Towards Resource-friendly, Extensible and Stable Incomplete Multi-view Clustering

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Motivation

Current Incomplete multi-view clustering methods suffer from three limitations: (1) intense time and/or space overheads; (2) intractable hyper-parameters; (3) non-zero variance results.

- Not suitable for large-scale clustering tasks.
- Not scalable to other scenarios.
- Not stable data grouping effect.

Our solution

- Instead of self-expression affinity, we manage to construct prototype-sample affinity for incomplete data so as to decrease the memory requirements.
- To eliminate hyper-parameters, besides mining complementary features among views by view-wise prototypes, we also attempt to devise cross-view prototypes to capture consensus features for jointly forming worth-having clustering representation.
- To avoid the variance, we successfully unify representation learning and clustering operation, and directly optimize the discrete cluster indicators from incomplete data.

Framework

- We learn the prototypes for each incomplete view, and build prototype-sample affinity with small size, thereby decreasing the complexity. We also skip the fusion stage by directly gathering all prototype information using one aggregation matrix.
- We introduce two types of prototypes for incomplete data to jointly explore multi-view features so as to form high-quality clustering representation without the help of hyper-parameters.
- We integrate representation learning and clustering operation together, and directly optimize the discrete labels from incomplete data, which not only well preserves the original diversity of samples but also generates stable results, decreasing the fuzziness.
- We give two equivalent solutions from perspectives of feasible region partitioning and objective transformation.

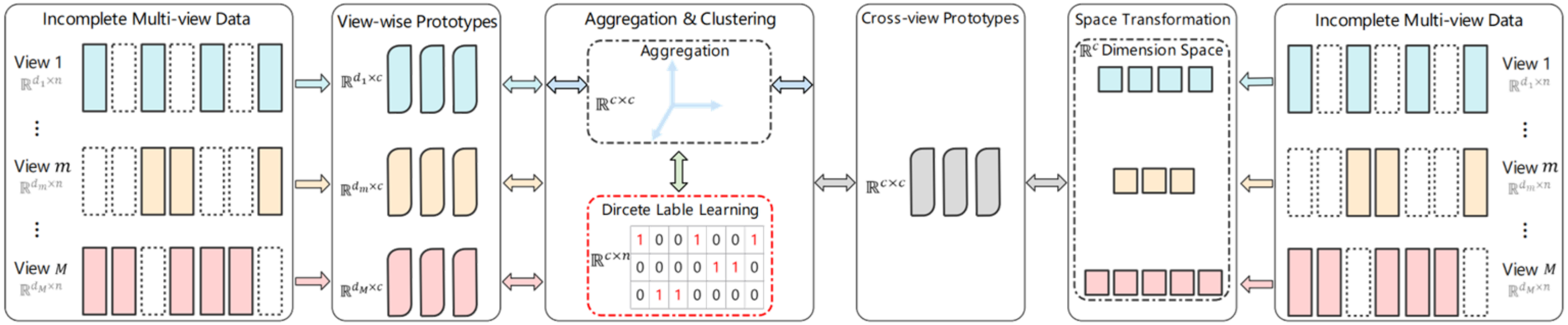


Figure 1. Framework of ToRES. It builds prototype-sample affinity with small size for incomplete views, and also does not involve the fusion stage like in Eq. (2). All prototype information is gathered via one aggregation matrix. To form desirable representation under without the help of hyper-parameters, it designs two types of prototypes, view-wise and cross-view, to jointly explore multi-view data features. To output stable results, it concurrently learns representation and performs clustering, and optimizes the cluster labels directly.

Methodology

- Objective function

$$
\begin{aligned}
\min_{\mathbf{G}_m,\mathbf{E},\mathbf{O},\mathbf{L},\mathbf{E}_m} \sum_{m=1}^{M} & \|\mathbf{G}_m\mathbf{D}_m\mathbf{W}_m - \mathbf{EOLW}_m\|_F^2 \\
& + \|\mathbf{D}_m\mathbf{W}_m - \mathbf{E}_m\mathbf{OLW}_m\|_F^2 \\
\text{s.t. } & \mathbf{G}_m\mathbf{G}_m^\top = \mathbf{I}_c, \mathbf{O}^\top\mathbf{O} = \mathbf{I}_c, \\
\mathbf{L} \in \{0,1\}^{c\times n}, & \|\mathbf{L}_{:,j}\|_1 = 1, j \in \{1,2,\ldots,n\},
\end{aligned}
$$

- Proposed algorithm ToRES

Algorithm 3 ToRES

Input: Original data $\{\mathbf{D}_m\}_{m=1}^M$, index vectors $\{w_m\}_{m=1}^M$.
Output: Discrete cluster label matrix $\mathbf{L}$.
Initialize: $\mathbf{O}$, $\mathbf{L}$, $\mathbf{G}_m$, $\mathbf{E}$, $\mathbf{E}_m$.
Construct indicator matrices $\{\mathbf{W}_m\}_{m=1}^M$.

repeat
  Update $\mathbf{O}$ by Algorithm 1 or Algorithm 2.
  Update $\mathbf{L}$ by Eq. (14).
  Update $\mathbf{G}_m$ by Eq. (16).
  Update $\mathbf{E}$ by Eq. (19).
  Update $\mathbf{E}_m$ by Eq. (21).
until convergent

- Scheme 1: Feasible region partitioning

Algorithm 1 Scheme 1 for solving Eq. (4)

Input: $\mathbf{G}_m$, $\mathbf{E}$, $\mathbf{L}$, $\mathbf{E}_m$, $\mathbf{D}_m$, $\mathbf{W}_m$.
Output: $\mathbf{O}$.
Construct $\mathbf{A}_m$, $\mathbf{B}_m$, $\mathbf{C}_m$.

for $j = 1$ to c **do**
  Update $\mathbf{O}_{:,j}$ by Eq. (8).
end for

- Scheme 2: Objective transformation

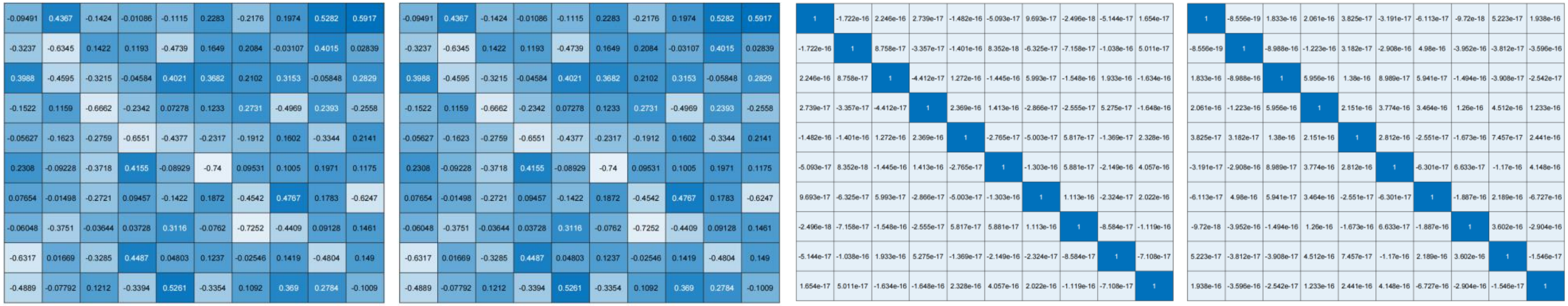
Algorithm 2 Scheme 2 for solving Eq. (4)

Input: $\mathbf{G}_m$, $\mathbf{E}$, $\mathbf{L}$, $\mathbf{E}_m$, $\mathbf{D}_m$, $\mathbf{W}_m$.
Output: $\mathbf{O}$.
Construct the function $f(\mathbf{O})$.
t=1.

repeat
  Calculate $\nabla f(\mathbf{O}_t)$.
  Perform SVD on $\nabla f(\mathbf{O}_t)$ to generate $\mathbf{U}_t$ and $\mathbf{V}_t^\top$.
  $\mathbf{O}_{t+1} = \mathbf{U}_t\mathbf{V}_t^\top$.
  t=t+1.
until $\|\mathbf{O}_t - \mathbf{O}_{t-1}\|_F / \|\mathbf{O}_{t-1}\|_F \le 1e-5$.

Experimental Performance

✓ Equivalence of two schemes



(a) $\mathbf{O}_{Algo1}$ (b) $\mathbf{O}_{Algo2}$ (c) $\mathbf{O}_{Algo1}^{\top}\mathbf{O}_{Algo1}$ (d) $\mathbf{O}_{Algo2}^{\top}\mathbf{O}_{Algo2}$

✓ Clustering Results

Dataset	Algorithm	NoHp	20%			40%			60%		
			ACC	NMI	Purity	ACC	NMI	Purity	ACC	NMI	Purity
AwAfea	IMSC-AGL	3	\	\	\	\	\	\	\	\	\
	AWP	0	8.25±0.00	9.37±0.00	9.29±0.00	8.71±0.00	9.14±0.00	10.00±0.00	8.25±0.00	8.81±0.00	9.15±0.00
	APMC	2	\	\	\	\	\	\	\	\	\
	IMG	3	\	\	\	\	\	\	\	\	\
	TMBSD	2	\	\	\	\	\	\	\	\	\
	IKMKC	2	\	\	\	\	\	\	\	\	\
	IMVTSC-MVI	3	\	\	\	\	\	\	\	\	\
	CPM-Nets	1	\	\	\	\	\	\	\	\	\
	LSIMVC	4	\	\	\	\	\	\	\	\	\
	GSRIMC	3	\	\	\	\	\	\	\	\	\
	COMPLETER	3	7.00±0.94	7.62±0.31	7.73±0.11	6.63±0.22	7.41±0.49	7.71±0.38	6.93±0.96	7.95±0.90	7.99±0.49
	TCIMC	3	\	\	\	\	\	\	\	\	\
	LRGR-IMVC	2	\	\	\	\	\	\	\	\	\
	BGIMVSC	2	\	\	\	\	\	\	\	\	\
	NGSP-CGL	3	6.46±0.19	6.00±0.31	7.24±0.26	5.90±0.19	5.22±0.31	6.72±0.24	5.70±0.17	5.06±0.27	6.65±0.19
	PIMVC	2	\	\	\	\	\	\	\	\	\
	ProImp	2	7.73±0.09	9.67±0.23	9.86±0.55	7.40±0.27	9.04±0.04	9.54±0.16	7.09±0.68	8.35±0.79	9.07±0.06
	HCP-IMSC	2	\	\	\	\	\	\	\	\	\
	APADC	2	4.92±0.00	3.05±0.76	5.82±0.09	4.72±0.21	3.02±0.53	5.91±0.43	4.52±0.58	2.81±0.46	5.54±0.45
	HCLS-CGL	2	\	\	\	\	\	\	\	\	\
	Ours-1	0	8.96±0.00	11.17±0.00	10.29±0.00	8.72±0.00	10.62±0.00	10.43±0.00	8.62±0.00	10.33±0.00	10.21±0.00
	Ours-2	0	8.96±0.00	11.17±0.00	10.29±0.00	8.72±0.00	10.62±0.00	10.43±0.00	8.62±0.00	10.33±0.00	10.21±0.00
EMNIST	All Compared Algorithms		\	\	\	\	\	\	\	\	\
	Ours-1	0	47.18±0.00	44.27±0.00	48.27±0.00	43.23±0.00	44.57±0.00	44.29±0.00	45.22±0.00	45.45±0.00	48.52±0.00
	Ours-2	0	47.18±0.00	44.27±0.00	48.27±0.00	43.23±0.00	44.57±0.00	44.29±0.00	45.22±0.00	45.45±0.00	48.52±0.00

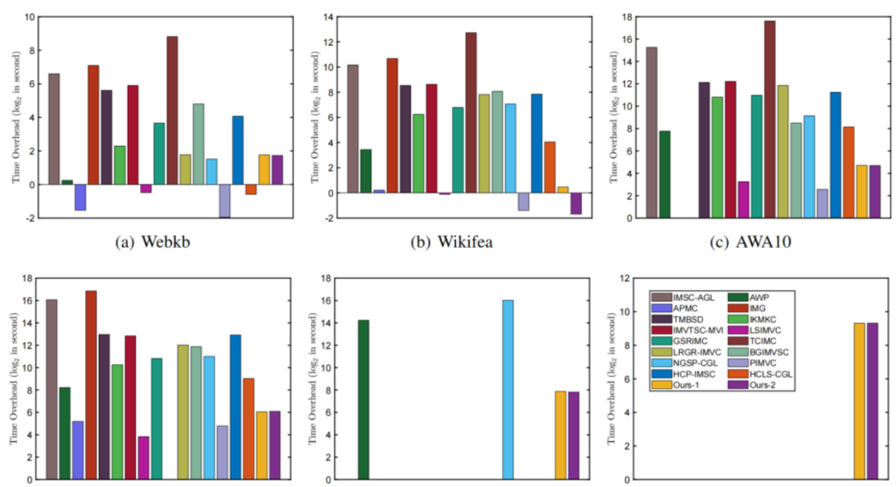
Experimental Performance

✓ Memory Overhead

Table 4. Memory Overhead Comparison (GB).

Methods	Webkb	Wikifea	AWA10	SUNRGB-D	AwAfea	EMNIST
IMSC-AGL	0.29	1.59	13.86	18.70	\	\
AWP	0.21	1.30	11.09	18.18	94.46	\
APMC	**0.09**	0.23	\	3.75	\	\
IMG	0.16	0.69	\	10.48	\	\
TMBSD	0.33	1.88	22.77	26.82	\	\
IKMKC	0.17	1.49	10.05	20.71	\	\
IMVTSC-MVI	0.27	1.57	20.78	23.04	\	\
LSIMVC	0.13	0.48	5.34	6.60	\	\
GSRIMC	0.30	2.52	29.81	33.91	\	\
TCIMC	0.49	2.92	33.18	\	\	\
LRGR-IMVC	0.20	1.09	11.34	15.92	\	\
BGIMVSC	0.17	0.92	8.88	20.13	\	\
NGSP-CGL	0.26	1.95	14.24	27.58	126.71	\
PIMVC	0.44	0.62	4.19	7.40	\	\
HCP-IMSC	0.35	1.61	19.59	24.03	\	\
HCLS-CGL	0.20	2.00	13.92	27.22	\	\
Ours	0.22	**0.20**	**2.36**	**3.68**	**11.58**	**26.34**

✓ Running Time



(a) Webkb (b) Wikifea (c) AWA10

(d) SUNRGB-D (e) AwAfea (f) EMNIST

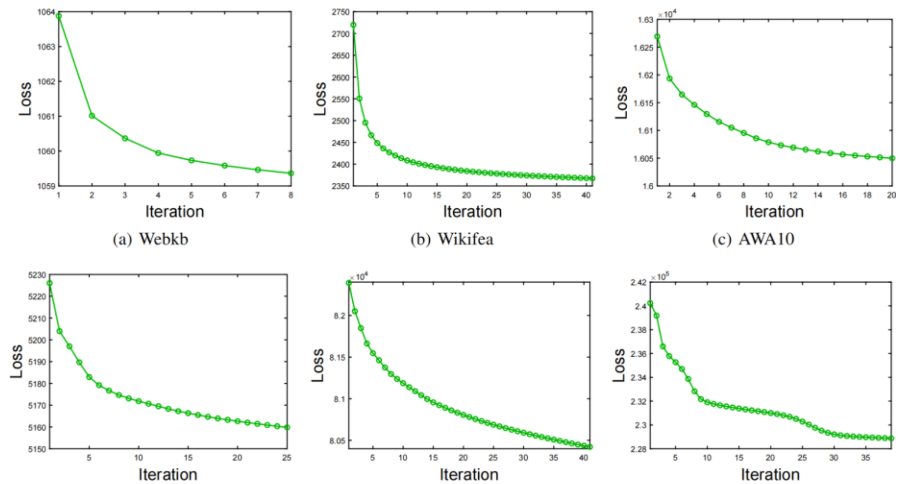
Ablation Study, Convergence

Ablation for two types of prototypes:

Table 5. Ablation Study for View-wise and Cross-view Prototypes

Dataset	Ablation Study	20%			40%			60%		
		ACC	NMI	Purity	ACC	NMI	Purity	ACC	NMI	Purity
Webkb	CVP	64.99	1.90	78.12	64.99	1.66	78.12	69.17	1.81	78.12
	VWP	51.19	3.50	78.12	51.95	2.72	78.12	56.90	2.94	78.12
	Ours	**86.20**	**32.61**	**86.20**	**85.35**	**31.95**	**85.35**	**73.83**	**4.11**	**78.12**
Wikifea	CVP	54.47	46.91	57.78	47.87	**41.79**	50.45	42.25	34.82	45.88
	VWP	52.51	46.18	57.50	48.89	40.09	51.31	**44.80**	33.04	46.19
	Ours	**56.28**	**47.90**	**58.86**	**48.95**	41.67	**51.74**	44.17	**35.63**	**47.07**
AWA10	CVP	16.51	2.84	21.53	15.63	2.59	21.04	14.55	2.00	20.55
	VWP	16.70	1.84	20.11	17.05	1.52	20.09	16.46	1.04	20.14
	Ours	**28.88**	**13.25**	**30.60**	**26.37**	**12.31**	**27.76**	**24.63**	**9.41**	**26.16**
SUNRGB-D	CVP	15.52	19.00	31.68	13.72	17.51	30.27	12.41	15.67	28.15
	VWP	15.40	5.98	16.23	13.50	4.01	14.11	12.52	3.57	13.11
	Ours	**20.93**	**25.73**	**37.16**	**19.82**	**23.87**	**35.94**	**19.75**	**20.98**	**32.31**
AwAfea	CVP	6.68	6.88	7.99	6.41	6.27	7.98	5.80	5.53	7.30
	VWP	4.23	1.44	4.34	4.38	1.42	4.63	4.20	1.36	4.32
	Ours	**8.96**	**11.17**	**10.29**	**8.72**	**10.62**	**10.43**	**8.62**	**10.33**	**10.21**
EMNIST	CVP	37.15	24.68	39.40	31.30	20.86	33.81	30.66	18.16	34.17
	VWP	13.31	2.76	13.31	16.91	5.92	16.91	16.50	5.36	16.60
	Ours	**47.18**	**44.27**	**48.27**	**43.23**	**44.57**	**44.29**	**45.22**	**45.45**	**48.52**

Convergence :



(a) Webkb (b) Wikifea (c) AWA10
(d) SUNRGB-D (e) AwAfea (f) EMNIST

Ablation for cluster indicator optimization:

Table 6. Clustering Result Comparison Between Two-step Strategy and Ours.

Dataset	Ablation Study	20%				40%				60%			
		ACC	NMI	Purity	Time	ACC	NMI	Purity	Time	ACC	NMI	Purity	Time
WebKB	Two-step	78.50±0.00	18.37±0.00	78.50±0.00	11.68	71.55±0.00	7.92±0.00	78.12±0.00	11.78	70.50±0.00	**6.78±0.00**	78.12±0.00	11.83
	Ours	**86.20±0.00**	**32.61±0.00**	**86.20±0.00**	**3.37**	**85.35±0.00**	**31.95±0.00**	**85.35±0.00**	**3.43**	**73.83±0.00**	4.11±0.00	**78.12±0.00**	**3.39**
Wiki	Two-step	52.75±1.61	45.46±0.88	56.90±1.42	53.30	48.60±2.60	38.75±1.76	**53.31±2.39**	54.72	43.74±2.19	31.15±0.93	**47.86±1.85**	46.93
	Ours	**56.28±0.00**	**47.90±0.00**	**58.86±0.00**	**1.36**	**48.95±0.00**	**41.67±0.00**	51.74±0.00	**1.36**	**44.17±0.00**	**35.63±0.00**	47.07±0.00	**1.44**
AWA10	Two-step	25.97±1.03	10.44±0.35	29.09±0.58	66.22	24.27±0.92	9.72±0.47	**28.02±0.76**	71.43	21.93±0.80	9.27±0.35	25.41±0.50	71.69
	Ours	**28.88±0.00**	**13.25±0.00**	**30.60±0.00**	**17.57**	**26.37±0.00**	**12.31±0.00**	27.76±0.00	**29.19**	**24.63±0.00**	**9.41±0.00**	**26.16±0.00**	**31.86**
SUNRGBD	Two-step	17.47±0.56	22.30±0.24	35.86±0.41	516.95	16.77±0.40	19.85±0.21	33.04±0.31	481.63	16.96±0.42	18.38±0.21	31.51±0.32	511.87
	Ours	**20.93±0.00**	**25.73±0.00**	**37.16±0.00**	**46.13**	**19.82±0.00**	**23.87±0.00**	**35.94±0.00**	**76.13**	**19.75±0.00**	**20.98±0.00**	**32.31±0.00**	**76.62**
AwAfea	Two-step	8.94±0.14	10.09±0.19	**10.39±0.19**	803.32	**8.85±0.08**	10.01±0.11	11.06±0.10	768.91	8.61±0.27	9.54±0.26	**10.50±0.28**	728.40
	Ours	**8.96±0.00**	**11.17±0.00**	10.29±0.00	**238.92**	8.72±0.00	**10.62±0.00**	**10.43±0.00**	**230.98**	**8.62±0.00**	**10.33±0.00**	10.21±0.00	**234.21**