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XI'AN JIAOTONG UNIVERSITY

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Institute of
Artificial Intelligence
and Robotics

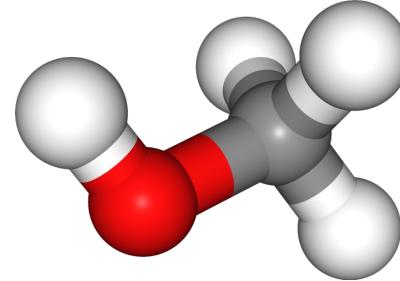


Self-Consistency Training for Density-Functional-Theory Hamiltonian Prediction

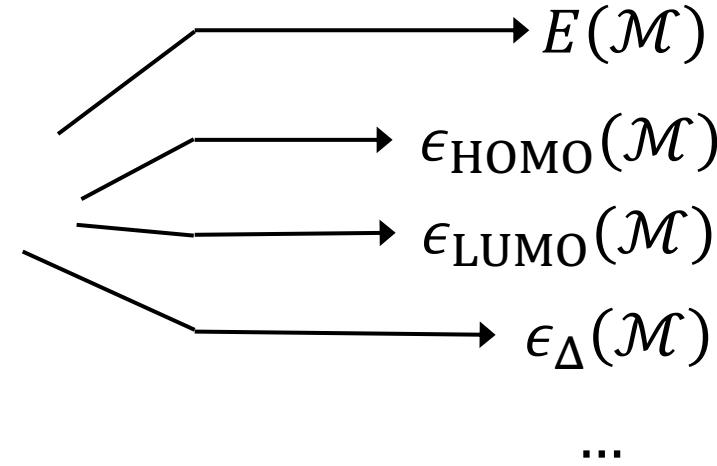
He Zhang^{1,2}, Chang Liu²#, Zun Wang², Xinran Wei², Siyuan Liu²,
Nanning Zheng¹#, Bin Shao², Tie-Yan Liu²

¹Xi'an Jiaotong University ²Microsoft AI for Science #Corresponding authors

Hamiltonian Prediction

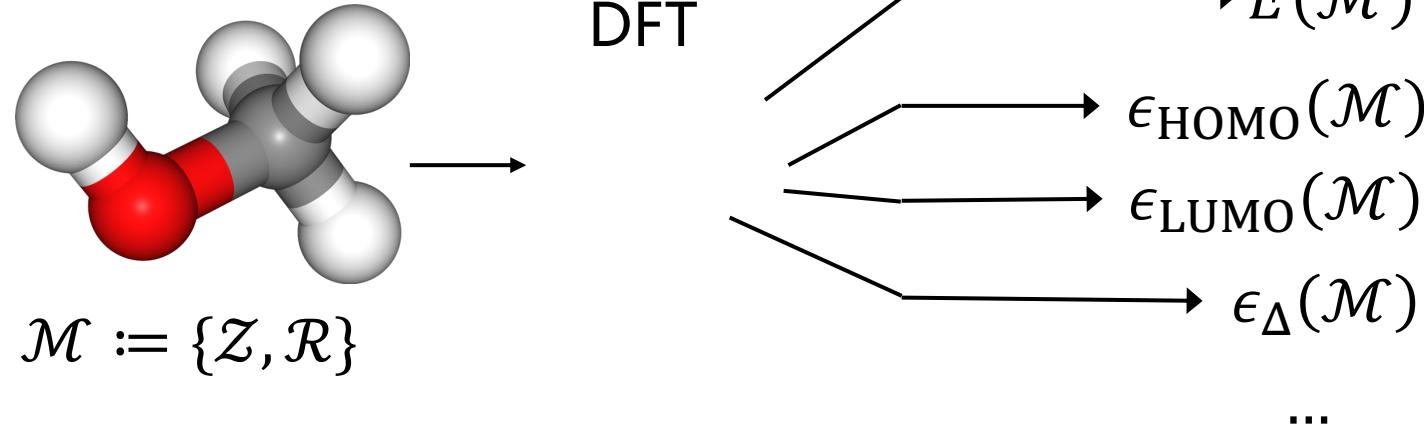


$$\mathcal{M} := \{\mathcal{Z}, \mathcal{R}\}$$



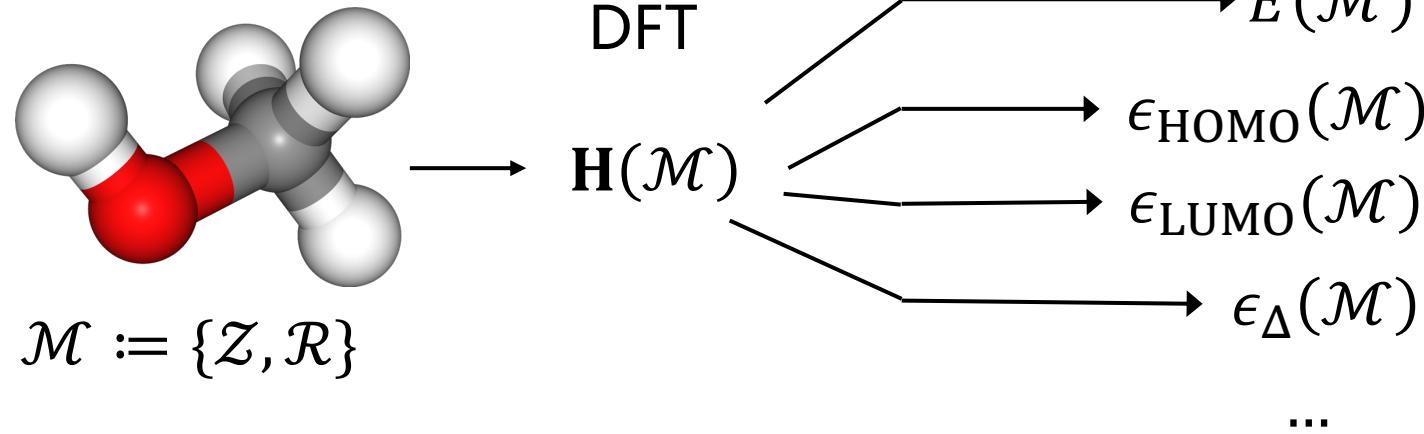
- Molecular properties: interaction among electrons and atomic nuclei

Hamiltonian Prediction



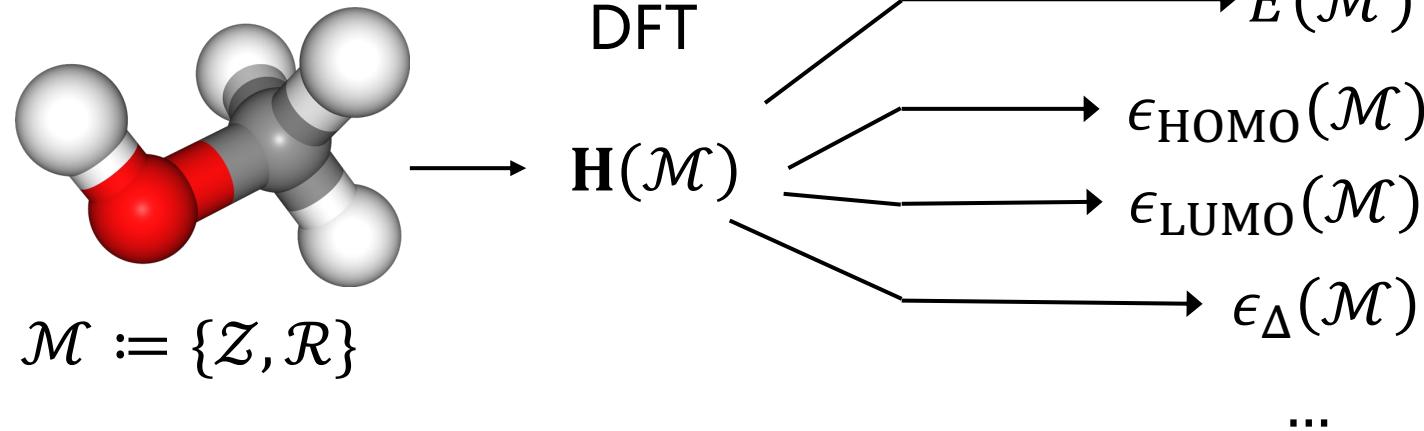
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Hamiltonian Prediction



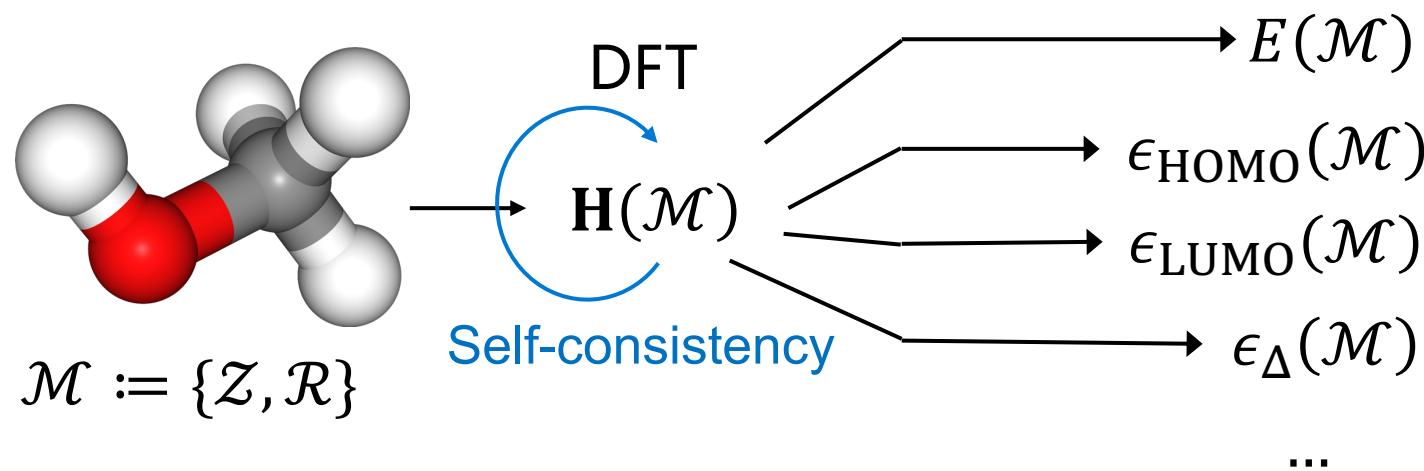
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Hamiltonian Prediction



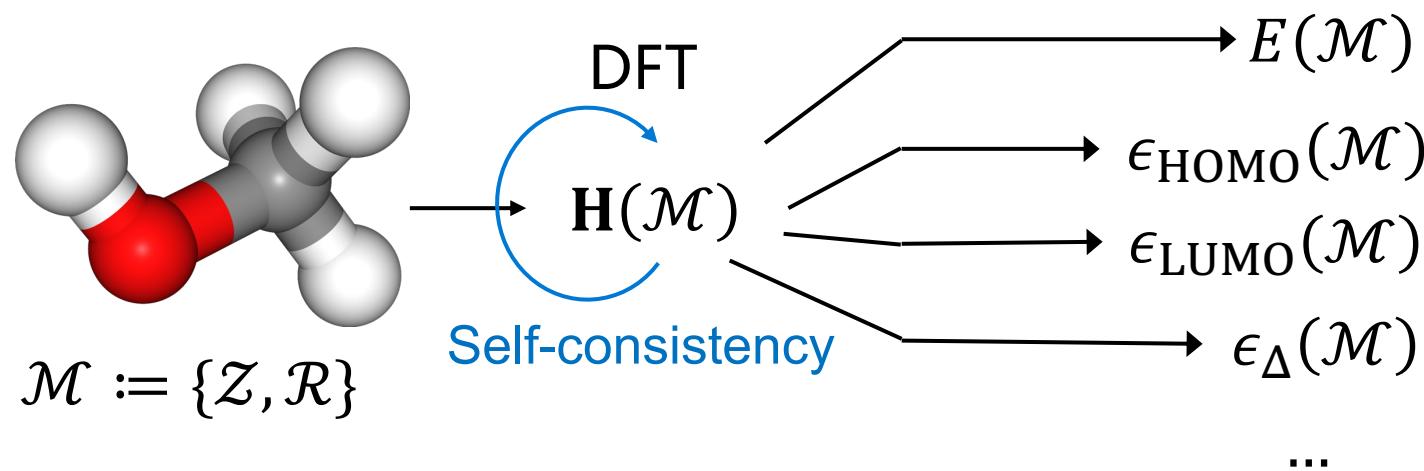
- Molecular properties: interaction among electrons and atomic nuclei
- DFT: solve electronic structure hence properties
- Hamiltonian: raw DFT solution, derive all properties
- Hamiltonian prediction:
“unified” predictor, provide all properties that DFT can

Hamiltonian Prediction



- Hamiltonian prediction has a **self-consistency** principle: **Training without label!**
 - Distinction from common property prediction: data-free training / self-improvement
 - Compensating **data scarcity** with scientific laws

Hamiltonian Prediction



- Hamiltonian prediction has a **self-consistency** principle: **Training without label!**
 - **Distinction** from common property prediction: data-free training / self-improvement
 - Compensating **data scarcity** with **scientific laws**
- Unique benefits:
 - Exact **generalization** to arbitrary workload beyond labeled data
 - Amortization of DFT calculation: **more efficient** than running DFT to generate labels

Background: DFT Formulation



- Describe N -electron state by orbitals $\{\phi_i(\mathbf{r})\}_{i=1}^N \rightarrow$ coefficients \mathbf{C} under a basis set
- Solve for the electron state \mathbf{C} of molecular structure \mathcal{M} by minimizing:

$$E_{\mathcal{M}}(\mathbf{C}), \text{ s.t. } \mathbf{C}^T \mathbf{S}_{\mathcal{M}} \mathbf{C} = \mathbf{I}.$$

- Solve the optimization problem:

$$\underbrace{\mathbf{H}_{\mathcal{M}}(\mathbf{C}) \mathbf{C} = \mathbf{S}_{\mathcal{M}} \mathbf{C} \epsilon}_{:= \frac{1}{2} \nabla_{\mathbf{C}} E_{\mathcal{M}}(\cdot)}$$

Kohn-sham equation

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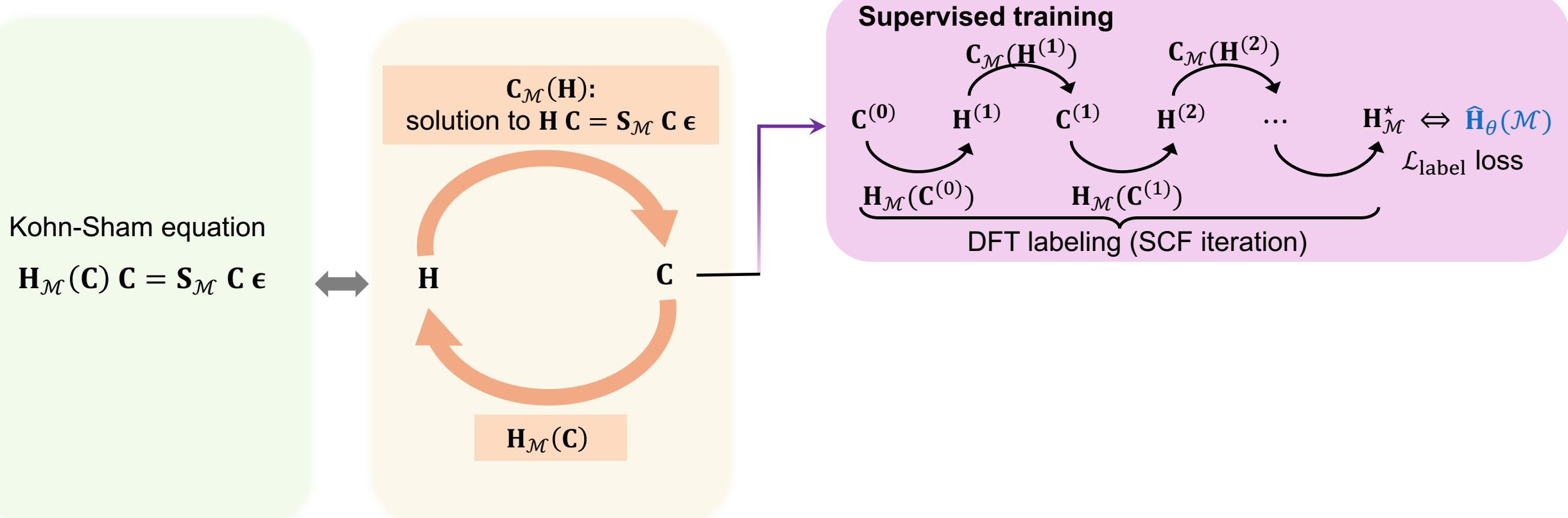


Self-Consistent Field (SCF) iteration:

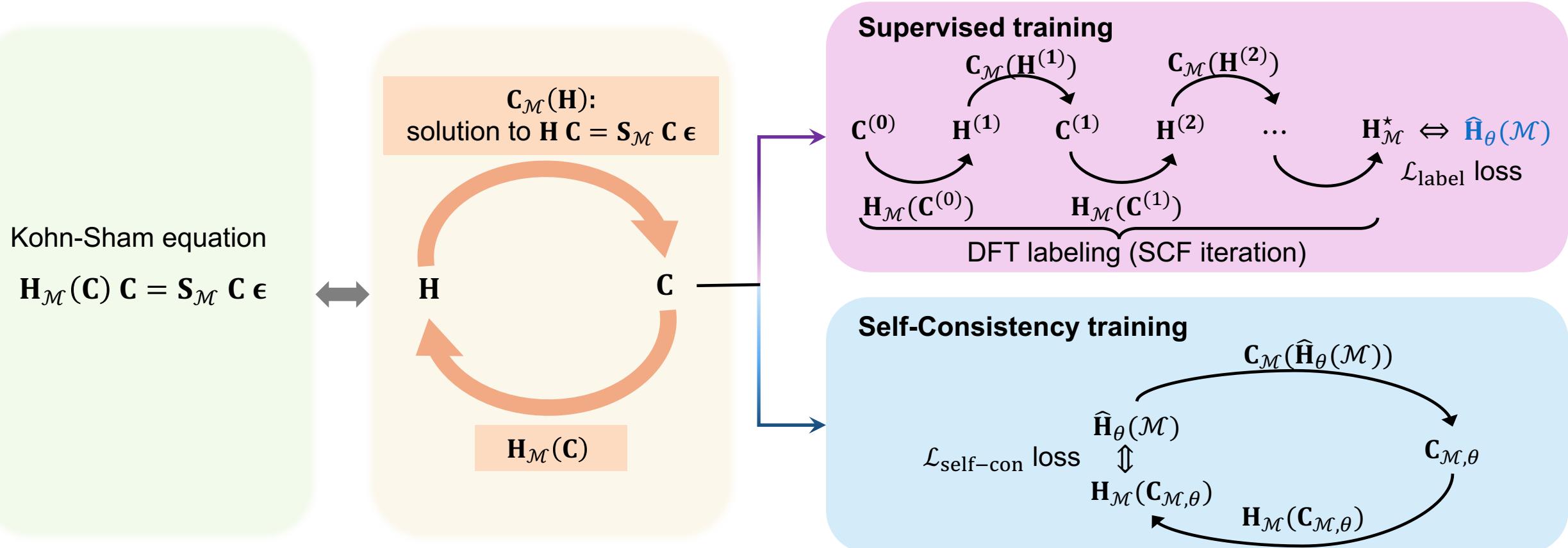
$$\mathbf{C}^{(k-1)} \rightarrow \mathbf{H}^{(k)} = \mathbf{H}_{\mathcal{M}}(\mathbf{C}^{(k-1)}) \rightarrow \mathbf{C}^{(k)} := \mathbf{C}_{\mathcal{M}}(\mathbf{H}^{(k)}) \text{ which solves } \mathbf{H}^{(k)} \mathbf{C} = \mathbf{S}_{\mathcal{M}} \mathbf{C} \epsilon$$

$\rightarrow \mathbf{H}_{\mathcal{M}}^*$ after convergence

DFT Calculation \rightarrow Self-Consistency Training



DFT Calculation \rightarrow Self-Consistency Training



$$L_{\text{sc}}(\theta; \mathcal{D}) = \frac{1}{|\mathcal{D}|} \sum_{\mathcal{M} \sim \mathcal{D}} \left\| \hat{\mathbf{H}}_{\theta}(\mathcal{M}) - \mathbf{H}_{\mathcal{M}} \left(\mathbf{C}_{\mathcal{M}} \left(\hat{\mathbf{H}}_{\theta}(\mathcal{M}) \right) \right) \right\|_{\text{F}}^2$$

Self-Consistency Training



- Self-consistency loss

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- Not just a regularization: it determines the DFT solution (label).
- Minimizing the gap unnecessarily drives $\widehat{\mathbf{H}}_{\theta}(\mathcal{M})$ towards $\mathbf{H}_{\mathcal{M}} \left(\mathbf{c}_{\mathcal{M}} \left(\widehat{\mathbf{H}}_{\theta}(\mathcal{M}) \right) \right)$.
 - The latter may even be farther from the solution, in which case both are driven to the solution.
 - Should not apply stop-gradient to the latter.

Self-Consistency Training



- Hamiltonian prediction
 - Roto-translational/SE(3) equivariance
 - QHNet [Yu'23]: an SE(3)-equivariant GNN balance efficiency and accuracy

Self-Consistency Training



- Hamiltonian prediction
 - Roto-translational/SE(3) equivariance
 - QHNet [Yu'23]: an SE(3)-equivariant GNN balance efficiency and accuracy
- Hamiltonian reconstruction
$$L_{sc}(\theta; \mathcal{D}) = \frac{1}{|\mathcal{D}|} \sum_{\mathcal{M} \sim \mathcal{D}} \left\| \widehat{\mathbf{H}}_\theta(\mathcal{M}) - \mathbf{H}_\mathcal{M} \left(\mathbf{C}_\mathcal{M} \left(\widehat{\mathbf{H}}_\theta(\mathcal{M}) \right) \right) \right\|_{\text{F}}^2$$
 - Numerically stable implementation of differentiation through eigensolver $\mathbf{C}_\mathcal{M}(\mathbf{H})$.
 - GPU implementation of Hamiltonian construction $\mathbf{H}_\mathcal{M}(\mathbf{C})$

Unique Benefits



- Generalization beyond labeled data: $\mathcal{L}_{\text{label}}(\theta; \underbrace{\mathcal{D}^{(1)}}_{\text{limited labeled dataset}}) + \lambda \mathcal{L}_{\text{self-con}}(\theta; \underbrace{\mathcal{D}^{(2)})}_{\text{unlimited unlabeled dataset}}$

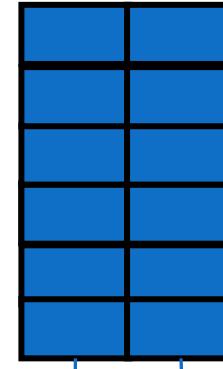
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- Amortization effect: efficiency over DFT labeling

Cost of one iteration:



DFT calculation:



supervision

Training molecules: $\mathcal{M}^{(1)} \mathcal{M}^{(2)} \mathcal{M}^{(3)} \mathcal{M}^{(4)} \mathcal{M}^{(5)} \mathcal{M}^{(6)} \mathcal{M}^{(7)} \mathcal{M}^{(8)} \mathcal{M}^{(9)} \mathcal{M}^{(10)} \mathcal{M}^{(11)} \mathcal{M}^{(12)}$

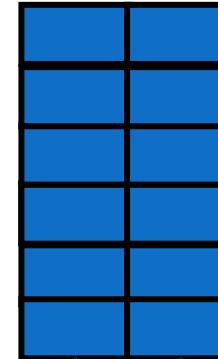
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Self-consistency training:



supervision

Results: Generalization beyond Labeled Data

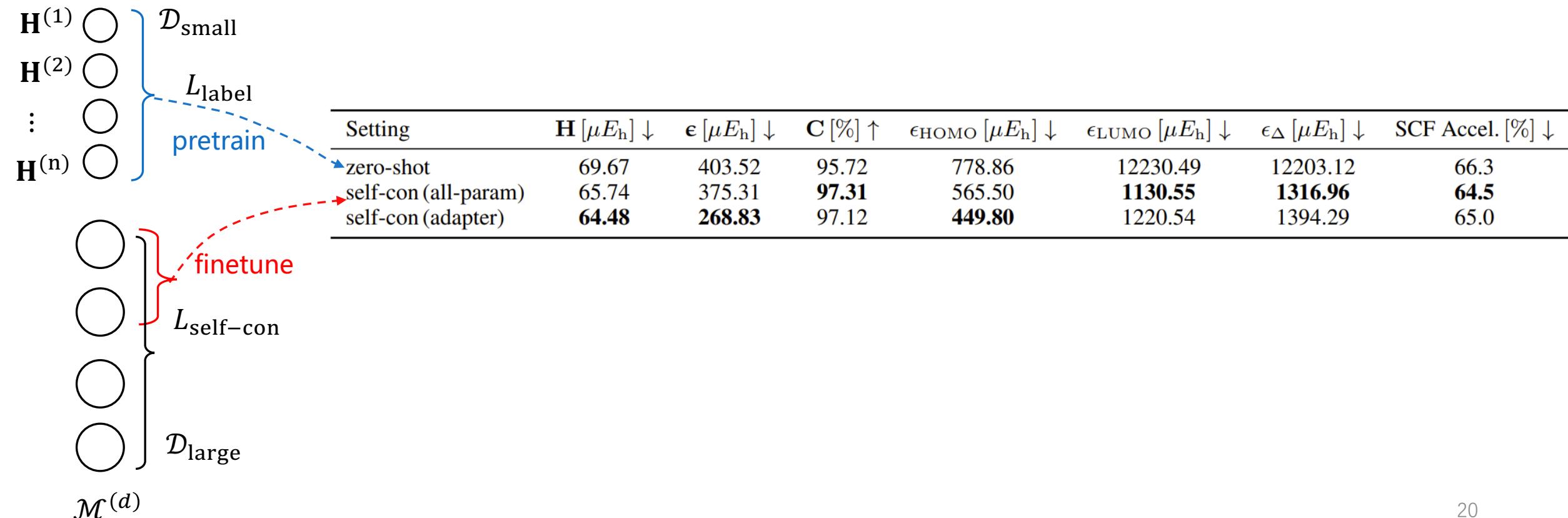


- Data-scarce scenario (MD17): $|\overline{\mathcal{D}}^{(1)}| = 100, |\mathcal{D}^{(2)}| = 24,900$

$H^{(1)}$	$\overline{\mathcal{D}}^{(1)}$		Direct prediction			Derived molecular properties			As DFT init		
$H^{(n)}$	L_{label}		Molecule	Setting	$H [\mu E_h] \downarrow$	$\epsilon [\mu E_h] \downarrow$	$C [\%] \uparrow$	$\epsilon_{HOMO} [\mu E_h] \downarrow$	$\epsilon_{LUMO} [\mu E_h] \downarrow$	$\epsilon_{\Delta} [\mu E_h] \downarrow$	SCF Accel. [%] \downarrow
	Ethanol			label	160.36	712.54	99.44	911.64	6800.84	6643.11	68.3
				label + self-con	75.65	285.49	99.94	336.97	1203.60	1224.86	61.5
	Malondi-aldehyde		label		101.19	456.75	99.09	471.92	1093.22	1115.94	69.1
			label + self-con		86.60	280.39	99.67	274.45	279.14	324.37	62.1
	Uracil		label		88.26	1079.51	95.83	1217.17	12496.1	11850.56	65.8
			label + self-con		63.82	315.40	99.58	359.98	369.67	388.30	54.5

Legend: $\overline{\mathcal{D}}^{(1)}$ (blue bracket), L_{label} (blue bracket), $L_{self-con}$ (red dashed bracket), $\mathcal{D}^{(2)}$ (red bracket), $\mathcal{M}^{(d)}$ (red bracket).

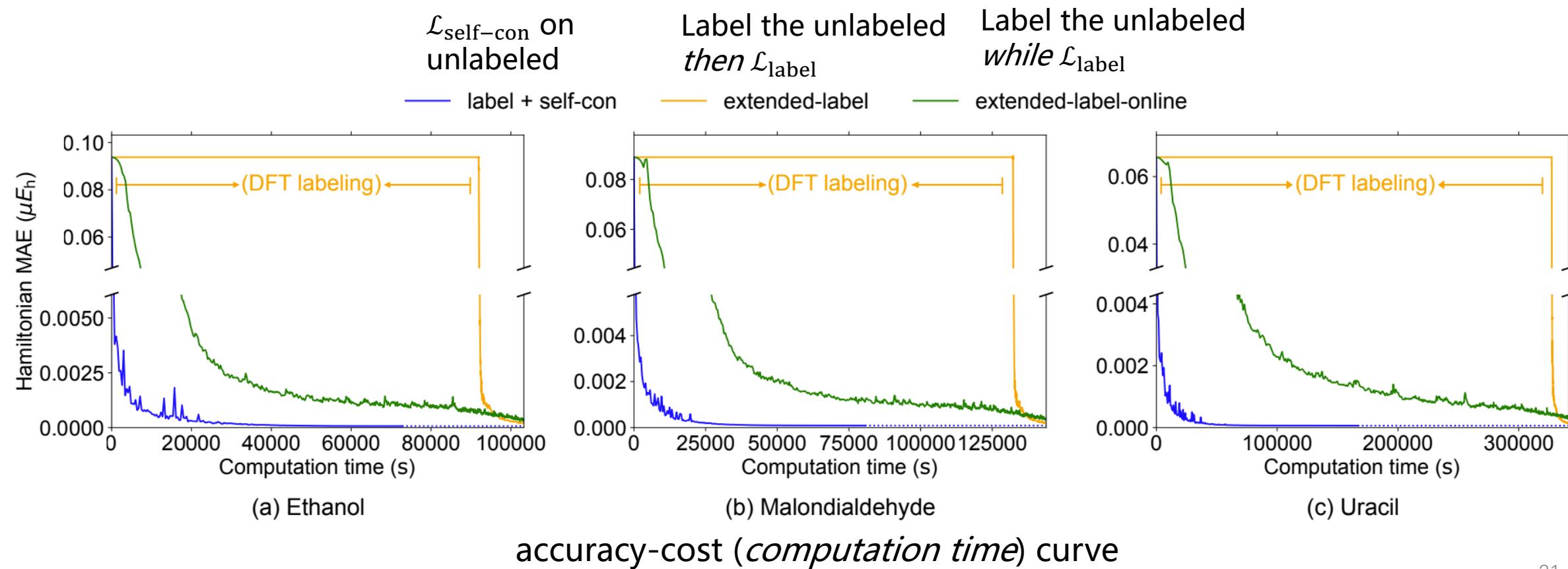
- Out-of-distribution (OOD) scenario (QH9)
labeled small molecules + **finetune on unlabeled large molecules** → test on large molecules



Results: Training Efficiency by Amortization



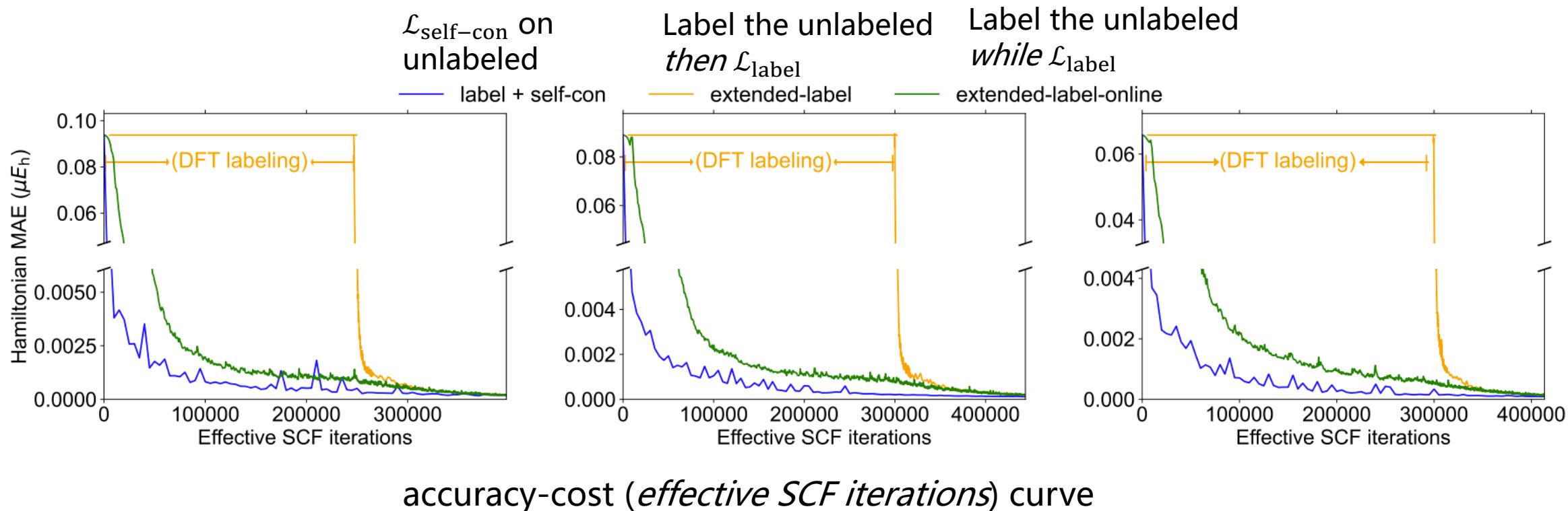
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Results: Amortization of DFT

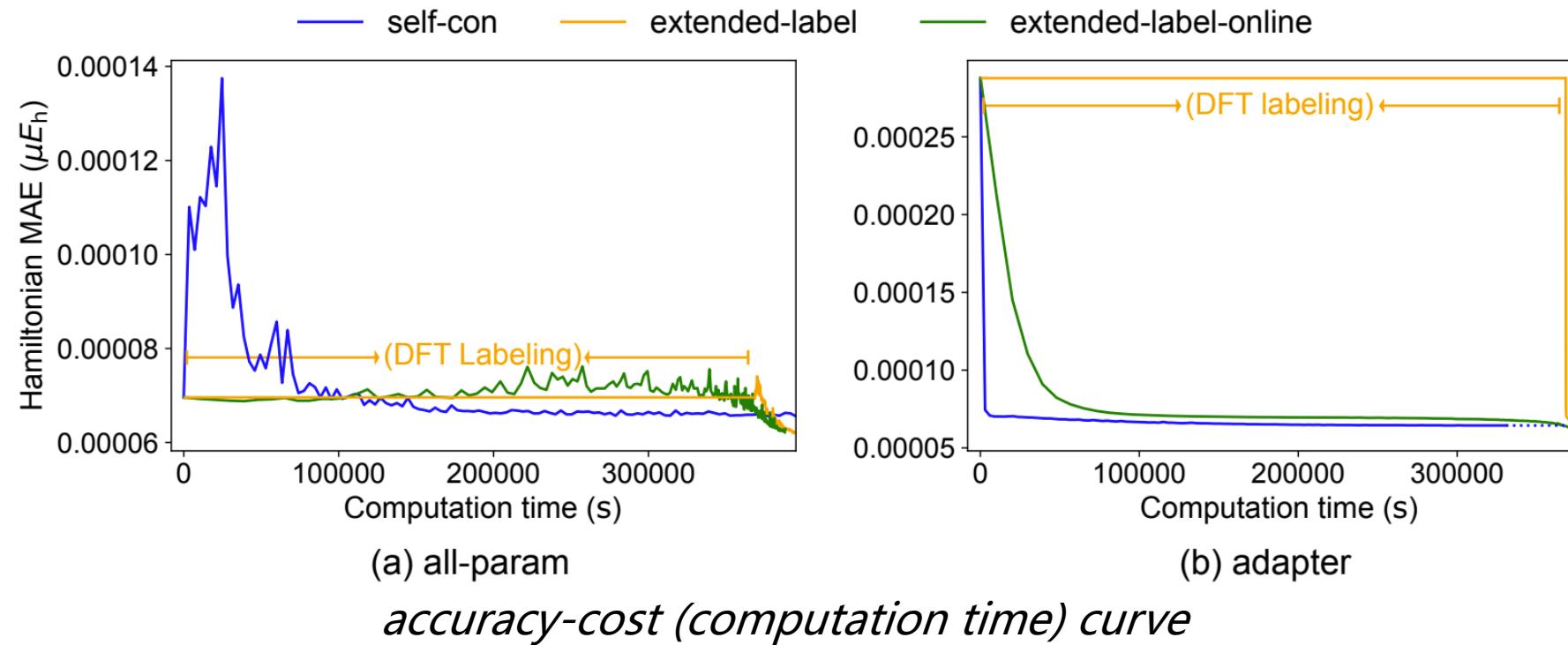


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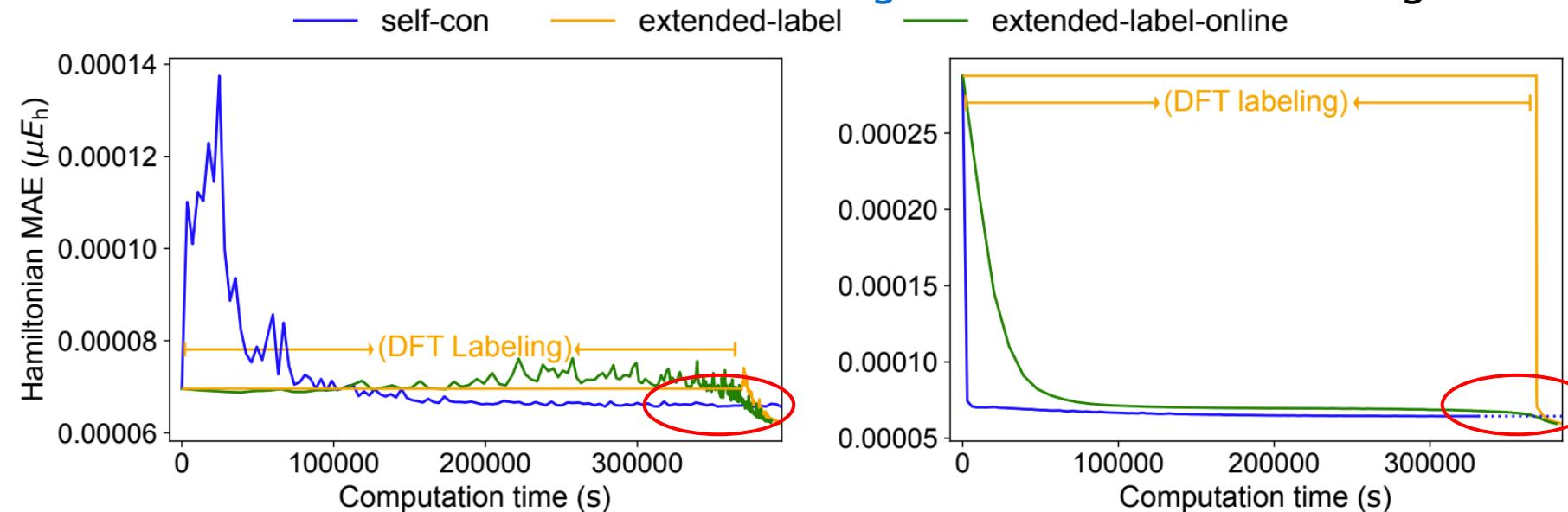
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Results: Amortization of DFT

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Final performance: self-consistency training gives better derived molecular properties!

FT mode	Setting	$H [\mu E_h] \downarrow$	$\epsilon [\mu E_h] \downarrow$	$C [\%] \uparrow$	$\epsilon_{HOMO} [\mu E_h] \downarrow$	$\epsilon_{LUMO} [\mu E_h] \downarrow$	$\epsilon_{\Delta} [\mu E_h] \downarrow$	SCF Accel. [%] \downarrow
all-param	extended-label	62.13	365.66	96.89	577.46	5962.16	6137.66	65.0
	self-con	65.74	375.31	97.31	565.50	1130.55	1316.96	64.5
adapter	extended-label	59.67	330.05	96.63	541.92	6372.12	6445.33	65.2
	self-con	64.48	268.83	97.12	449.80	1220.54	1394.29	65.0

Results: Amortization of DFT

- Direct acceleration over DFT calculation
 - Self-consistency training time vs. DFT computation time to reach the same level of electronic energy accuracy

Molecule	criterion [μE_h]	$t_{\text{self-con}}$ [s]	t_{DFT} [s]
Ethanol	31.0	4.50×10^4	6.40×10^4
Malondialdehyde	88.9	4.81×10^4	1.05×10^5
Uracil	177.2	1.23×10^5	2.15×10^5

Results: Extending Applicable Scale of Hamiltonian Prediction

- Labeled QM9 (≤ 31 atoms) + Finetune on unlabeled larger molecules \rightarrow test on MD22 (ALA3: 42 atoms, DHA: 56 atoms)
 - vs. zero-shot generalization
 - vs. SOTA end-to-end property predictors

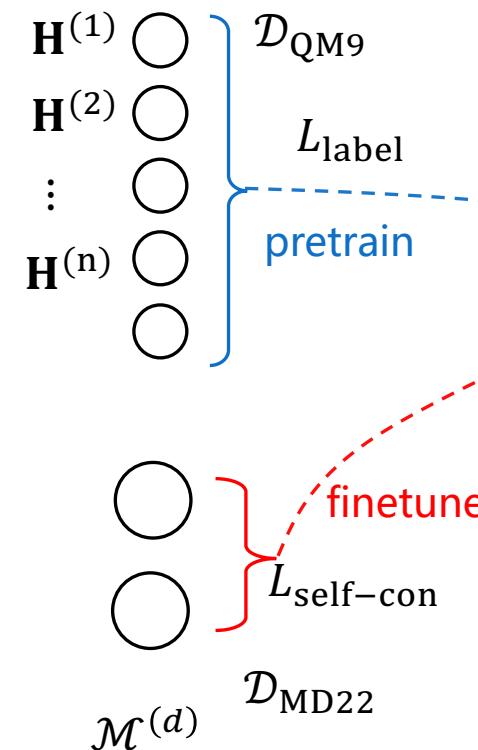


Table 5. Generalization results on large-scale molecules. All metrics are calculated on MD22 test structures.

Molecule	Setting	$H [\mu E_h] \downarrow$	$\epsilon [\mu E_h] \downarrow$	$C [\%] \uparrow$	$\epsilon_{HOMO} [\mu E_h] \downarrow$	$\epsilon_{LUMO} [\mu E_h] \downarrow$	$\epsilon_{\Delta} [\mu E_h] \downarrow$	SCF Accel. [%] \downarrow
ALA3	zero-shot	237.71	6.54×10^3	52.24	6.90×10^3	9.51×10^4	9.79×10^4	84.6
	self-con	52.49	1.22×10^3	94.46	2.07×10^3	3.76×10^3	2.69×10^3	64.7
	e2e (ET)	N/A	N/A	N/A	1.74×10^5	7.72×10^3	2.38×10^5	N/A
	e2e (Equiformer)	N/A	N/A	N/A	2.38×10^5	1.16×10^4	2.27×10^5	N/A
DHA	zero-shot	397.87	1.84×10^4	20.15	1.11×10^4	1.90×10^5	1.85×10^5	170.8
	self-con	56.12	1.81×10^3	83.51	1.99×10^3	4.01×10^3	2.34×10^3	67.0
	e2e (ET)	N/A	N/A	N/A	2.92×10^5	2.58×10^4	3.39×10^5	N/A
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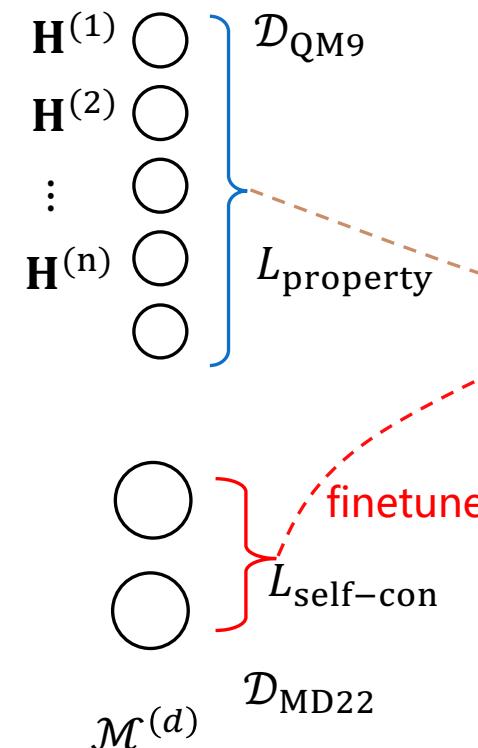


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and Robotics



Thank you

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