Roping in Uncertainty: Robustness and Regularization in Markov Games

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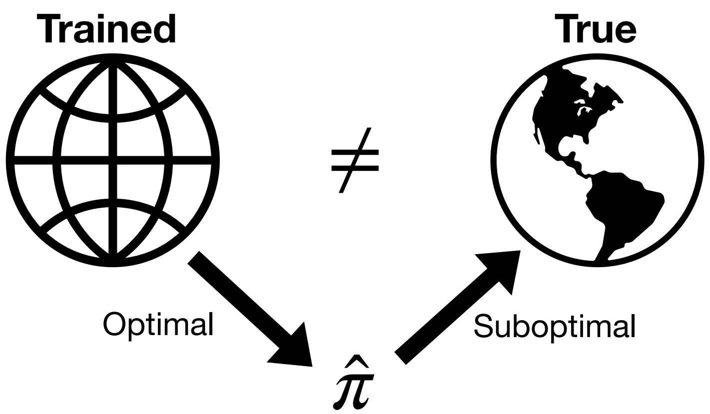
Robust Nash Equilibrium can be computed *efficiently* via Game Regularization!

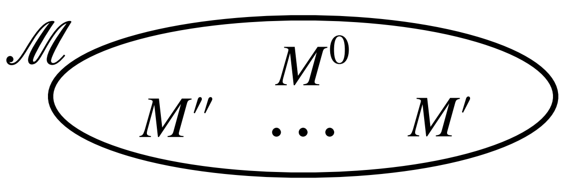
Motivation



Offline RL must be robust!

Robust Markov Games

RMG = model M^0 and uncertainty set $\mathcal{M}$:



Example [Ball Uncertainty]:

$$\mathcal{R}_{i,s,h} = \{R \in \mathbb{R}^{A_1 \times A_2} \mid ||R - R^0||_p \le \alpha_{i,s,h}\}$$

Robust NE do well for all models in $\mathcal{M}$:

$$\pi_i^* \in \arg\max_{\pi_i} \min_{M \in \mathcal{M}} V_{M,i}^{\pi_i^*, \pi_{-i}^*}$$

Regularized Games

RGs use a *Regularized Value*:

$$V_{i,h}^{\pi}(s) = E_M^{\pi}\left[\sum_{t=h}^{H} r_{i,t}(s_t, a_t) - \Omega_{i,t}(s_t, \pi_t) \mid s_h = s\right]$$

Example Regularizers:

1. Entropy: $\Omega_{i,h}(s,\pi) = \alpha_i \sum_{a_i \in \mathcal{A}_i} \pi_i(a_i) \log \pi_i(a_i)$
2. Norm: $\Omega_{i,h}(s,\pi) = \alpha_{i,s,h} ||\pi_i||_p ||\pi_{-i}||_q$

Equivalence of RMG and RG

Robust Markov Game

⇕

Characteristic function

$$\Omega_{i,h}(s,\pi) := \sigma_{\mathcal{R}_{i,s,h}}\left(-\pi_i \pi_{-i}^\top\right)$$

⇕ $\sigma_X(y) := \sup_{x \in X} \langle x, y \rangle$

Regularized Game

*Similar results hold for Transition Uncertainty

Computational Complexity

$-\sigma_{\mathcal{R}_1}(-\pi_1 \pi_2^\top) = \pi_1^\top R_1 \pi_2$

$-\sigma_{\mathcal{R}_2}(-\pi_1 \pi_2^\top) = \pi_1^\top R_2 \pi_2$

→ General Sum

Theorem: *Computing RNE is PPAD-hard even for (s,a)-rectangular zero-sum matrix games*.

Key to Tractability: Decomposability

$$\sigma_{\mathcal{R}_{i,s,h}}(-\pi_i \pi_{-i}^\top) = \Omega_{i,i}^h(\pi_i) + \Omega_{i,-i}^h(\pi_{-i})$$

Theorem: *If σ is decomposable, the equivalent RG is zero-sum so can be solved in poly time.*

Conclusions

A <u>general</u> and <u>tractable</u> equivalence:

Robustness ⟷ Regularization

- Robust policies can be found using regularization, sometimes efficiently.
- Regularizers provide robustness.