Gibbs Sampling of Continuous Potentials on a Quantum Computer

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Motivation

Problem. Given an ‘energy’ function $E : \mathbb{R}^d \to \mathbb{R}$, generate samples from the corresponding Gibbs distribution

$$p(x) = \frac{e^{-\beta E(x)}}{Z_\beta}$$

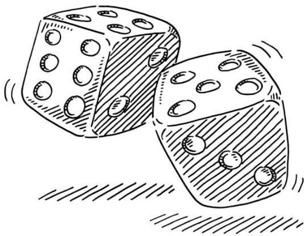

image source: istockphoto.com

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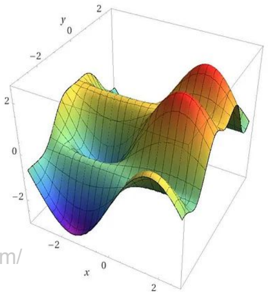
Optimization



https://medium.com/

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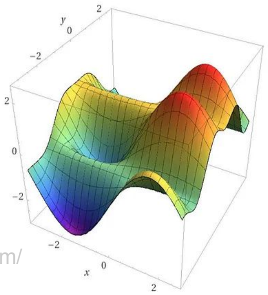

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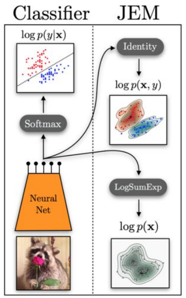

Training EBMs

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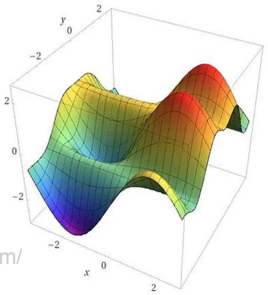

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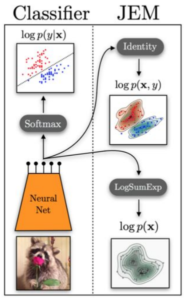

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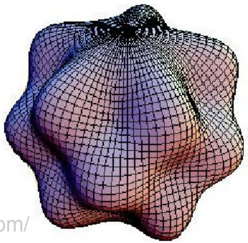
Training EBMs

Counting



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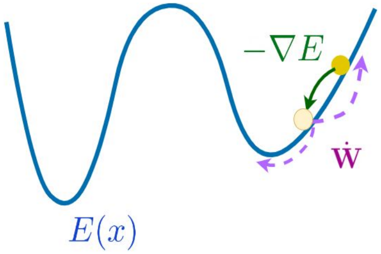
Results

Method	Potential type	Sampling complexity	Mean estimation complexity
Our work	non-convex periodic	$\widetilde{O}\left(\kappa_{E/2}e^{\Delta/2}d^7\right)$	$\widetilde{O}\left(\kappa_{E/2}e^{\Delta/2}d^7\Delta_f\varepsilon^{-1}\right)$
Rejection sampling	non-convex	$O\left(e^{\Delta}\right)$	$O\left(e^{\Delta}\Delta_f^2\varepsilon^{-2}\right)$
Our work	Morse and periodic	$\widetilde{O}\left(\lambda^{-2}e^{\Delta/2}d^7\right)$	$\widetilde{O}\left(\lambda^{-2}e^{\Delta/2}d^7\Delta_f\varepsilon^{-1}\right)$
[LE20]	Morse and periodic	$\widetilde{O}\left(\lambda^{-4}L^4d^3\varepsilon^{-2}\right)$	$\widetilde{O}\left(\lambda^{-4}L^4d^3\Delta_f^2\varepsilon^{-4}\right)$
[DCWY18]	convex	$\widetilde{O}\left(L^2d^3\varepsilon^{-2}\right)$	$\widetilde{O}\left(L^2d^3\Delta_f^2\varepsilon^{-4}\right)$
[CLL$^+$22]	strongly convex	$\widetilde{O}\left(\mu^{-1/2}L^{1/2}d\right)$	$\widetilde{O}\left(\mu^{-1/2}L^{1/2}d\Delta_f\varepsilon^{-1}\right)$
[LST20]	strongly convex	$\widetilde{O}\left(\mu^{-1}Ld\right)$	$\widetilde{O}\left(\mu^{-1}Ld\Delta_f^2\varepsilon^{-2}\right)$

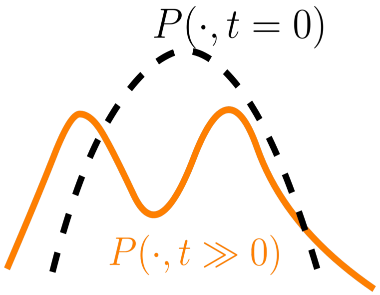
Techniques: main idea

SDE
Langevin dynamics
Good for classical computers



$$\mathrm{d}X_t = -\nabla E(X_t) + \sqrt{2}\,\mathrm{d}W_t$$

PDE
Fokker–Planck equation
Good for quantum computers



$$\partial_t p_t = \nabla(e^{-E}\nabla(e^{E} p_t))$$

- Due to recent quantum algorithms, we can efficiently solve differential equations over exponentially many points [CPO21]
- Normalization constant comes at the cost of post-selection → relates to mixing time!

Techniques: interpolation

Theorem. We can quantumly interpolate

$$N \text{ point discretization} \mapsto e^{\Omega(N)} \text{ point discretization}$$

with **poly(N)** gate complexity.

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f is (semi)-**analytic** $\Longleftrightarrow$ $\dfrac{|\widehat{f}|^2}{||\widehat{f}||^2}$ is a **sub-exp** distribution

Thank You!