ICML
International Conference On Machine Learning

MOHAMED BIN ZAYED UNIVERSITY OF ARTIFICIAL INTELLIGENCE

On the Recoverability of Causal Relations from Temporally Aggregated I.I.D. Data

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Motivation

What is temporal aggregation? Summation/Averaging/…

$\sum_{t=1}^{k} x_t$, $\frac{1}{k}\sum_{t=1}^{k} x_t$, $\frac{1}{\sqrt{k}}\sum_{t=1}^{k} x_t$ …

Temporal aggregation is so common in real-world observation: Daily/Weekly/Monthly stock prices, hourly/daily/monthly temperature, GDP, electricity consumption…

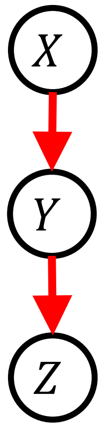
Motivation

Temporal aggregation in non-temporal causal discovery

- Instantaneous causation:

“temporal aggregation is a realistic, plausible, and well-known reason for observing apparent instantaneous causation”

---Granger, C. W. (1988). Some recent development in a concept of causality. *Journal of econometrics*



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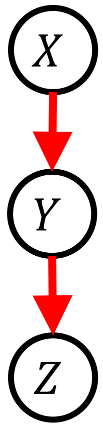
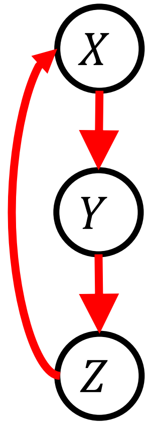


- Cyclic causation:

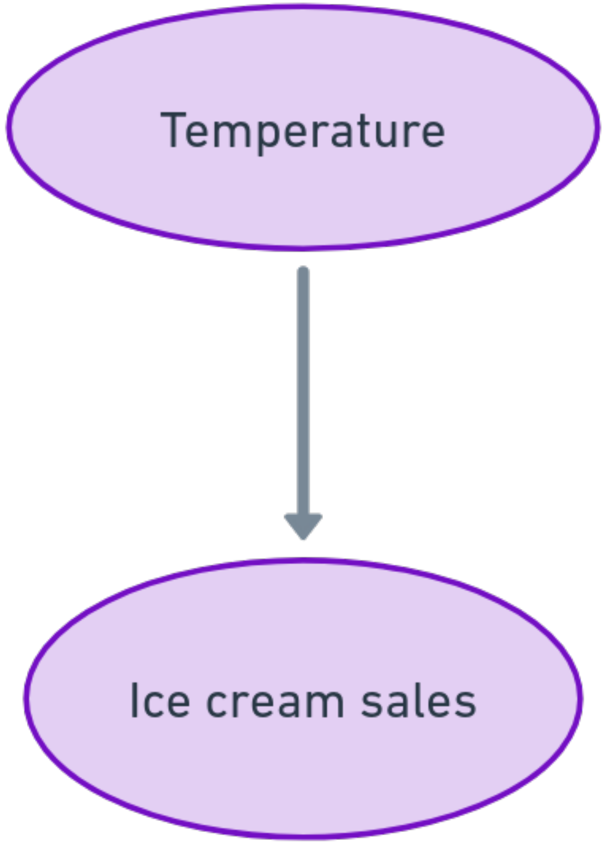
"The observed variable is the vector $\bar{\mathbf{x}}[t]$, defined as the average of $\mathbf{x}$ over the observation period starting at t:"

$$\bar{\mathbf{x}}[t] \equiv \frac{1}{n}\sum_{k=1}^{n} \mathbf{x}[t + k\Delta\theta]$$

---Lacerda, G., Spirtes, P. L., Ramsey, J., & Hoyer, P. O. (2008). Discovering cyclic causal models by independent components analysis. UAI 2008

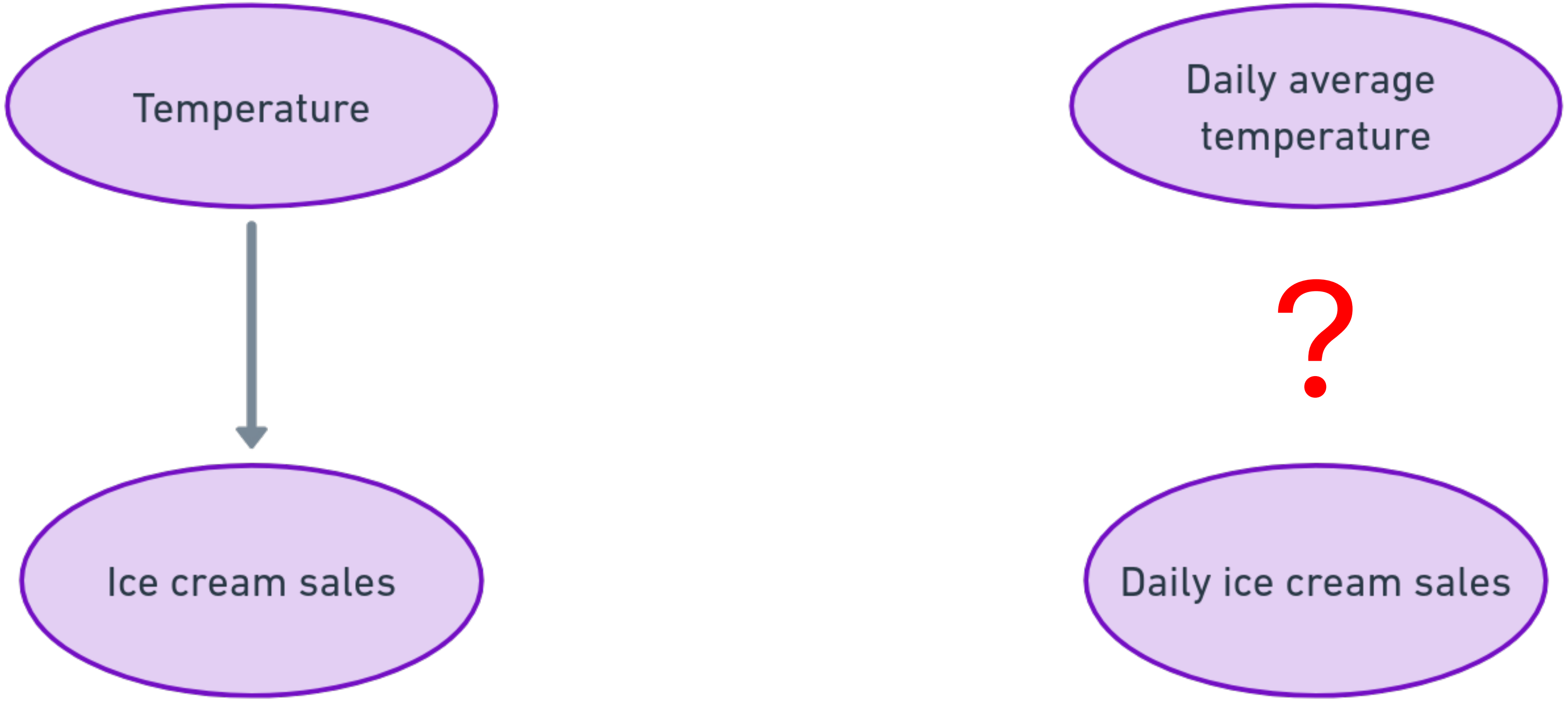


Motivation



Motivation

Is the causal relationship consistent across different observational levels?



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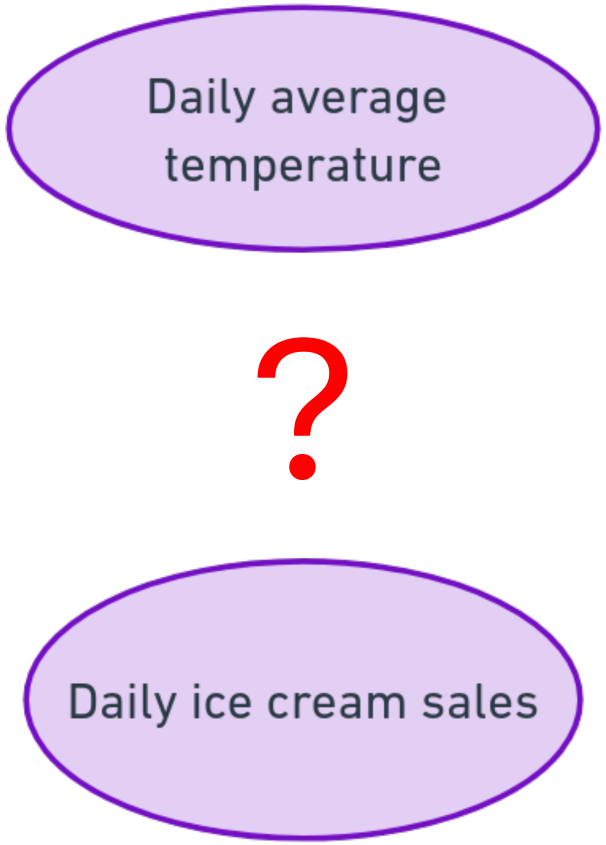
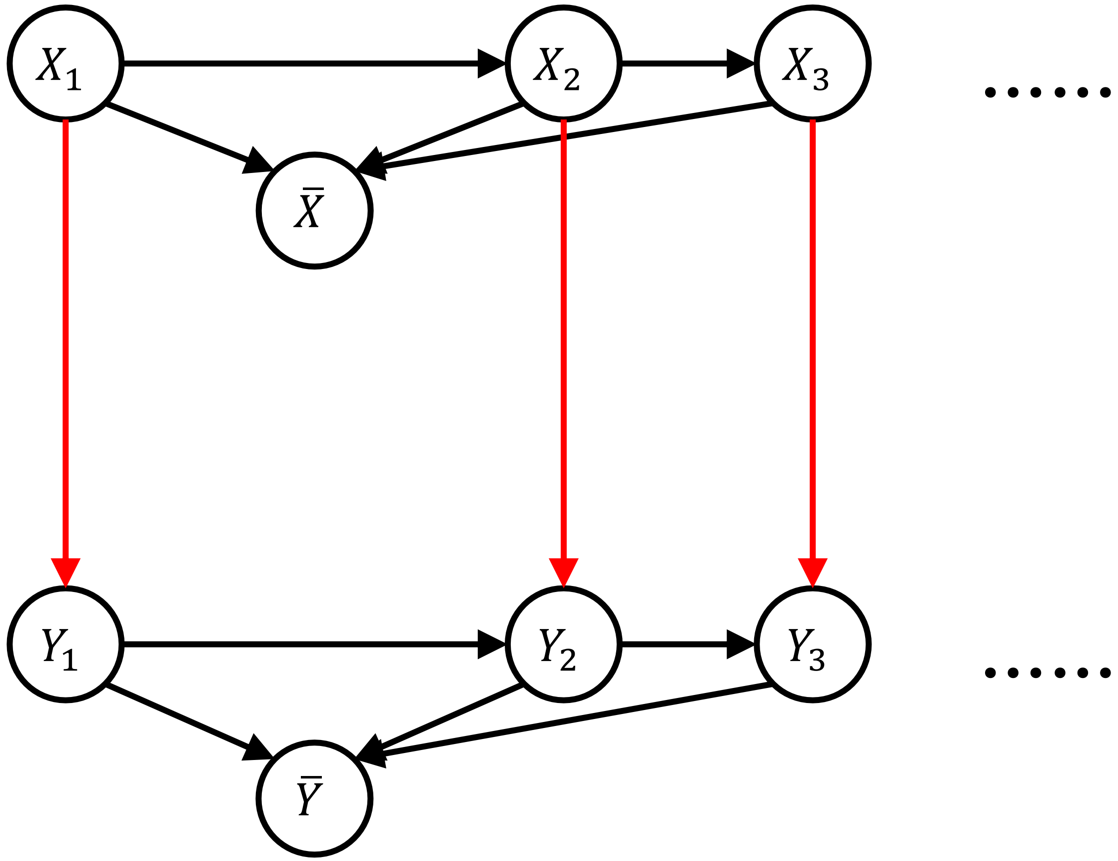
Is the causal relationship consistent across different observational levels?





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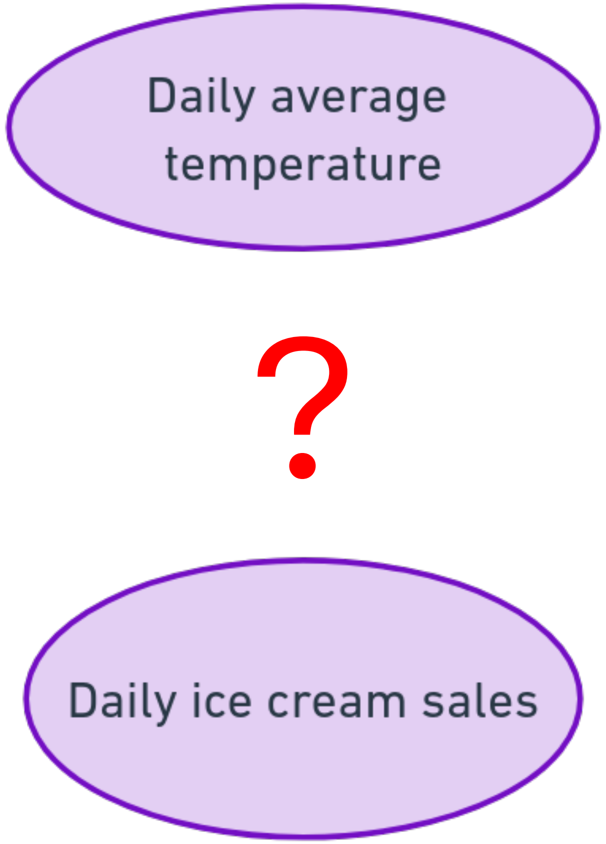

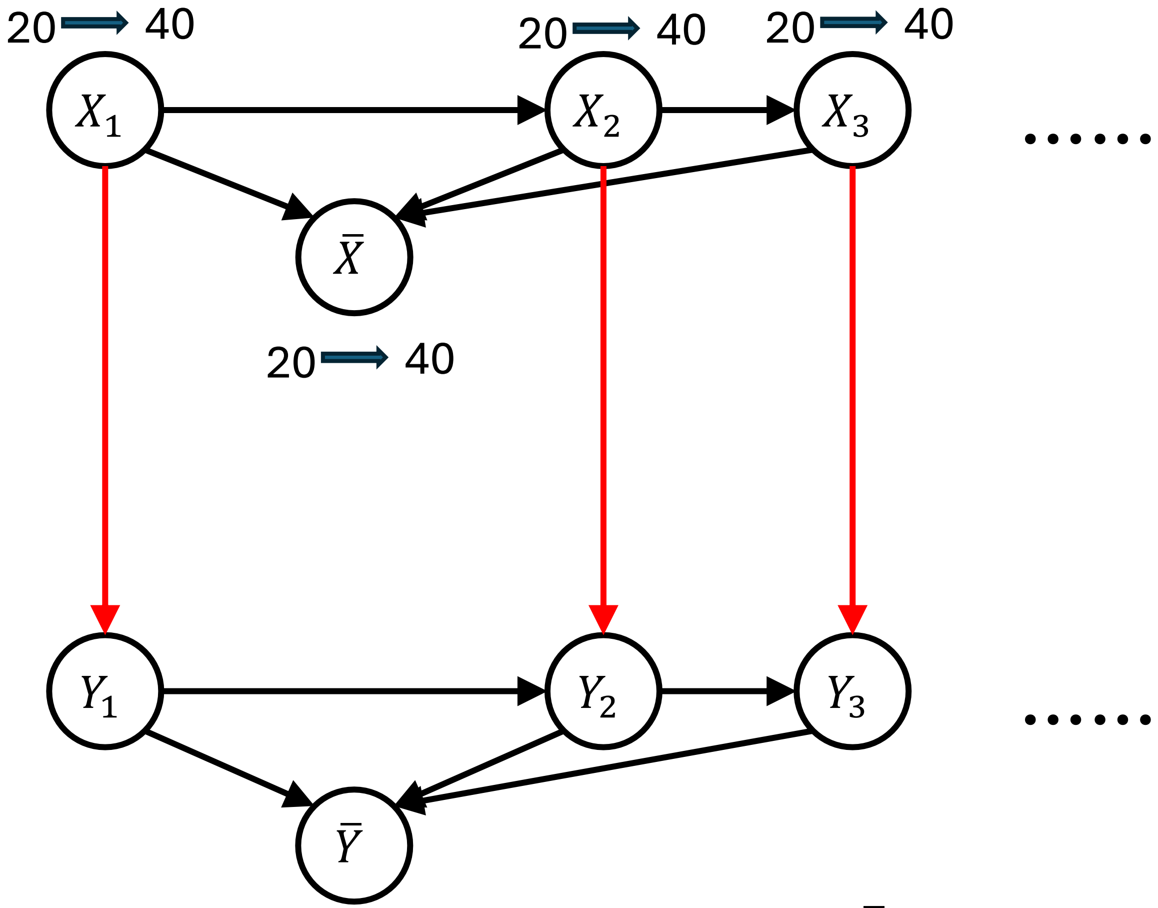

Intervention on $\bar{X}$ is not well-defined

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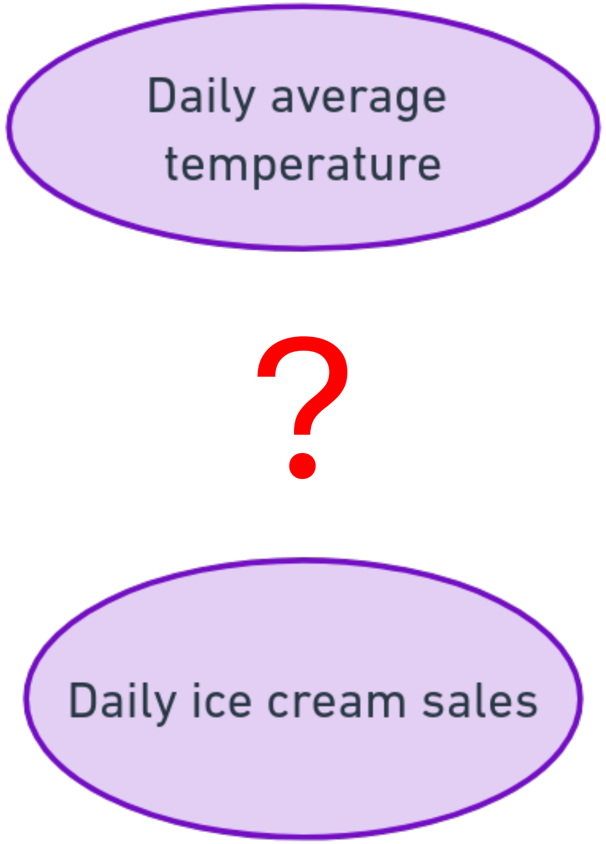
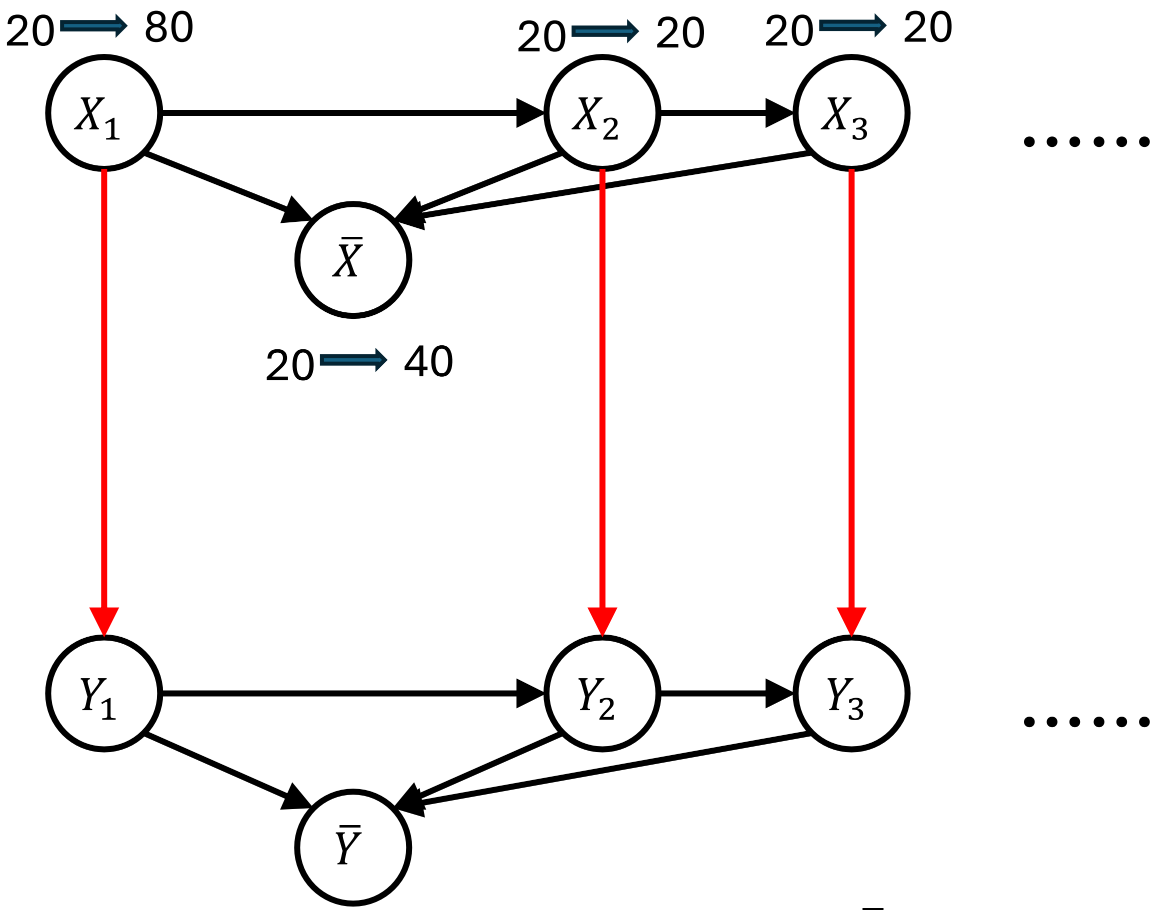
Is the causal relationship consistent across different observational levels?





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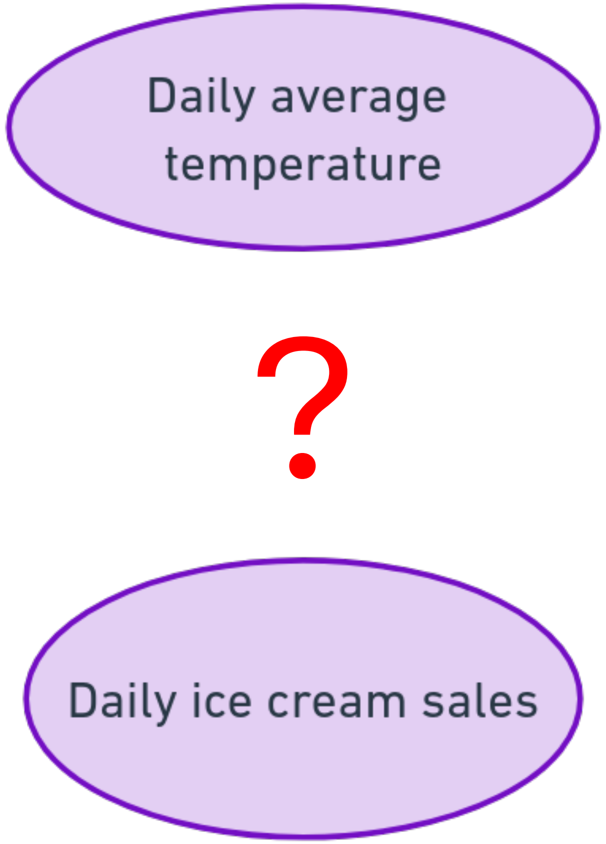
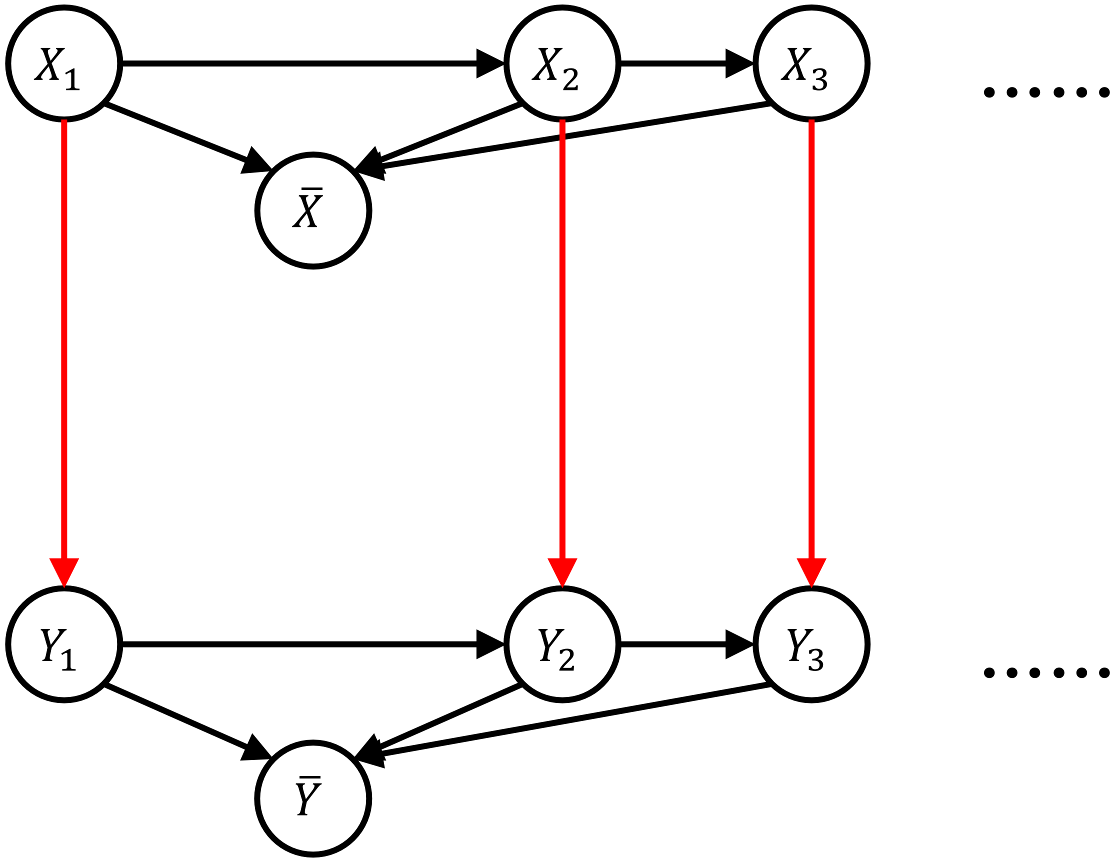
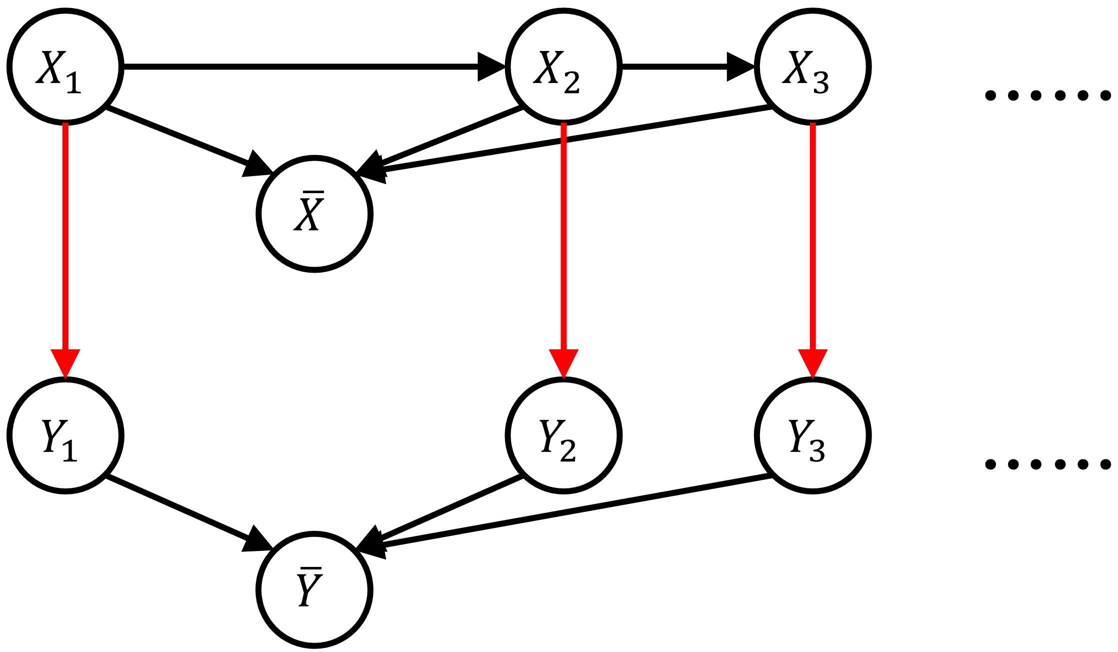




X_t is the cause of Y_t, but $\bar{X}$ may not be the cause of $\bar{Y}$.

Linear

$$Y_t = aX_t + N_t$$

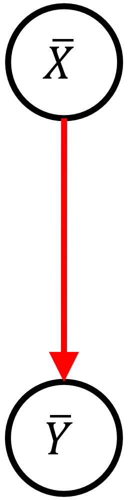


Linear

$$Y_t = aX_t + N_t$$

Thanks to additivity and homogeneity of linear transformation:

$$\bar{Y} = a\bar{X} + \bar{N}$$

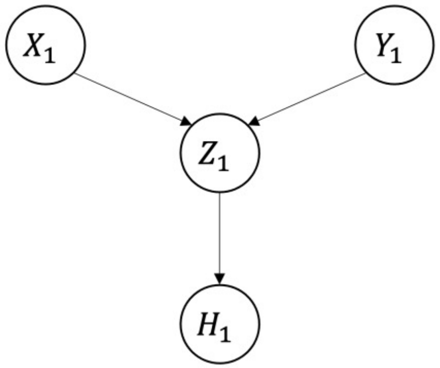


Gong, M., Zhang, K., Schölkopf, B., Glymour, C., & Tao, D. (2017, August). Causal discovery from temporally aggregated time series. UAI 2017

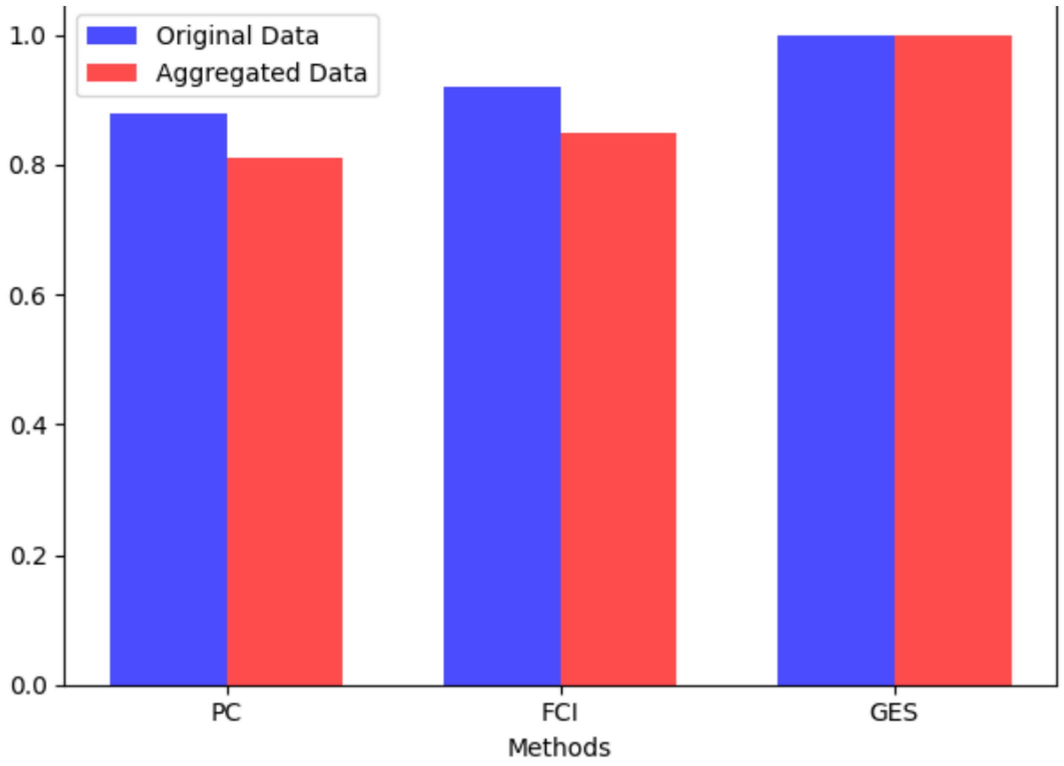
Causal model:

$$
\begin{aligned}
Z &= X + Y + N_Z, \\
H &= Z + N_H \quad \text{(for linear);} \\
Z &= X^2 + Y^2 + N_Z, \\
H &= Z^2 + N_H \quad \text{(for nonlinear).}
\end{aligned}
$$

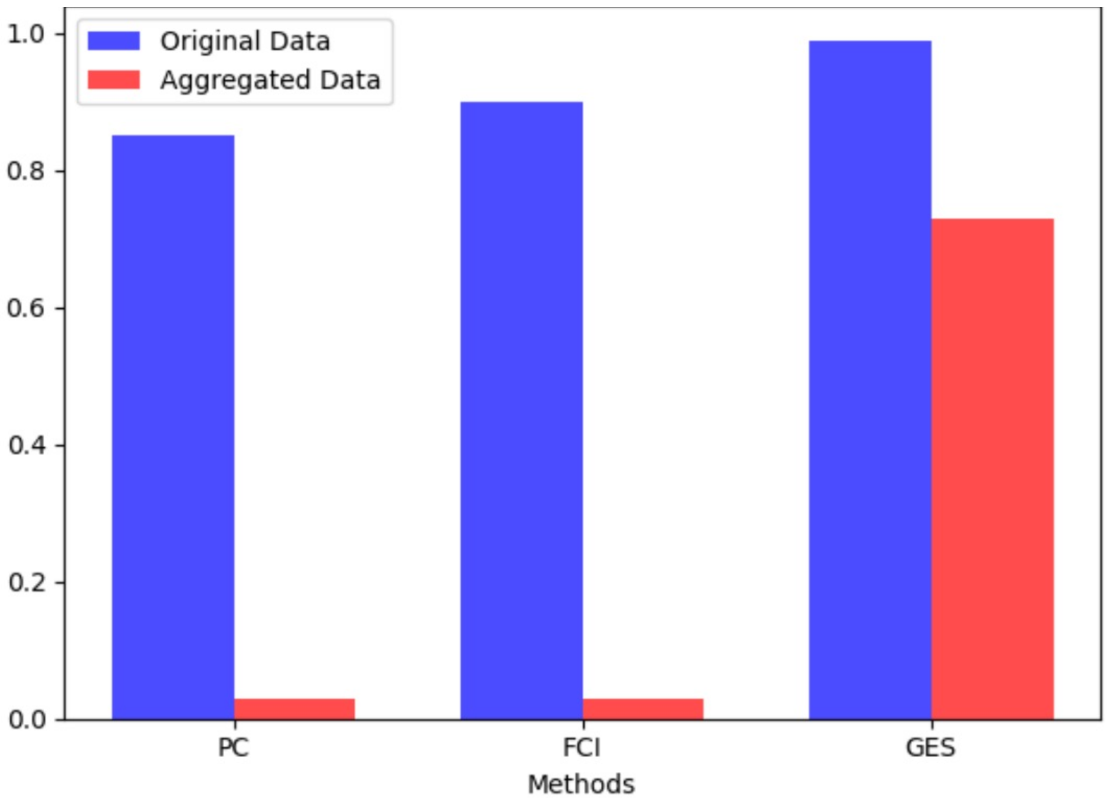
Ground truth:



Accuracy in linear case



Accuracy in non-linear case



Motivation

Many real-world observational data can be considered as the result of aggregation from fine-grained, micro-level, non-linear causal processes. How can we trust the real-world results from non-linear causal discovery methods given that the causal relationship may be inconsistent across different levels?

Our work focuses on:

- When will the causal discovery fail or succeed on aggregated data?
- How the causal discovery results go wrong?

Functional Consistency (LiNGAM, ANM…)

Definition 3.2 (Functional Consistency Regarding Additive Noise). Consider the bivariate aligned model defined in 3.1 incorporates additive noise: $Y_t = f(X_t) + N_{Y,t}$. This process exhibits functional consistency regarding additive noise if there exists a function $\hat{f}$ such that the aggregated variables can be represented as $\overline{Y} = \hat{f}(\overline{X}) + N$, where N is independent of $\overline{X}$, and such $\hat{f}$ exists only in the correct causal direction.

Functional Consistency

Theorem 3.3 (Construction of $\hat{f}$). *If such $\hat{f}$, as defined in Definition 3.2, exists, then $\hat{f}$ must take the form:*

$$\hat{f}(T) = \mathbb{E}\left(\sum_{i=1}^{k} f(X_i) \mid \overline{X} = T\right) + c, \tag{2}$$

where c is any constant (which can be incorporated into the noise term) and the expression $\mathbb{E}(\cdot \mid \overline{X} = T)$ denotes the conditional expectation. For simplicity, we set $c = 0$. Consequently, this implies:

$$\mathbb{E}(\hat{f}(\overline{X})) = \mathbb{E}\left(\sum_{i=1}^{k} f(X_i)\right). \tag{3}$$

Theorem 3.4 (Necessary and Sufficient Condition). *The necessary and sufficient condition for the existence of the additive noise causal model defined in Definition 3.2 is that $N = \sum_{i=1}^{k} N_{Y,i} + \left(\sum_{i=1}^{k} f(X_i) - \hat{f}(\overline{X})\right)$ is independent of $\overline{X}$, where $\hat{f}$ is defined by Eq. 2.*

Functional Consistency

Direct LiNGAM

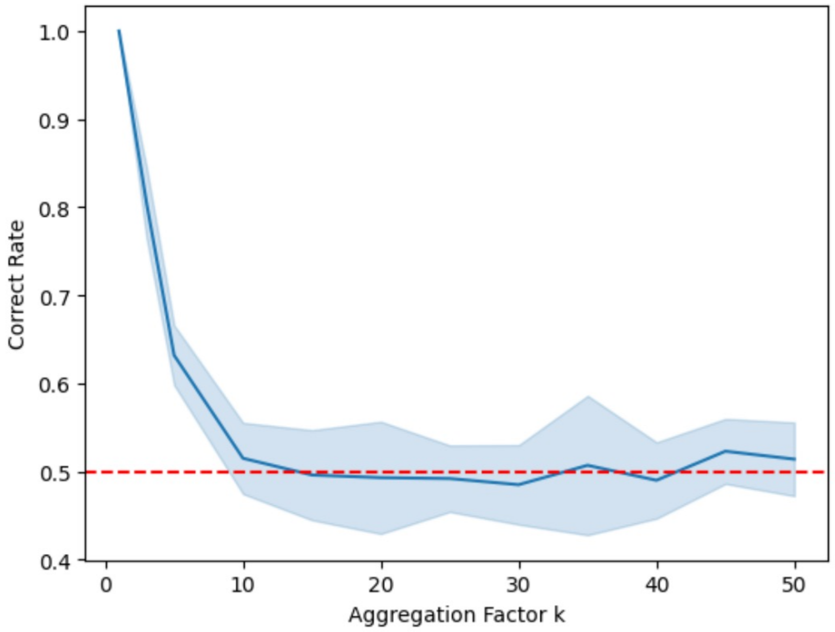


Figure 7. Linear Case: Direct LiNGAM Correction Rate with Different Aggregation Factors k. The blue area represents the standard deviation. The red line represents the random guess baseline.

Additive noise model:

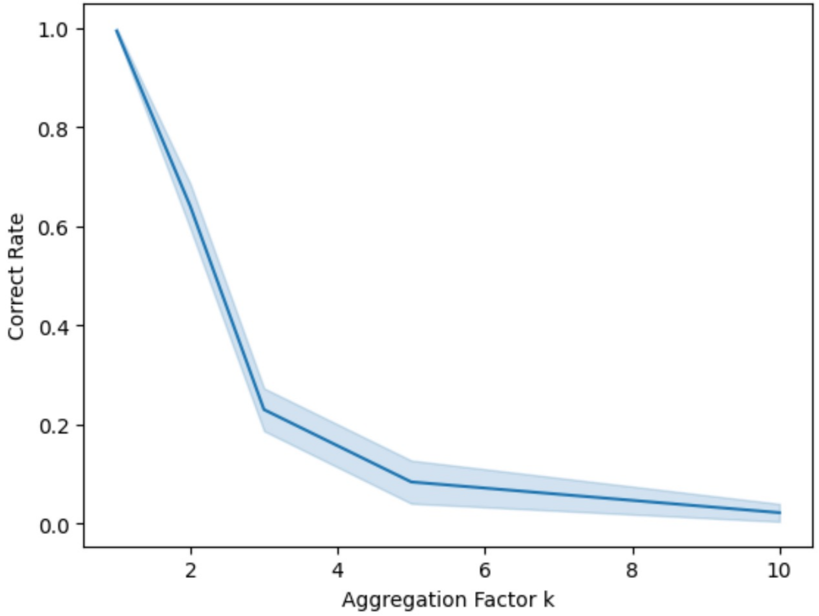
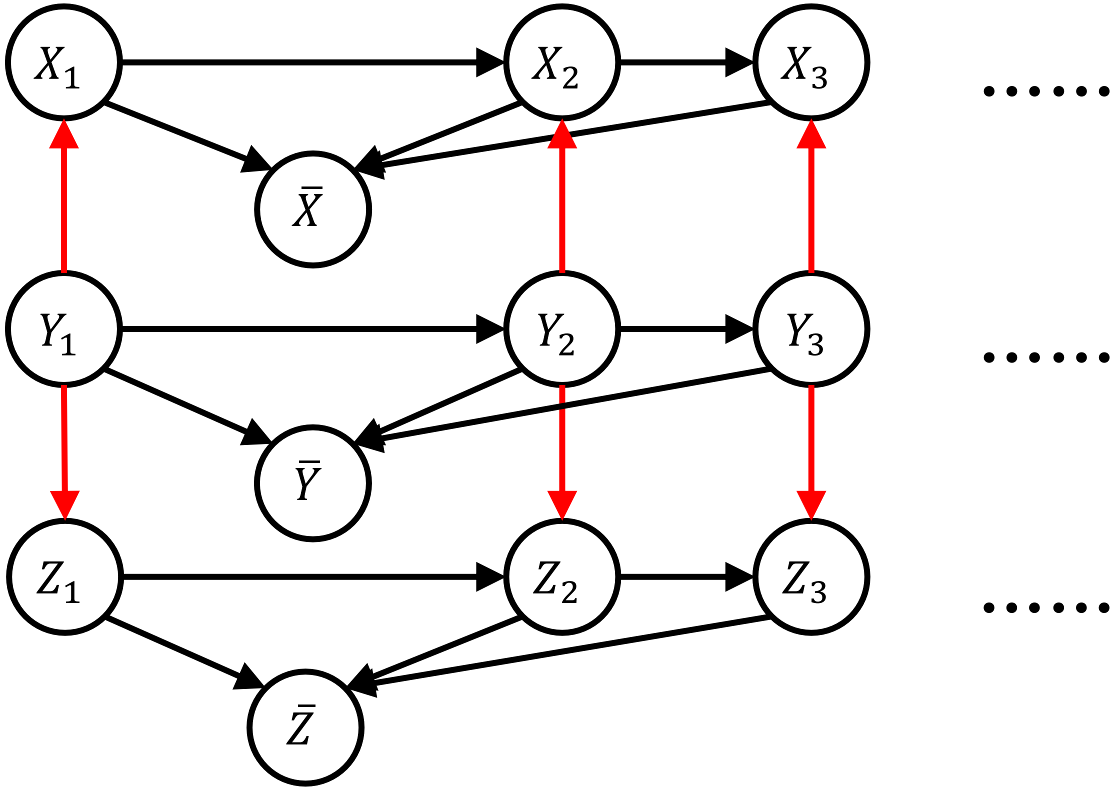
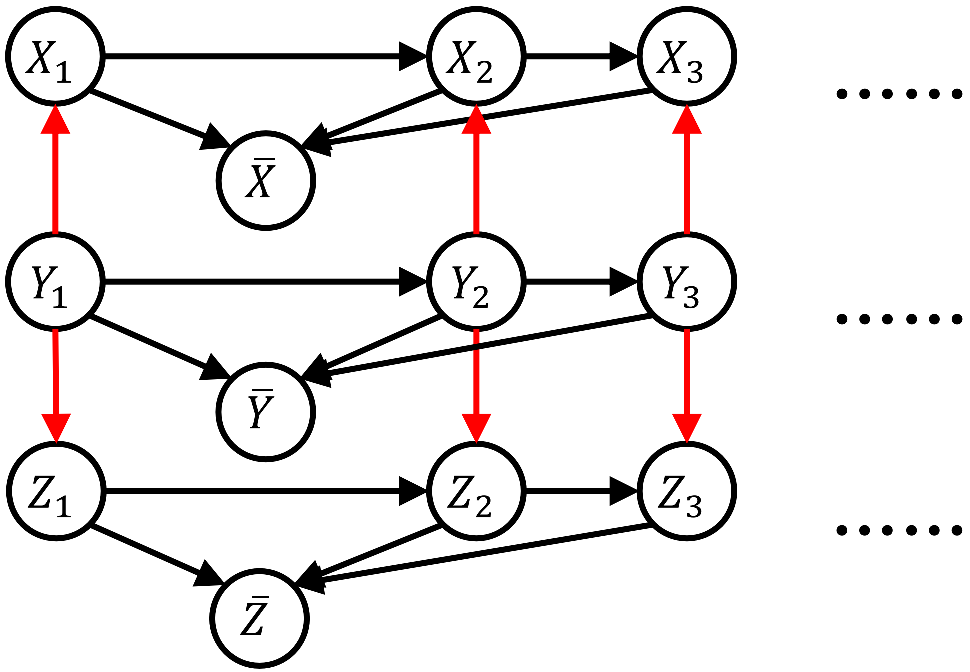


Figure 8. Nonlinear Case: ANM Correction Rate with Different Aggregation Factors k. The blue area represents the standard deviation.

Conditional Independence Consistency (PC, FCI...)



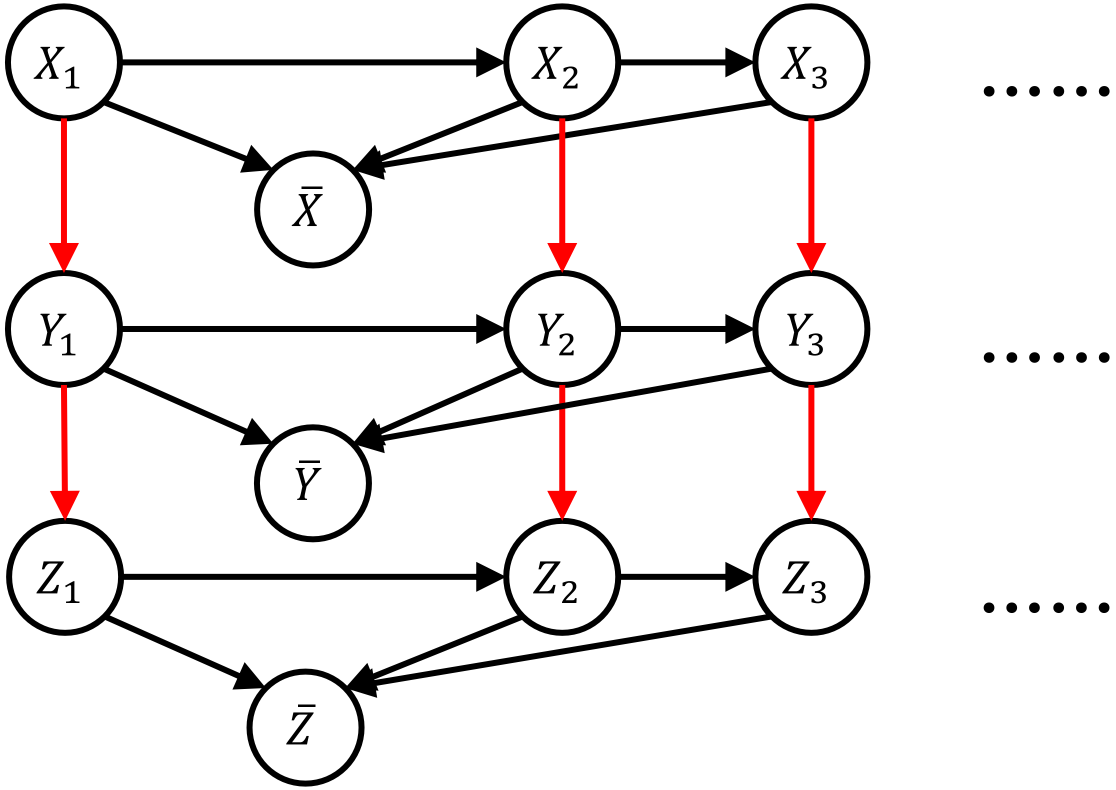


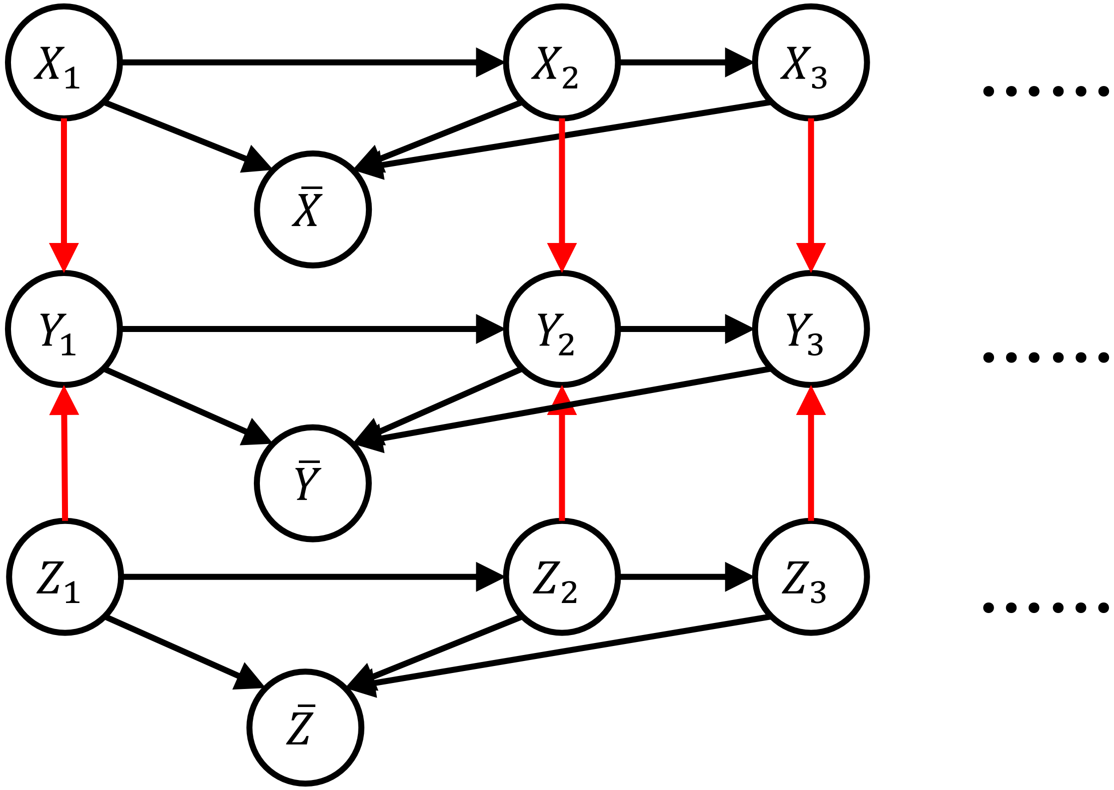
(I) $\overline{X} \perp\!\!\!\perp \overline{Y}$, (II) $\overline{Y} \perp\!\!\!\perp \overline{Z}$, (III) $\overline{X} \perp\!\!\!\perp \overline{Z}$, (IV) $\overline{X} \perp\!\!\!\perp \overline{Y} \mid \overline{Z}$, (V) $\overline{Y} \perp\!\!\!\perp \overline{Z} \mid \overline{X}$, (VI) $\overline{X} \perp\!\!\!\perp \overline{Z} \mid \overline{Y}$.

Rejection Rate for CIT:

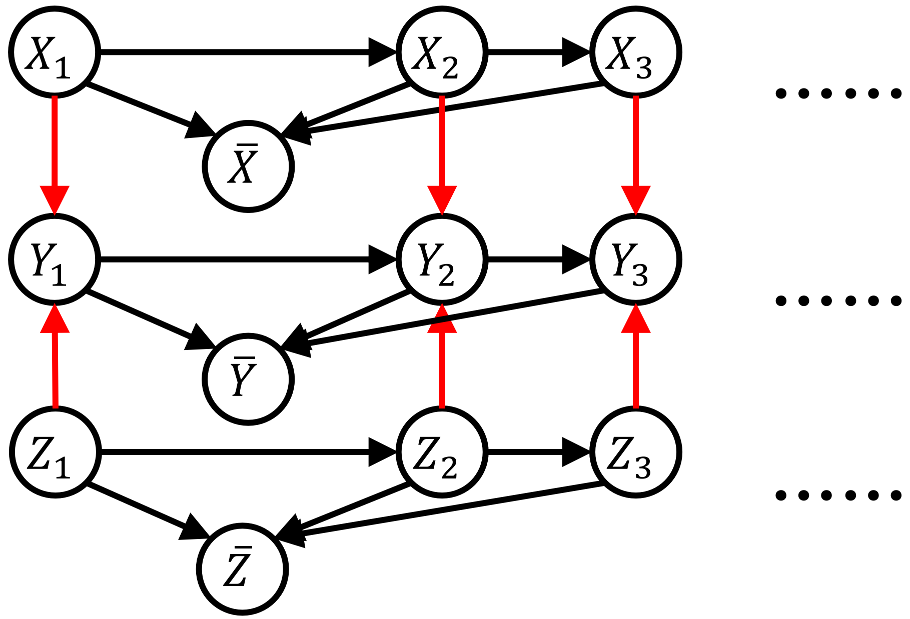
(b) Fork Structure

$X_t \to Y_t$	$Y_t \to Z_t$	I	II	III	IV	V	VI
Linear	Linear	100%	100%	100%	100%	100%	5%
Nonlinear	Linear	92%	100%	84%	92%	100%	5%
Linear	Nonlinear	100%	93%	85%	100%	93%	5%
Nonlinear	Nonlinear	92%	93%	72%	86%	87%	58%


X_1
X_2
X_3
$\bar{X}$
Y_1
Y_2
Y_3
$\bar{Y}$
Z_1
Z_2
Z_3
$\bar{Z}$
……
……
……



X_1
X_2
X_3
$\bar{X}$
......
Y_1
Y_2
Y_3
$\bar{Y}$
......
Z_1
Z_2
Z_3
$\bar{Z}$
......



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Rejection Rate for CIT:

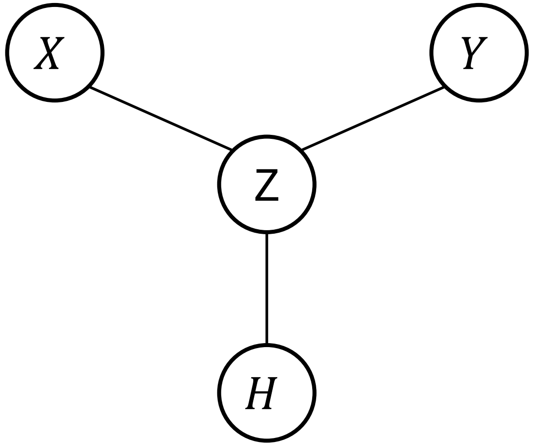
(c) Collider Structure

$X_t \to Y_t$	$Y_t \to Z_t$	I	II	III	IV	V	VI
Linear	Linear	100%	100%	5%	100%	100%	99%
Nonlinear	Linear	95%	89%	5%	96%	91%	51%
Linear	Nonlinear	90%	95%	5%	91%	96%	48%
Nonlinear	Nonlinear	81%	81%	6%	83%	81%	29%

PC algorithm:

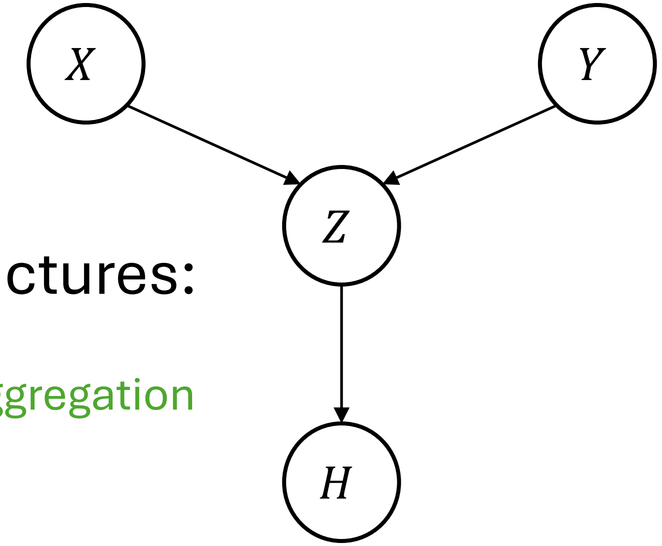
1.Skeleton Discovery:

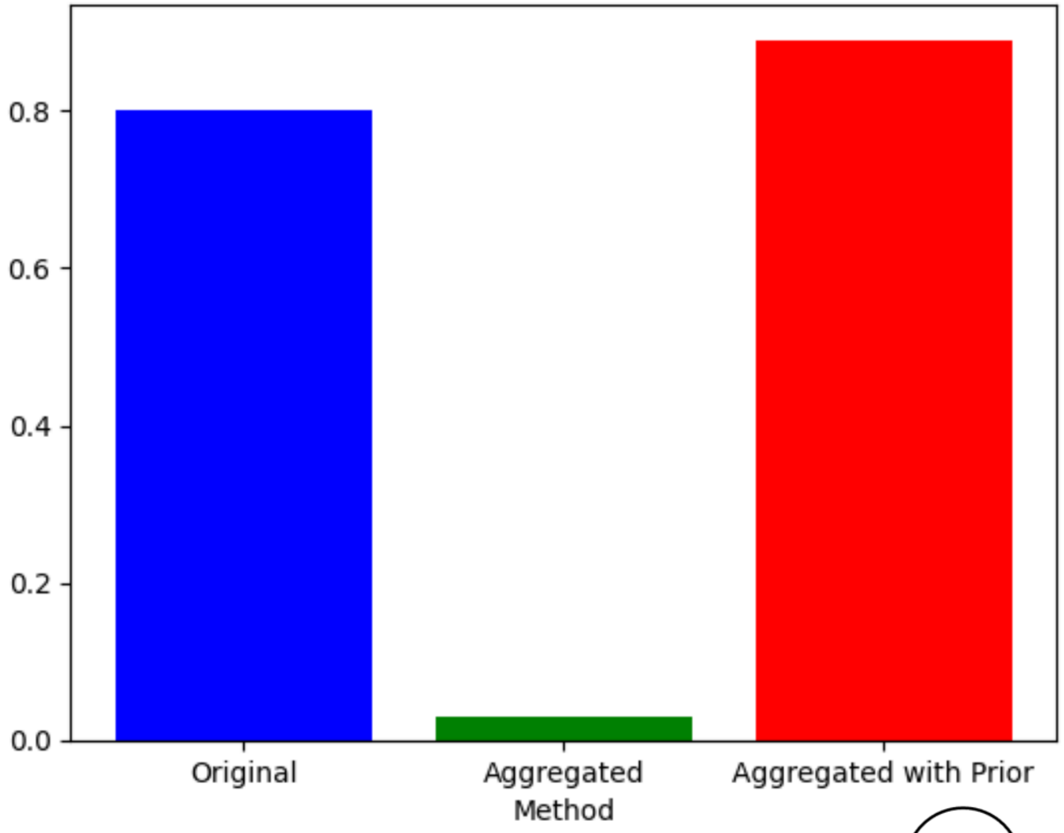
Suffer from aggregation

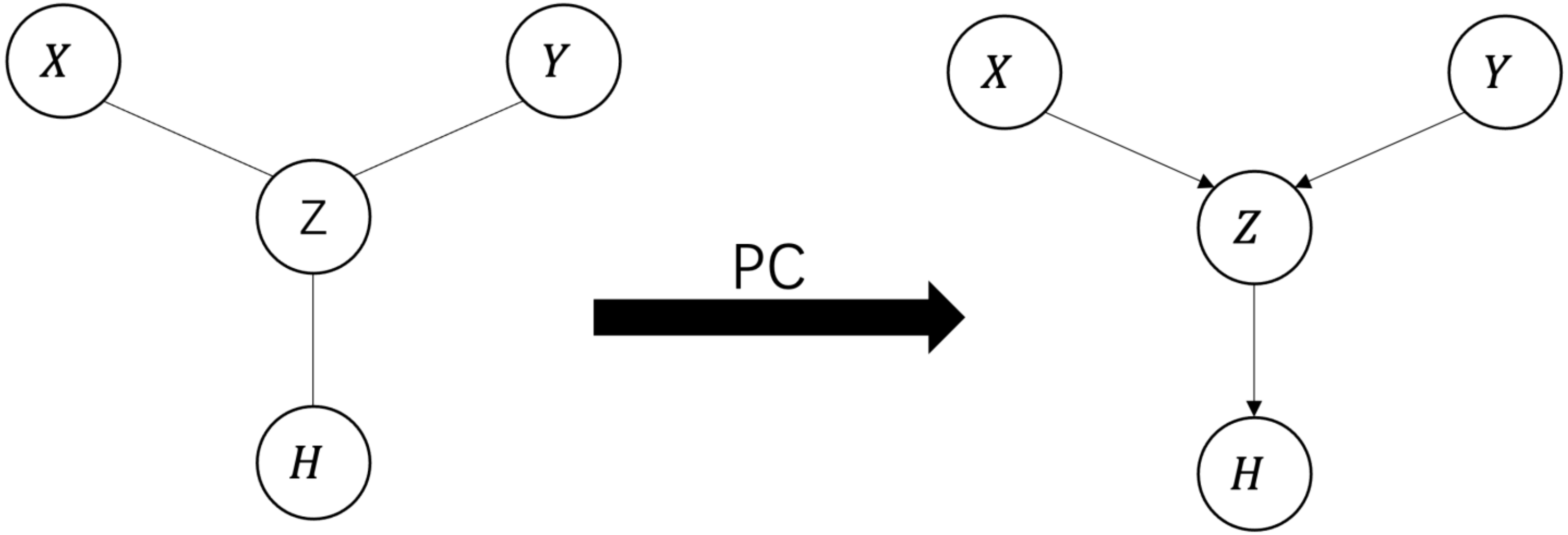


2.Finding V-structures:

Not affected by aggregation




0.8
0.6
0.4
0.2
0.0
Original
Aggregated Method
Aggregated with Prior


X
Y
Z
H
PC
X
Y
Z
H

Thanks for listening!