

# Exploiting Code Symmetries for Learning Program Semantics

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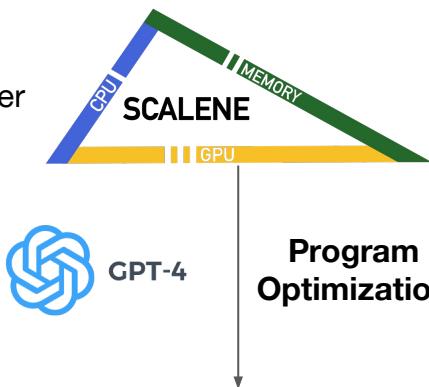


# Machine Learning Shows Promise for Analyzing Programs



## Detecting and Exploiting Vulnerabilities

Python Profiler



```
⚡ z1 = [i for i in range(0, 300000)][299999]
```

15

```
# Proposed optimization:  
# This code can be optimized by using the built-in function max()  
z1 = max(range(0, 300000)) # ~10x faster
```



Brandon Rossi  
@0xConda

I told ChatGPT to pentest a scope of IPs and it found me \$69,420 worth of bugs.



GitHub  
Copilot

### Explain Code

The screenshot shows the GitHub Copilot interface with the "EXPLAIN" section open. It displays a FizzBuzz code example in JavaScript:

```
const donut = Number(process.argv[2])  
  
if ((donut % 3 === 0) && (donut % 5 === 0))  
{  
  console.log('FizzBuzz')  
}
```

Below the code, there are buttons for "Explain code" and "Advanced". A large green "Ask Copilot" button is prominently displayed.

#### RESULT

/\* Here is the explanation for the code above:  
1. We are taking the argument from the command line and converting it to a number.  
2. We then use an if statement to check if the number is divisible by 3 and 5.  
3. If the number is divisible by 3 and 5, we output FizzBuzz.  
4. If the number is not divisible by 3 and 5, we output the number. \*/

### Translate Code

The screenshot shows the GitHub Copilot interface with the "LANGUAGE TRANSLATION" section open. It displays the same FizzBuzz code example in JavaScript, with the "Translate code into" dropdown set to "python". A large green "Ask Copilot" button is prominently displayed.

Translate code into:

python

Ask Copilot

# Limitations: Lack Understanding of Program Semantics

A code summarization based on GGNN (Li et al. 2017, Rabin et al. 2020, Bui et al. 2021)

Permute lines 7,8,9 (move line 9 before line 8).

```
1 private void FuncName(String modeStr, Matcher matcher) {  
2     boolean commaSepreated = false;  
3     for (int i = 0; i < 1 || matcher.end() < modeStr.length(); i++) {  
4         if (i > 0 && (!commaSepreated || !matcher.find())) {  
5             throw new IllegalArgumentException(modeStr);  
6         }  
7         String str = matcher.group(2);  
8         char type = str.charAt(str.length() - 1);  
9         boolean user, group, others, stickyBit; ->  
10        user = group = others = stickyBit = false;  
11    }
```

“Run”

```
1 private void FuncName(String modeStr, Matcher matcher) {  
2     boolean commaSepreated = false;  
3     for (int i = 0; i < 1 || matcher.end() < modeStr.length(); i++) {  
4         if (i > 0 && (!commaSepreated || !matcher.find())) {  
5             throw new IllegalArgumentException(modeStr);  
6         }  
7         boolean user, group, others, stickyBit;   
8         String str = matcher.group(2);  
9         char type = str.charAt(str.length() - 1);  
10        user = group = others = stickyBit = false;  
11    }
```

“Update”

## Consequences: Lacking Robustness and Generalization

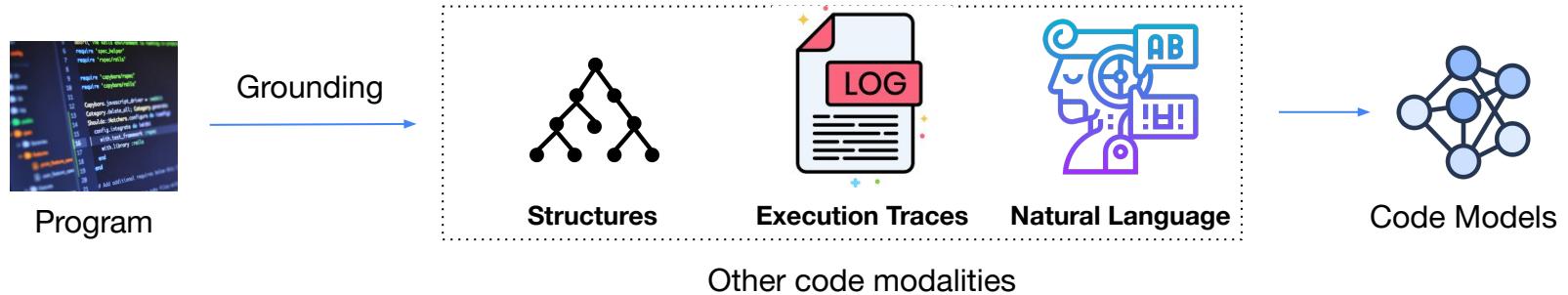
- I. Overfit to spurious textual and task-specific patterns
- II. Distribution shift: program syntax and task requirement changes

# Existing Practice of Teaching ML Code Semantics

## Common practice 1



## Common practice 2



## Limitation: Expensive and No Guarantee

- I. **Expensive:** Enumerate samples with different transformations and modalities
- II. **No guarantee:** the model can still make mistakes on seen transformed samples

# How to Specify Code Semantics and Build ML Models Respecting it?

## Operational Semantics

$$\text{PRE} \frac{}{\alpha.t \xrightarrow{\text{true}, \alpha} t}$$

$$\text{COND} \frac{t \xrightarrow{b', \alpha} t'}{\text{if } b \text{ then } t \xrightarrow{b' \wedge b, \alpha} t'} \text{fv}(b) \cap \text{bv}(\alpha) = \emptyset$$

$$\text{SUM1} \frac{t \xrightarrow{b, \alpha} t'}{t + u \xrightarrow{b, \alpha} t'}$$

$$\text{SUM2} \frac{u \xrightarrow{b, \alpha} u'}{t + u \xrightarrow{b, \alpha} u'}$$

$$\text{PAR1} \frac{t \xrightarrow{b, \alpha} t' \quad bv(\alpha) \cap \text{fv}(u) = \emptyset}{t \mid u \xrightarrow{b, \alpha} t' \mid u} \quad \text{PAR2} \frac{u \xrightarrow{b, \alpha} u' \quad bv(\alpha) \cap \text{fv}(t) = \emptyset}{t \mid u \xrightarrow{b, \alpha} t' \mid u'}$$

$$\text{COM} \frac{t \xrightarrow{b, c?x} t' \quad u \xrightarrow{b', c!e} u'}{t \mid u \xrightarrow{b \wedge b', \tau} t'[e/x] \mid u'}$$

$$\text{RES} \frac{t \xrightarrow{b, \alpha} t'}{t \setminus L \xrightarrow{b, \alpha} t' \setminus L} \text{Chan}(\alpha) \notin L$$

$$\text{REN} \frac{t \xrightarrow{b, \alpha} t'}{t[f] \xrightarrow{b, f(\alpha)} t'[f]}$$

$$\text{REC} \frac{t[\bar{e}/\bar{x}] \xrightarrow{b, \alpha} t' \quad P(\bar{x}) \Leftarrow t}{P(\bar{e}) \xrightarrow{b, \alpha} t'}$$

- **Interpreter**
- **Static Analysis**
- ...

Bound by

- execution
- interpretation
- fix-point
- syntax

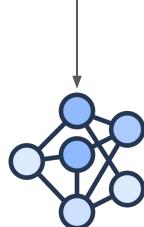
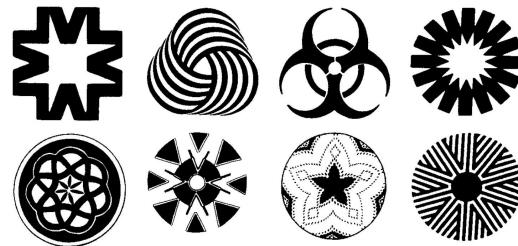


**ML Models**  
e.g., LLMs

# A New Group-Theoretic Framework to Formalize Learning Code Semantics

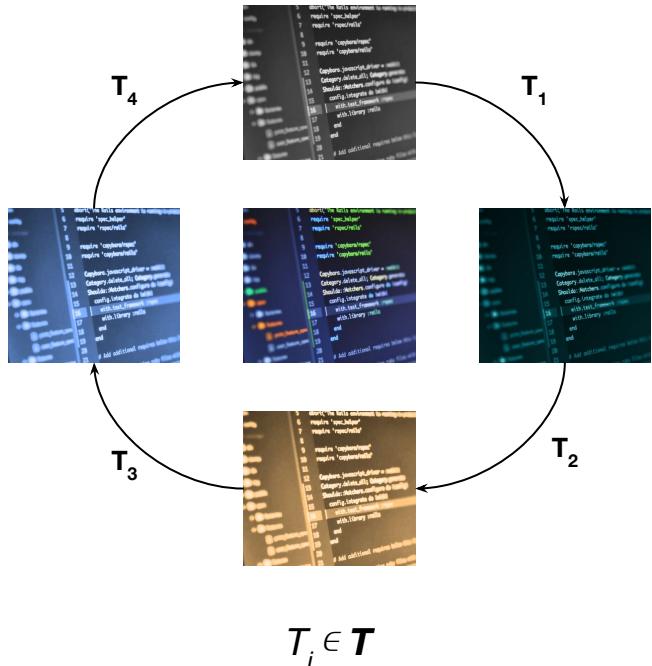


## Symmetry Groups



## Code Models

# Code Symmetry Group



$T$  is a **Group** iff

- **Associative:**  $T_i \cdot (T_j \cdot T_k) = (T_i \cdot T_j) \cdot T_k$
- **Inverse:**  $\forall T_i \in T, \exists T_j \in T, T_i \cdot T_j = \mathbf{1}$
- **Identity:**  $\forall \mathbf{1} \in T, \mathbf{1} \cdot T_i = T_i \cdot \mathbf{1}$
- **Closure:**  $\forall T_i, T_j \in T, (T_i \cdot T_j) \in T$

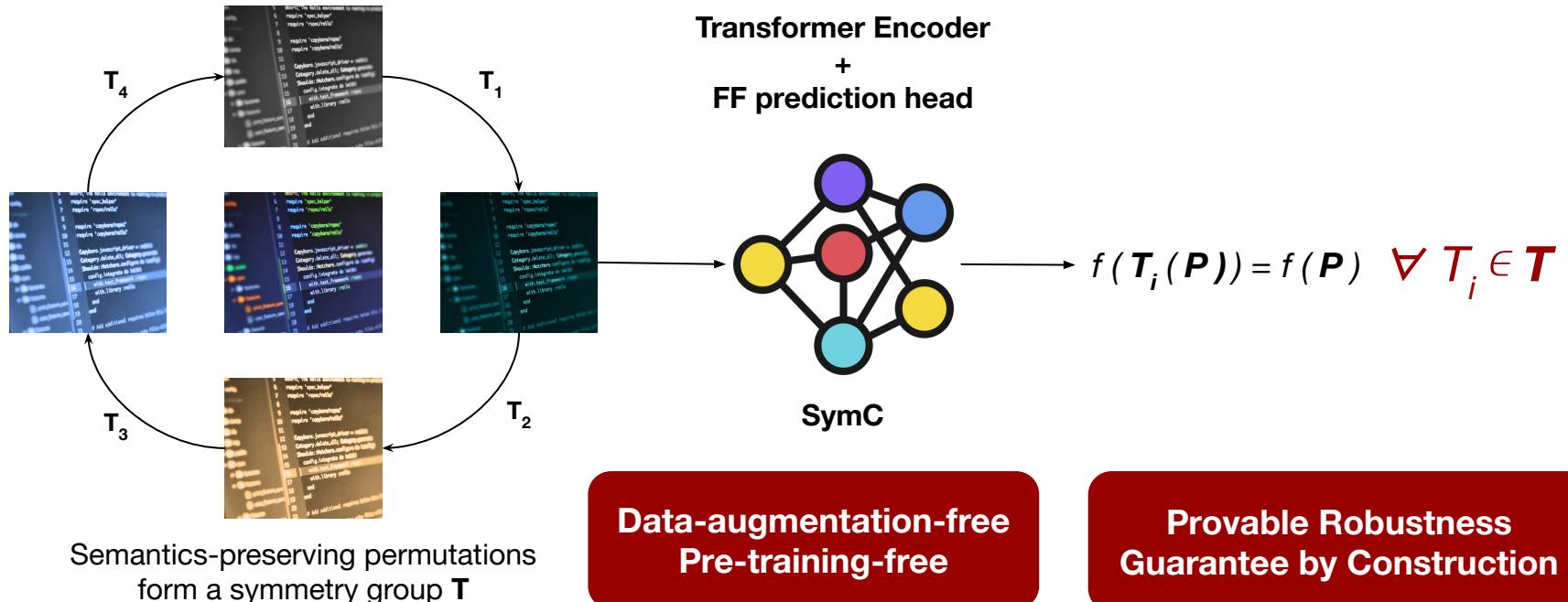
E.g., Permutation Group  $T$

$$\begin{array}{lll} 1: \mathbf{x=5} & \xrightarrow{\text{Instruction}} & 1: \mathbf{y=6} \\ 2: \mathbf{y=6} & & 2: \mathbf{x=5} \\ 3: \mathbf{z=x+y} & \xrightarrow{\text{Reordering}} & 3: \mathbf{z=x+y} \\ & & \dots \end{array}$$

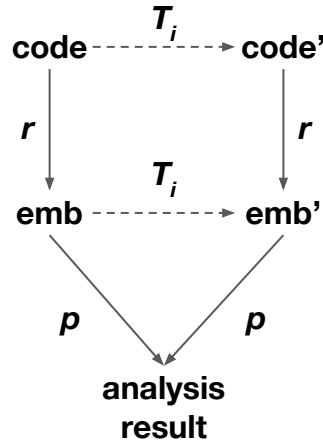
## Key Benefit

- Compositionality
  - Prove by Induction
- Amenable to Neural Architecture Design
  - AI4Science
  - Geometric DL

# SymC: Permutation-Group-Invariant Transformer

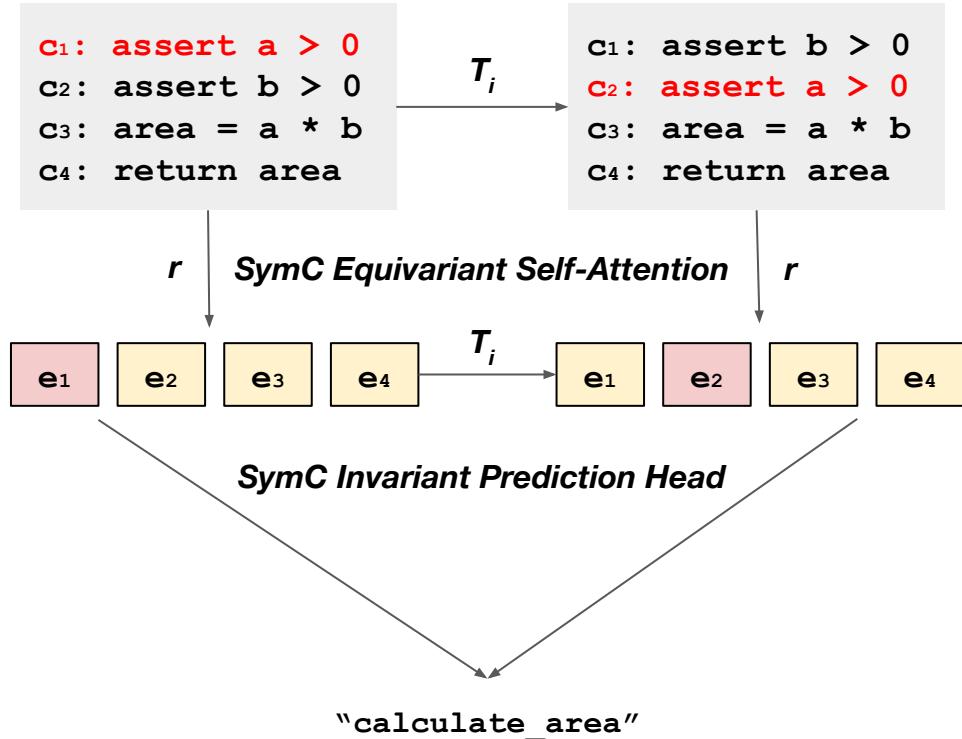


# Group Invariance and Equivariance in SymC



$r$ : representation learning module,  
e.g., self-attention layers

$p$ : predictive learning module,  
e.g., fully-connected prediction head

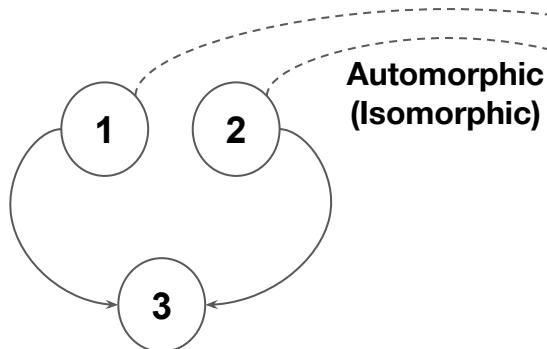


# Aut(PDG): The Automorphism Group of Program Dependence Graph

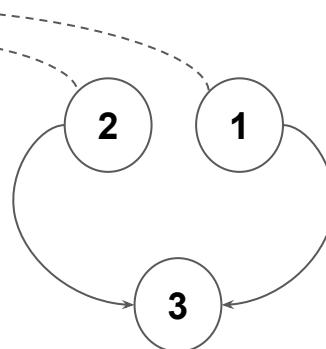
```
1 int a = 1;  
2 int b = 2;  
3 int c = a + b;
```

```
1 int b = 2;  
2 int a = 1;  
3 int c = a + b;
```

Program Dependence Graph



Program Dependence Graph



Automorphic  
(Isomorphic)

- **Data Dependency**

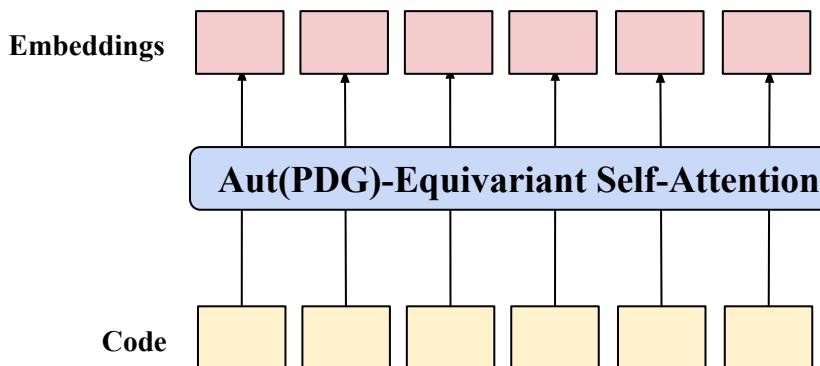
- **RAW**: Read after Write
- **WAR**: Write after Read
- **WAW**: Write after Write

- **Control Dependency**

Distance Matrix  $d$ :  
Relative distance to the lowest common ancestor (LCA)

0	inf	0
inf	0	0
1	1	0

# Aut(PDG)-Equivariant Self-Attention



$$\text{Attn}(x) = V \cdot (K^T \cdot Q)$$

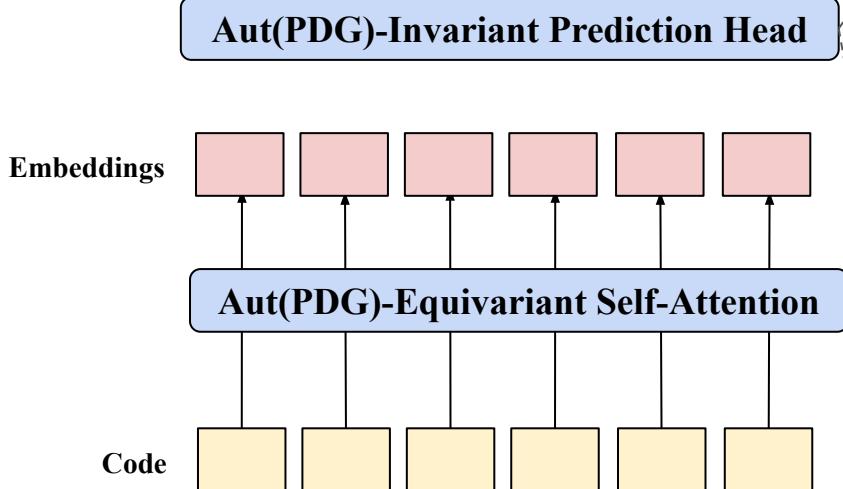
$$\begin{aligned}\text{Attn}(xp) &= Vp \cdot ((Kp)^T \cdot Qp) \\ &= V\cancel{p}p^T(K^T \cdot Q)p = V \cdot (K^T \cdot Q)p = \text{Attn}(x)p\end{aligned}$$

How to make it equivariant **only to** Aut(PDG), the semantics-preserving permutations?

$$G\text{-Attn}(x) = V \cdot (K^T \cdot Q + \cancel{d}_{\text{PDG}})$$

$$\begin{aligned}G\text{-Attn}(xp) &= Vp \cdot ((Kp)^T \cdot Qp + \cancel{d}_{\text{PDG}}) \\ &= V\cancel{p}p^T(K^T \cdot Q)p + Vp\cancel{d}_{\text{PDG}} \\ &= V \cdot (K^T \cdot Q + \cancel{d}_{\text{PDG}})p = G\text{-Attn}(x)p\end{aligned}$$

# Aut(PDG)-Invariant Prediction Head



**Pooling Based, e.g., Mean, Sum, etc.**

Sum( )

Avg( )

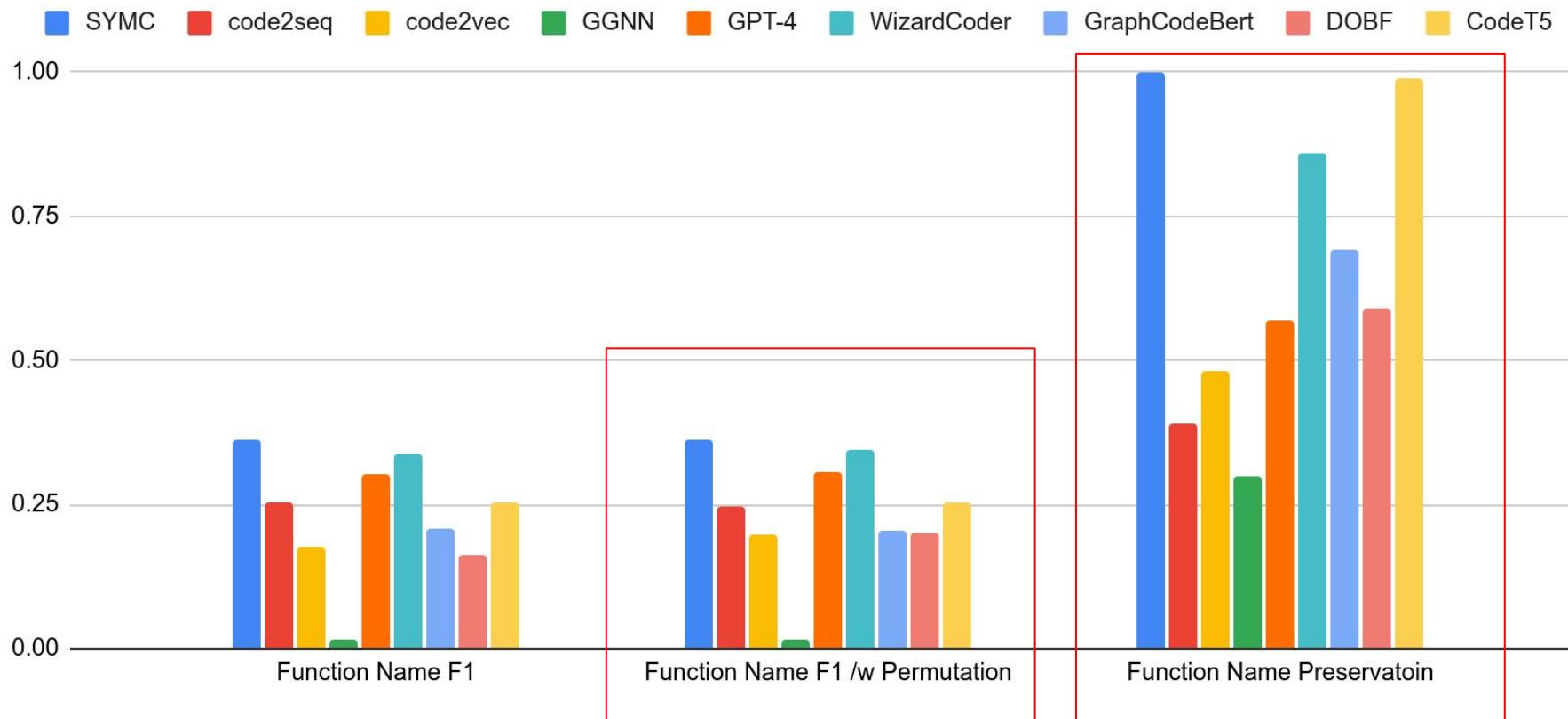
**Token-level Based, e.g., [CLS]**

$$GA = V \cdot (K^T \cdot \mathbf{q} + d_{[:,q]})$$

a single  
query token

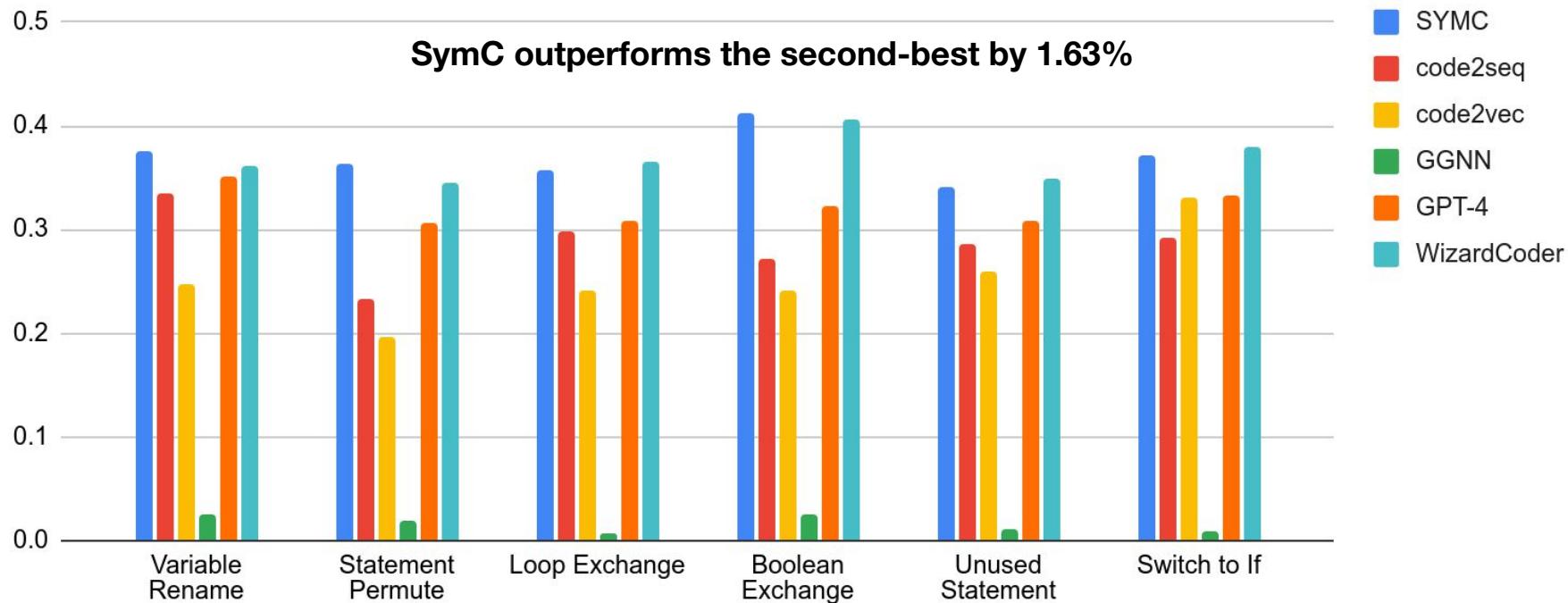
column of  
query token  
to all others

## Results: Robustness Against Permutation



# Results: Generalization against Unseen Transformations

The performance (F1) of SYMC and baselines against different unseen permuted code transformations.



# Results: Invariance and Generalization

■ SymC ■ Palmtree ■ PalmTree-O ■ PalmTree-N

