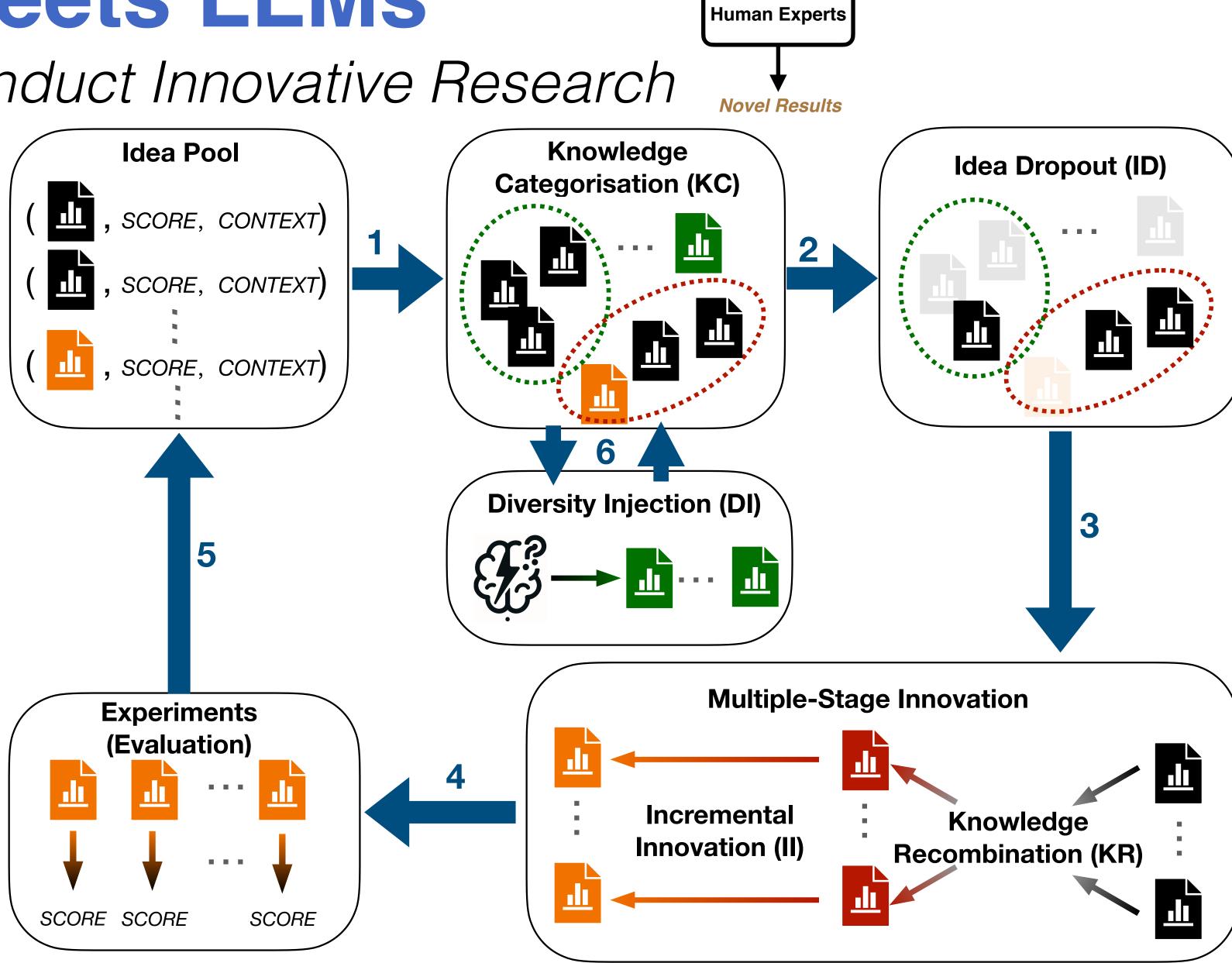
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Knowledge Recombination (KR): Generate new ideas by merging existing ones.



How Human Experts Conduct Innovative Research

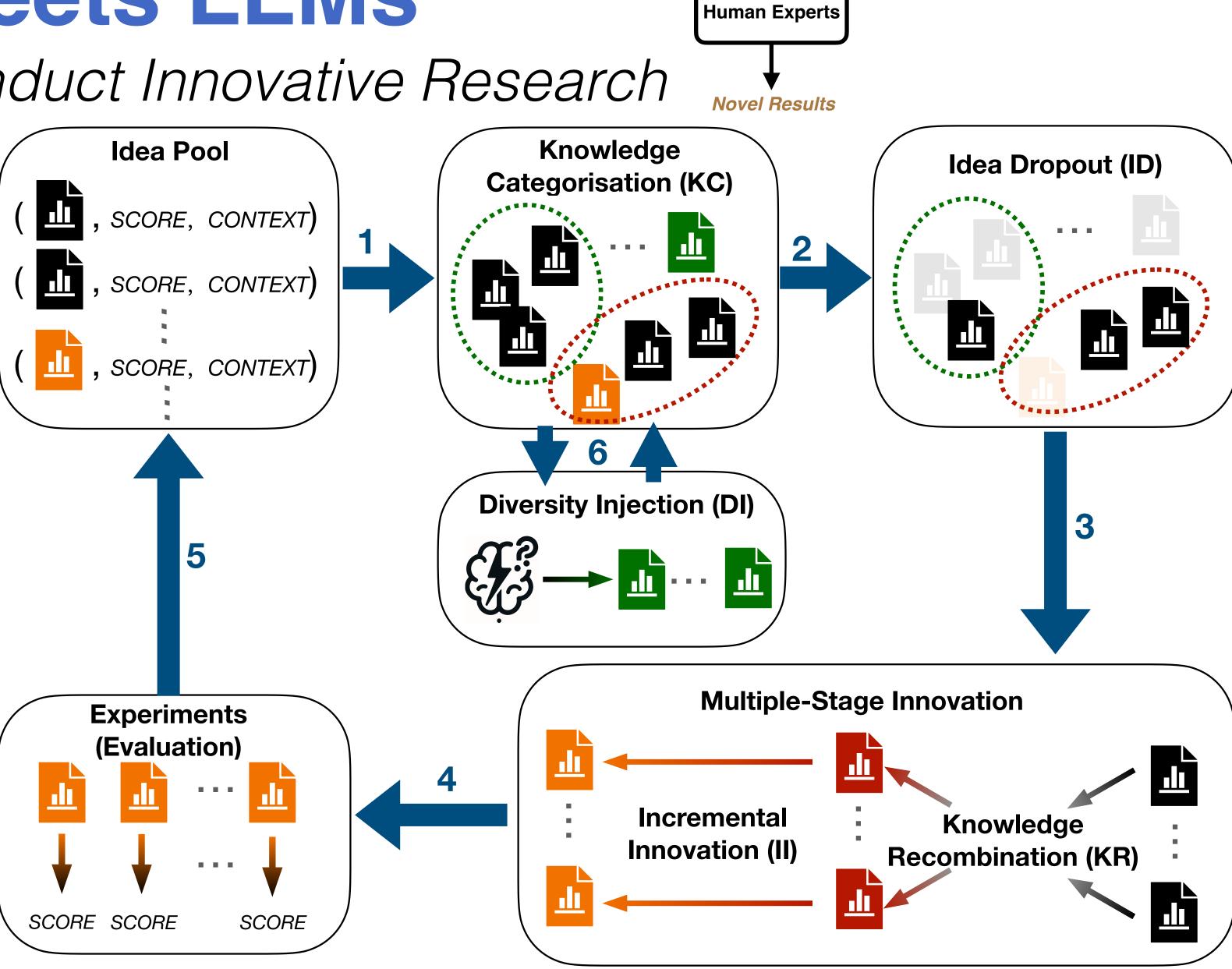
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Incremental Innovation (II): Make gradual improvements to existing ideas.



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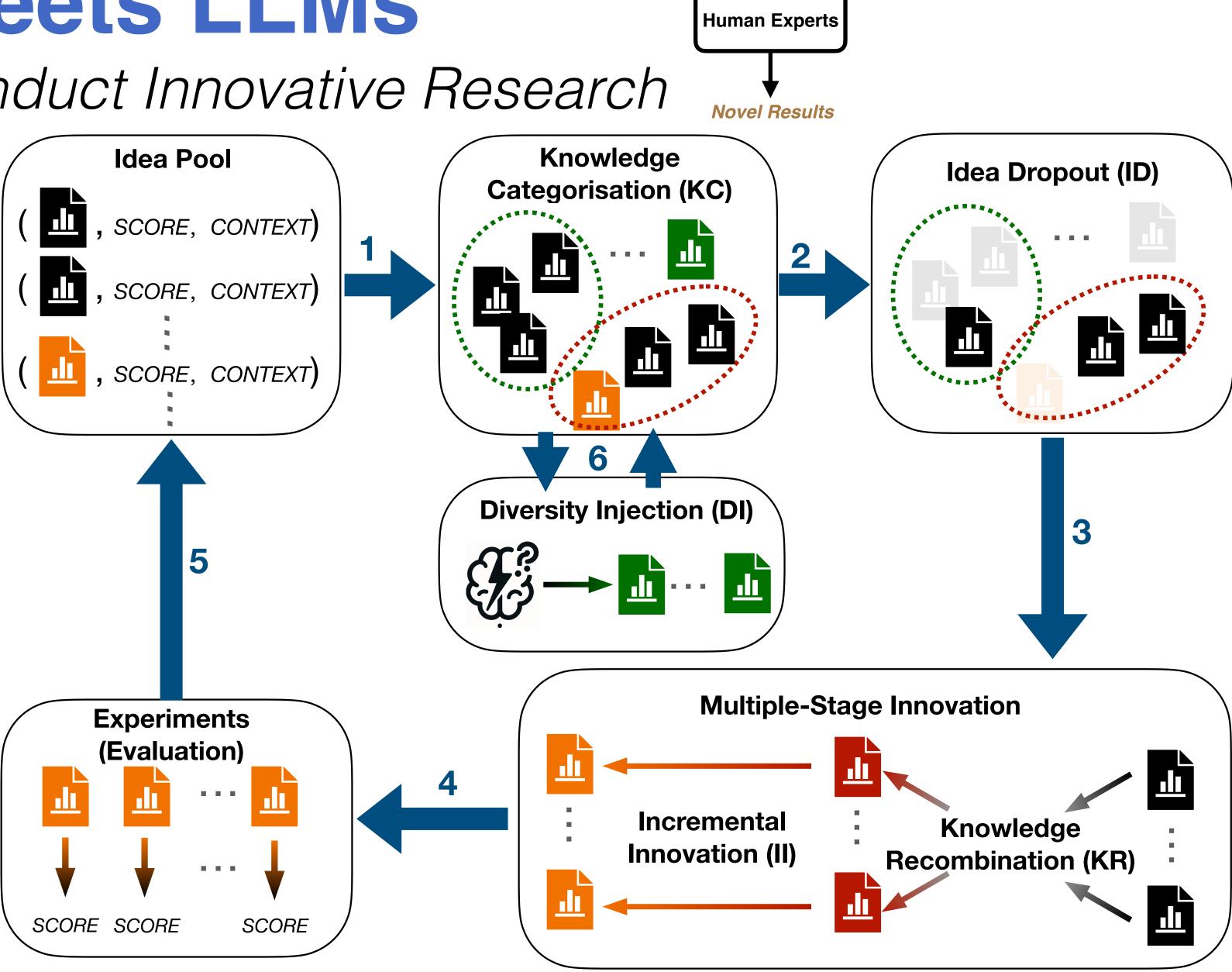
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Experiments (Evaluation): Test and score ideas to validate their potential.



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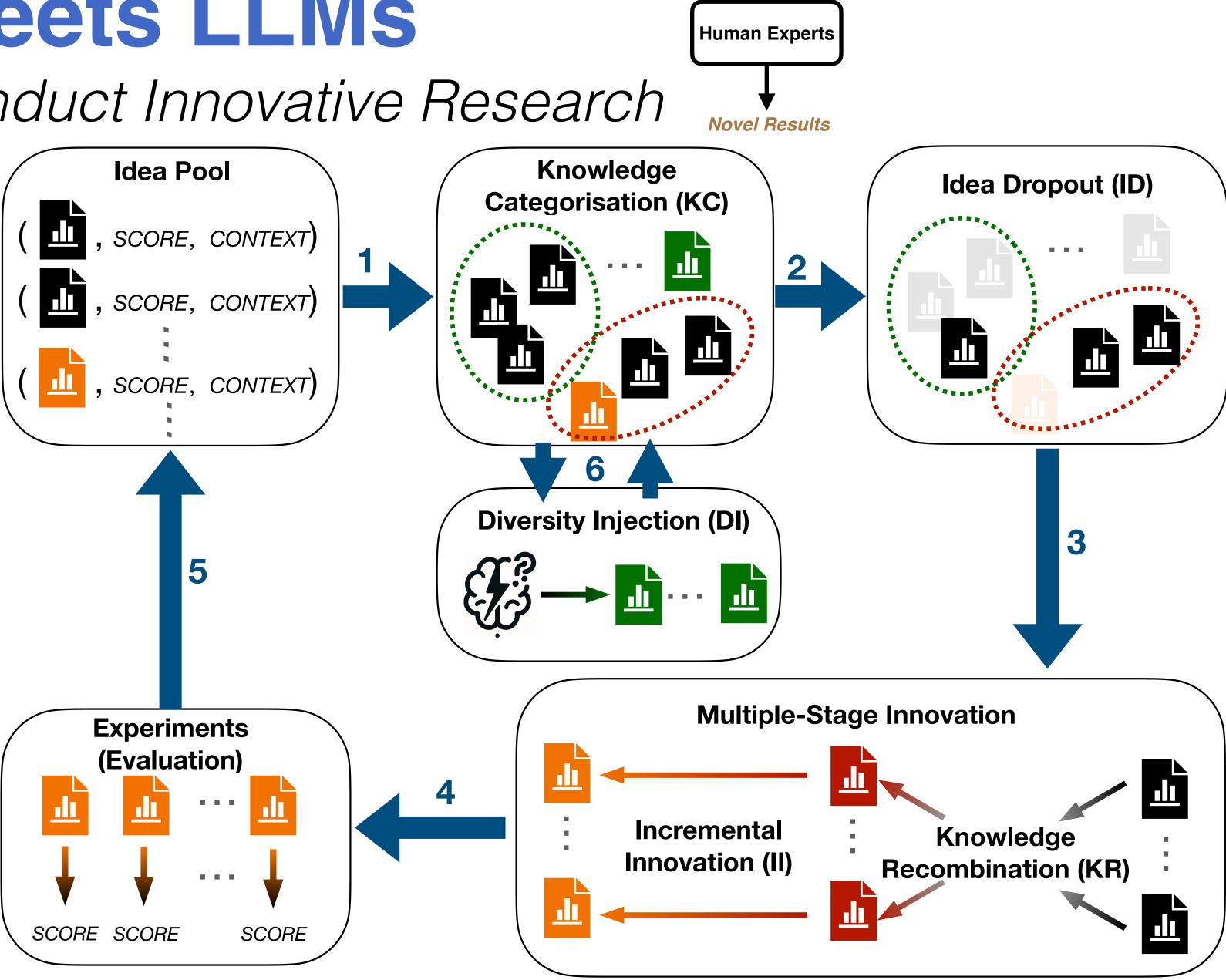
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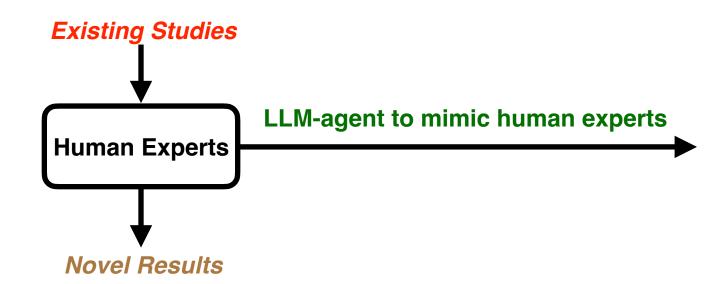
Diversity Injection (DI): Introduce new, orthogonal ideas through brainstorming or external feedback.



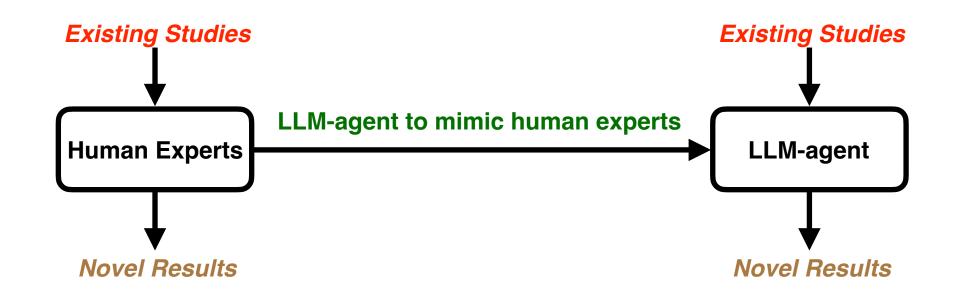
LLM-agent Driven Innovative Research



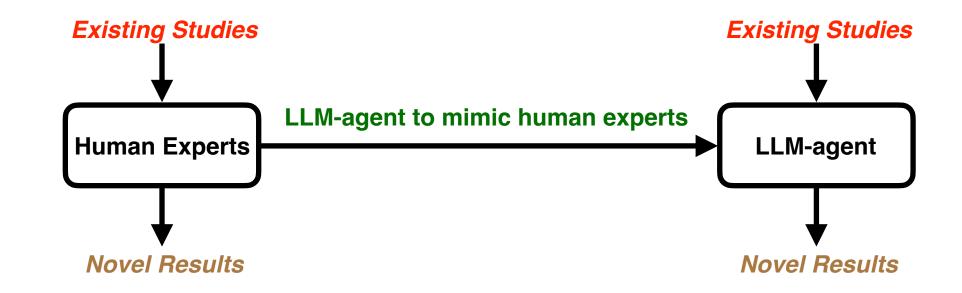
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LLM-agent Driven Innovative Research



Which algorithm in the above is methodologically most similar to the new algorithm? Just give me the function number with no other words.

Knowledge Categorization (KC) Prompt

Algorithm 1:

omitted)

Algorithm N:

Independently make improvements over these Algorithms that will increase their practical performance (not on the code efficiency, readability and parallel processing level). You are encouraged to be creative to incorporate novel ideas.

Algorithm 1:

Algorithm 1 score:

• (omitted)

Algorithm N: . . .

Algorithm N score:

Algorithms 1 to N are implementations of the 'GenerateSample' function. A lower score implies better performance.

Learning from their results, think about what works and what doesn't, provide *M* novel methods with lower scores. You are encouraged to be creative to incorporate novel ideas but do not simply stack methods together.

Knowledge Recombination (KR) Prompt

/ Algorithm 1: # centroid

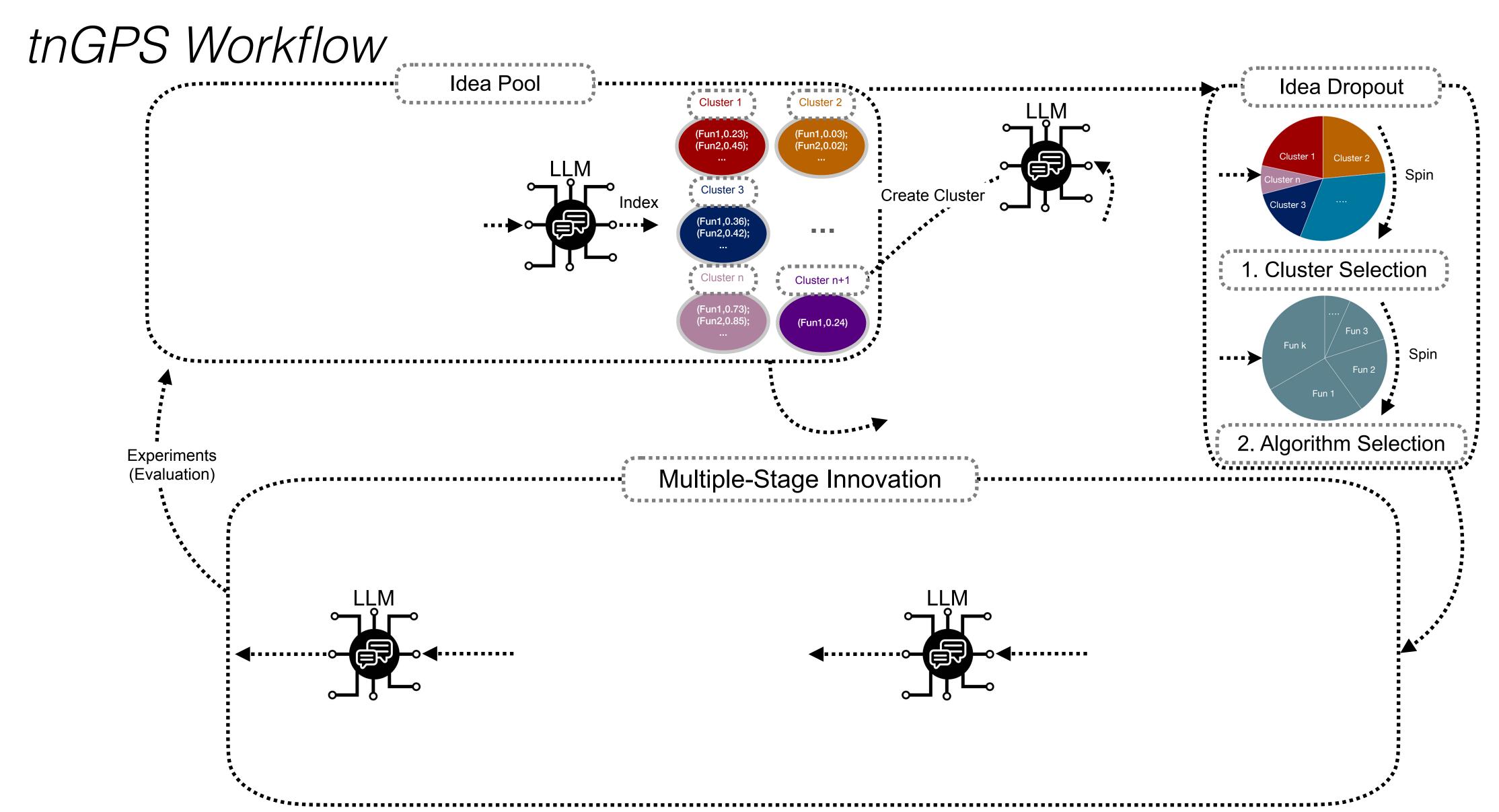
(omitted)

Algorithm N: # centroid

Give me a novel 'GenerateSample' that is methodologically different from the above algorithms. You are encouraged to be creative to incorporate novel ideas but do not simply stack methods together.

Incremental Innovation (II) Prompt

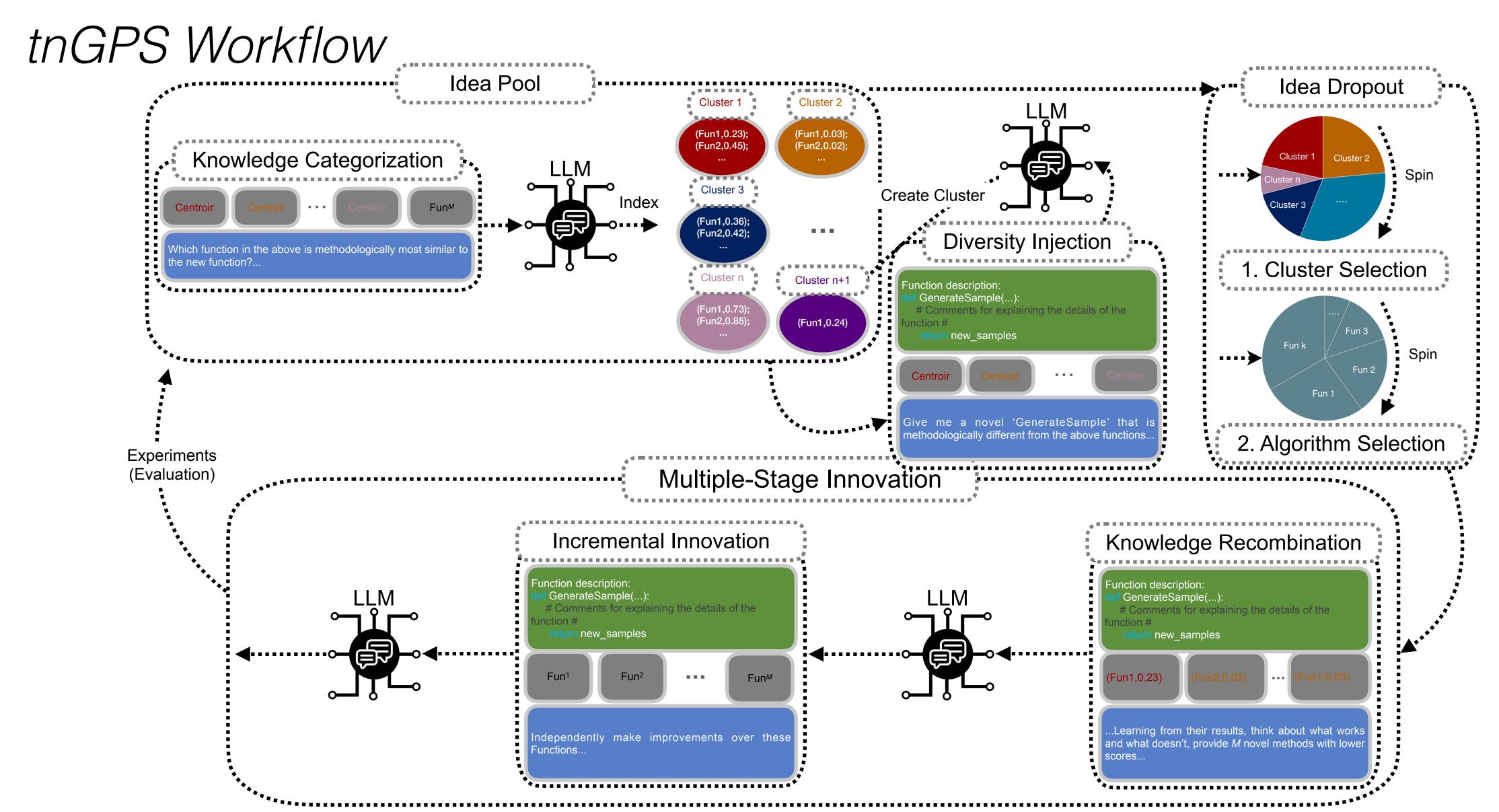
Diversity Injection (DI) Prompt



tnGPS Workflow Cluster 2 (Fun2,0.02); (Fun2, 0.45) **Knowledge Categorization** Cluster 3 (Fun2,0.42); 1. Cluster Selection Cluster n Cluster n+1 (Fun2, 0.85) (Fun1,0.24) 2. Algorithm Selection **Experiments** (Evaluation)

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Experimental Results

Natural Images Compression

Objective:

$$F(G, \mathbf{r}) = \underbrace{\frac{1}{\epsilon(G, \mathbf{r})}}_{\text{compression ratio (CR)}} + \lambda \cdot \underbrace{\min_{\mathcal{Z} \in TNS(G, \mathbf{r})} \|\mathcal{X} - \mathcal{Z}\|^2 / \|\mathcal{X}\|^2}_{\text{relative square error (RSE)}},$$

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Settings of tnGPS: four images for training, ten images for testing.

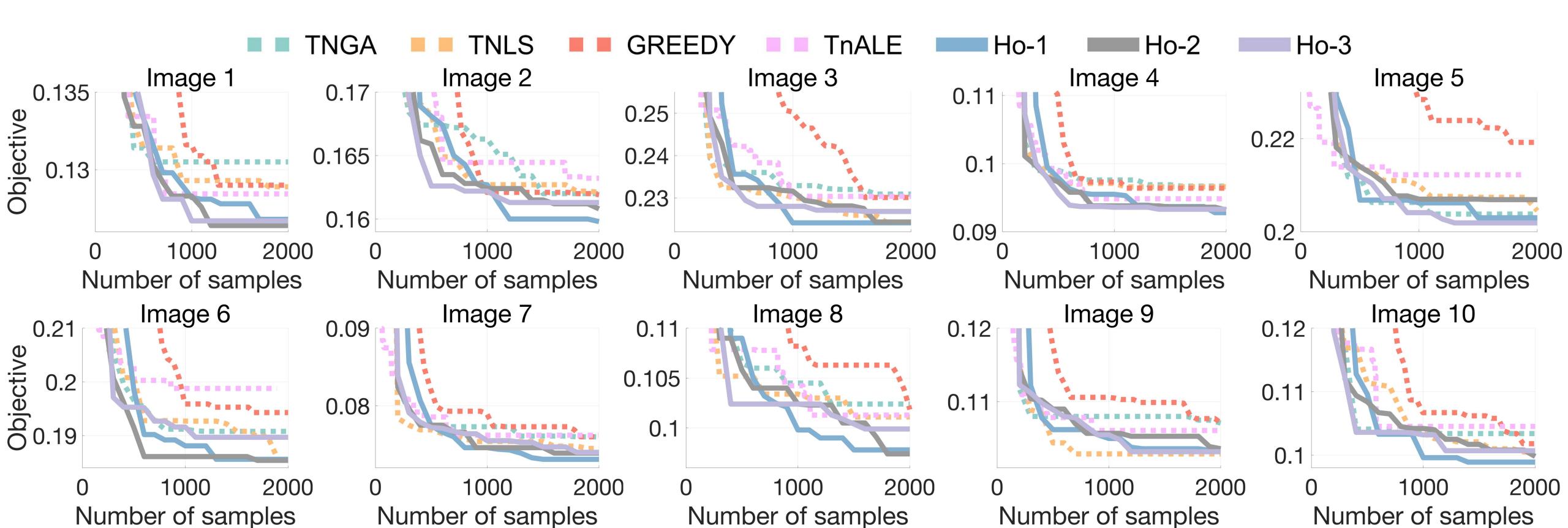
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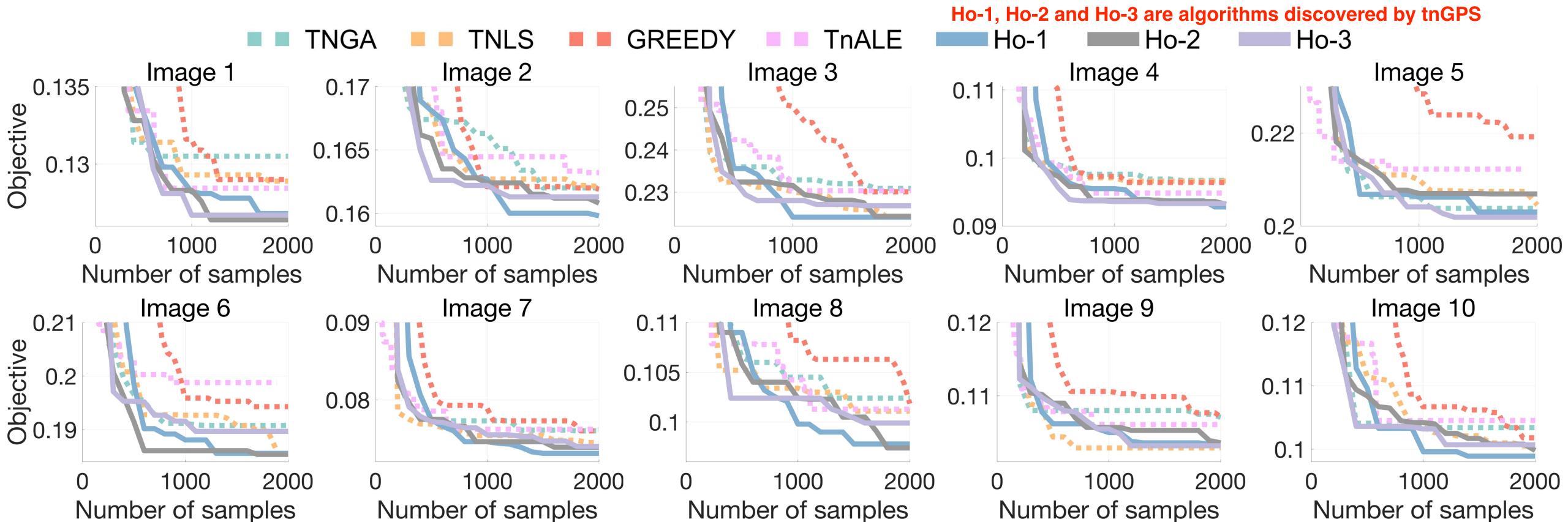


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Ablation Studies

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	baseline	tnGPS	KR	ΙΙ	DI
Objective	0.1558	0.1102	0.1308	0.1273	0.1239

The results highlight the importance of the 'KR', 'II', and 'DI' components.

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baseline	GPT-4	GPT-3.5	Claude-1	Claude-2	Incomplete descriptions
0.1847	0.1813	0.1842	0.1840	0.1834	0.1819

More powerful LLMs like GPT-4 enhance tnGPS performance.

Insights Gained from the Generated Algorithms

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tnGPS can leverage insights gained from the existing algorithms and the embedded knowledge in LLMs for novel algorithm generation

• Non-Markovian searching dynamic (Ho-1, Ho-2, Ho-3)

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Boundary mutation (Ablation 1)

Concluding Remarks

- tnGPS: a LLM-driven framework for discovering new TN-SS algorithms.
- tnGPS is designed by prompting LLMs to mimic human experts.
- LLMs provide us new ideas of solving more broad tensor problems.

Thank You