Transformer-based Stagewise Decomposition for Large-Scale Multistage Stochastic Optimization

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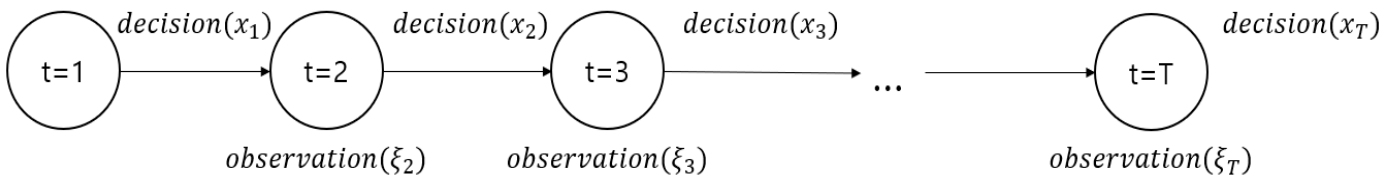
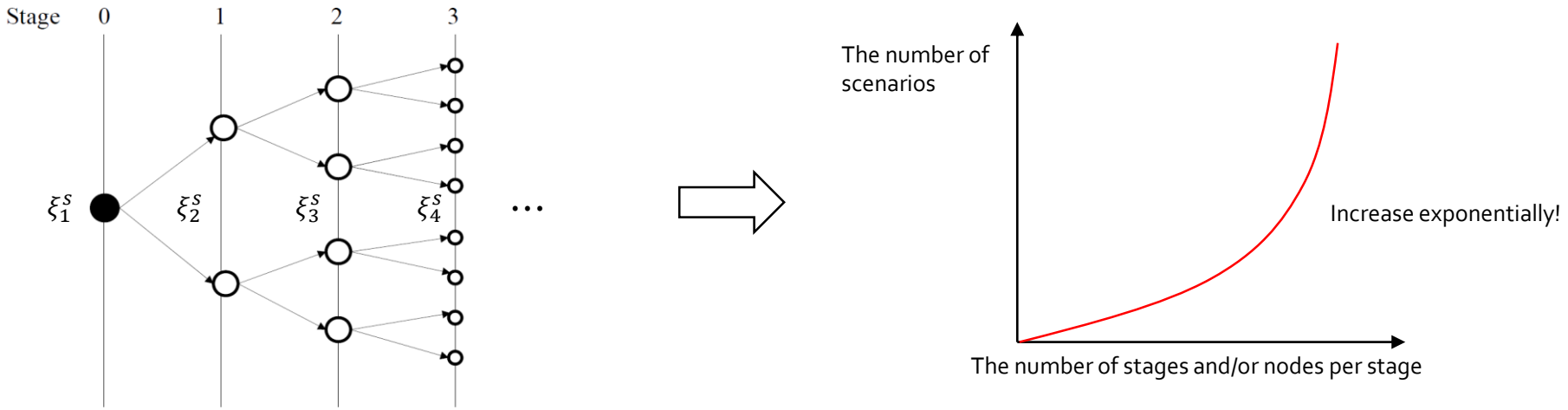
Multistage Stochastic Programming

- **Multi-period sequential decision-making problems under uncertainty**

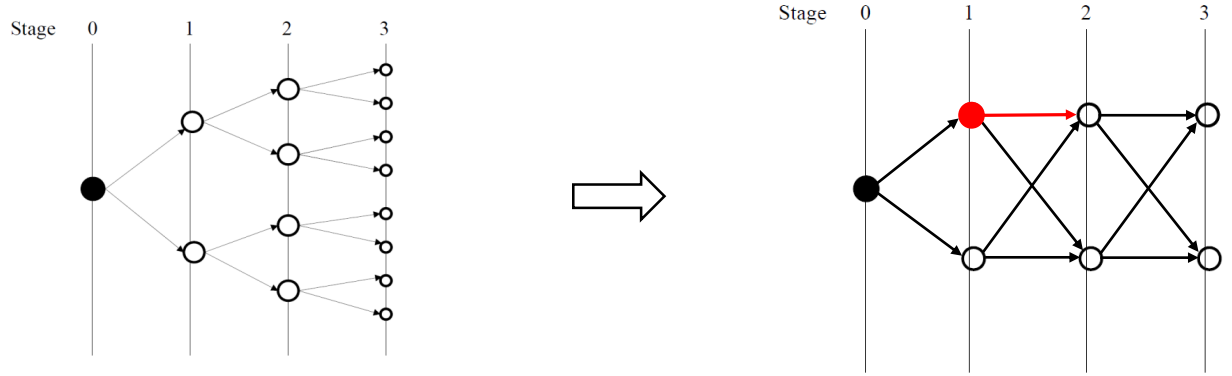


- **Multistage Stochastic Programming (MSP)**

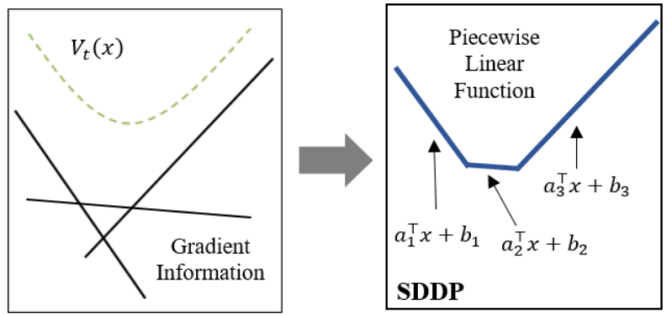


Stochastic Dual Dynamic Programming

- **Stagewise Decomposition**



- **Stochastic Dual Dynamic Programming (SDDP)**

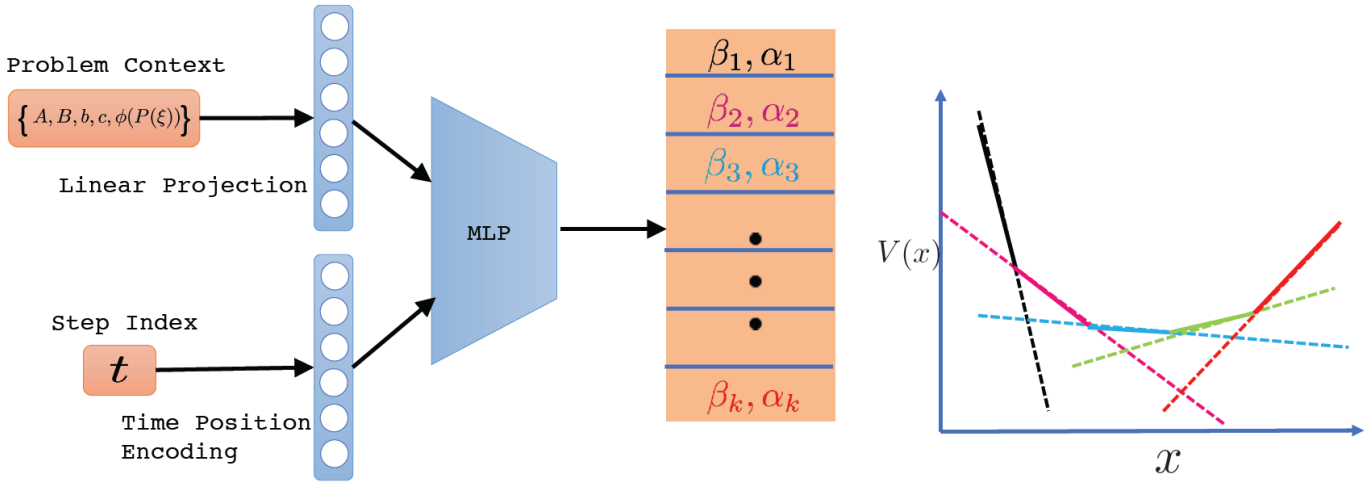


$\hat{V}_t(x) = max\{a_i^\top x + b_i\}$

- **Issues: computational inefficiency**
  - Computational time of subproblem increases
  - Slight perturbation in the problem requires a complete re-solving

Improving SDDP

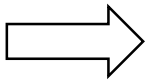
- **Neural SDDP (ν-SDDP)**



- **Drawbacks:**
  - Previously generated cuts are not considered to generate new ones
  - Only linear programs can be solved by ν-SDDP
  - The number of cuts to be generated is fixed

Motivation

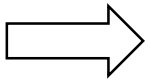
1. Generation of new cuts depends on previously generated cuts



Attention Networks?

2. Cuts are related to each other

3. Cuts are generated sequentially until the convergence condition is satisfied

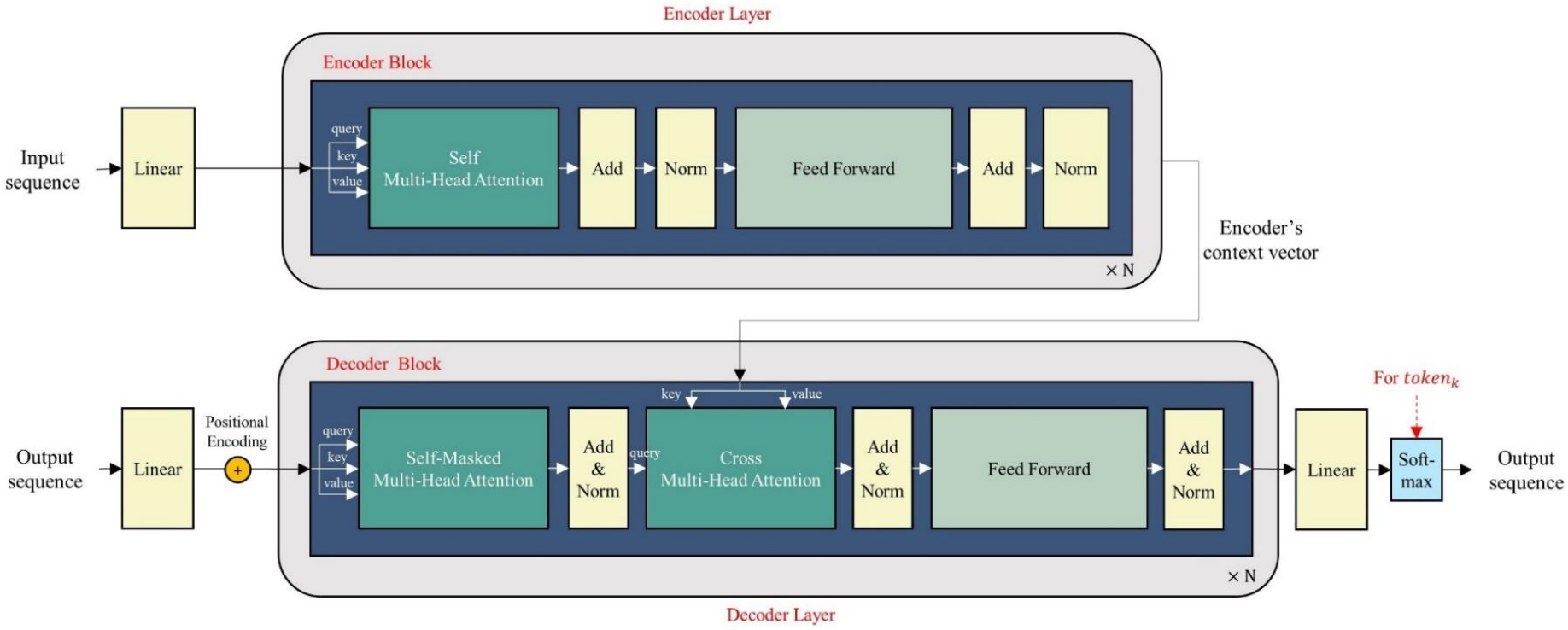


Sequence Models?

We propose a model, ***TranSDDP***, that uses architecture of **Transformer** to generate the cuts for approximating the value function in SDDP

Architecture – TranSDDP

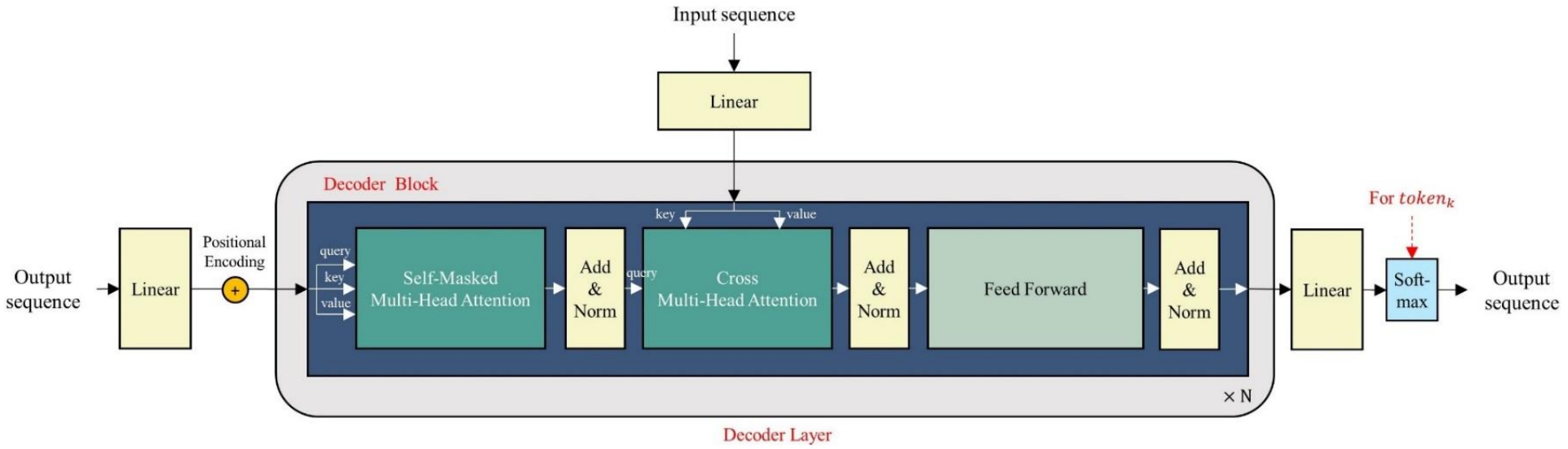
- **TranSDDP**



Architecture – TranSDDP-Decoder

- Herein, we note that:
  - The size of the input sequence is fixed
  - Interrelationships between input sequence are relatively insignificant

- **TranSDDP-Decoder: decoder part of the TranSDDP model**



TranSDDP Learning

- **Input sequence**: Parameters of probability distribution for stochastic elements + time
  - Parameters are sampled from the prior distribution (parameter space)
  - Enables solving for a family of problems, not limited to a single instance

- **Output sequence**: cut information + token

- **Dataset** $\mathcal{D}_s := \left\{ z_s := \left(\Lambda, \tilde{t}, \{\beta_k, \alpha_k, \tau_k\}_{k=1}^{K} \right)_s \right\}_{s=1}^{S}$

- **Loss function**: MSE Loss (for cuts) + CE Loss (for tokens)

Numerical Experiments

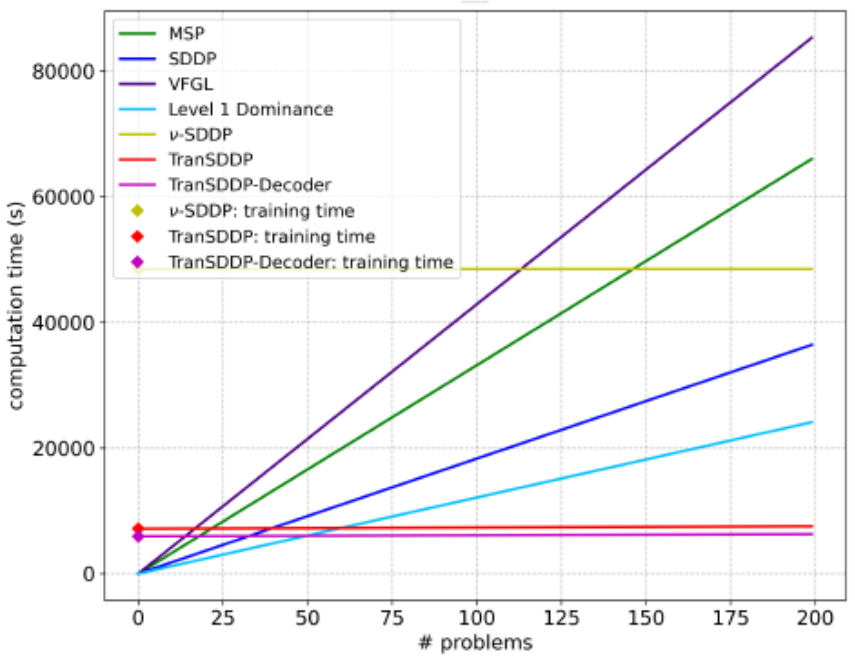
- **Tasks**
  - Task 1: Energy Planning
  - Task 2: Financial Planning
  - Task 3: Production Planning

- **Benchmarks**
  - MSP, SDDP
  - L1-Dominance, Value Function Gradient Learning (VFGL), Neural SDDP

Energy Planning

Table 2. Performance comparisons for EP problem

Task	Algorithm	Error ratio	Evaluation time (s)	Training time
	MSP	-	331.58	-
	SDDP	3.349 ± 2.698%	183.01	-
	L1	0.326 ± 0.379%	121.096	-
$T = 7$	VFGL	1.169 ± 0.822%	428.559	-
	ν-SDDP	40.410 ± 29.742%	0.02	13h 27m
	TranSDDP	1.191 ± 0.701%	1.990	1h 59m
	TranSDDP-Decoder	1.010 ± 0.548%	1.712	1h 39m
	MSP	-	902.73	-
	SDDP	3.441 ± 3.357%	450.12	-
	L1	0.35 ± 0.43%	179.776	-
$T = 10$	VFGL	1.796 ± 1.100%	438.237	-
	ν-SDDP	68.070 ± 5.459%	0.023	23h 22m
	TranSDDP	2.337 ± 1.736%	2.100	2h 30m
	TranSDDP-Decoder	3.826 ± 2.018%	1.784	2h 6m

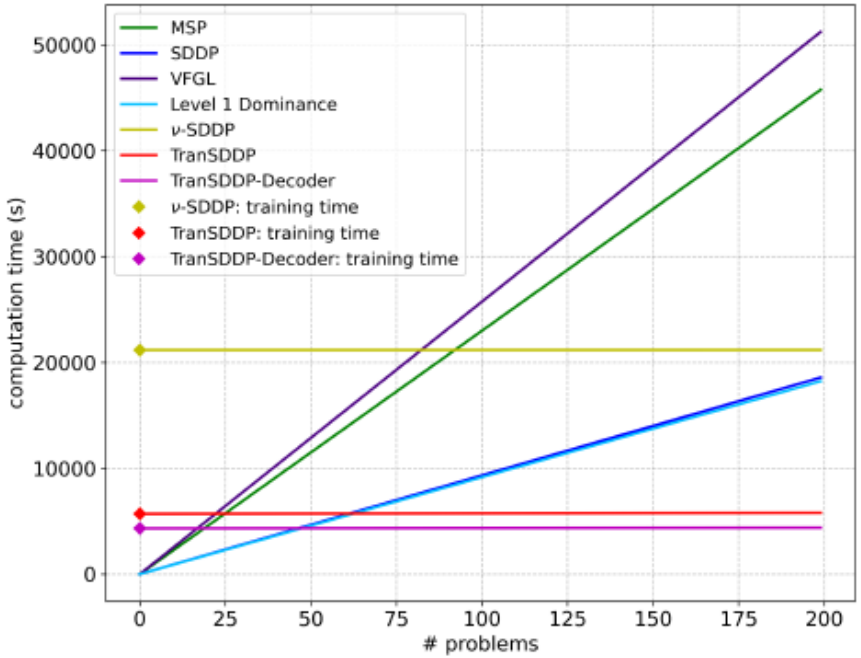


(a) Energy Planning

Financial Planning

Table 3. Performance comparisons for FP problem

Task	Algorithm	Error ratio	Evaluation time (s)	Training time
$T = 7$	MSP	-	229.93	-
	SDDP	$1.782 \pm 1.192\%$	93.35	-
	L1	$1.283 \pm 1.096\%$	91.595	-
	VFGL	$0.200 \pm 0.160\%$	257.387	-
	ν-SDDP	$515.722 \pm 0.802\%$	0.021	5h 52m
	TranSDDP	$0.962 \pm 0.199\%$	0.460	1h 35m
	TranSDDP-Decoder	$0.611 \pm 0.198\%$	0.335	1h 12m
$T = 10$	MSP	-	559.61	-
	SDDP	$2.848 \pm 1.647\%$	144.57	-
	L1	$1.630 \pm 1.360\%$	177.941	-
	VFGL	$0.110 \pm 0.082\%$	284.863	-
	ν-SDDP	$317.890 \pm 0.448\%$	0.024	11h 4m
	TranSDDP	$1.704 \pm 0.209\%$	0.630	1h 50m
	TranSDDP-Decoder	$1.364 \pm 0.208\%$	0.490	1h 17m

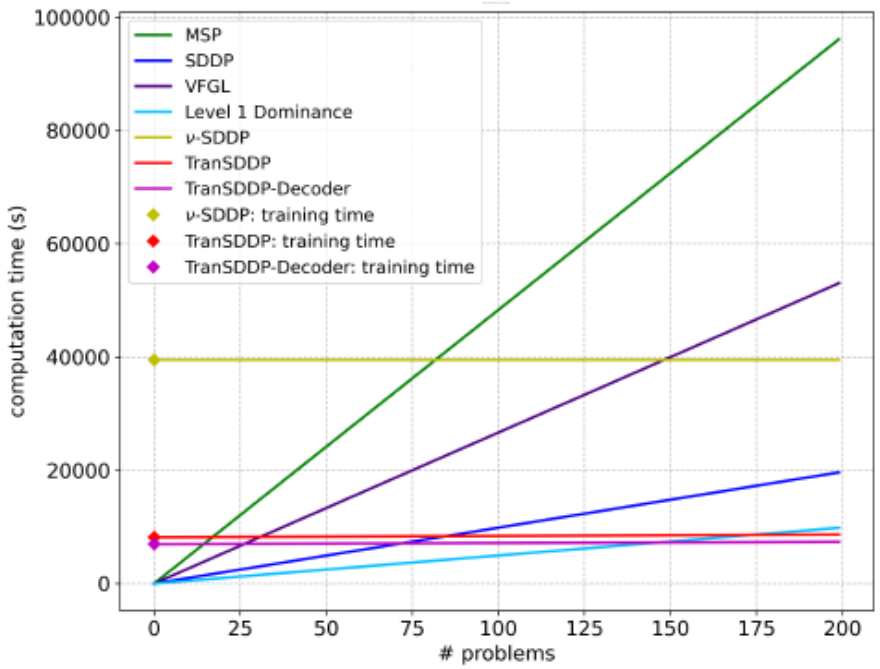


(b) Financial Planning

Production Planning

Table 4. Performance comparisons for PP problem

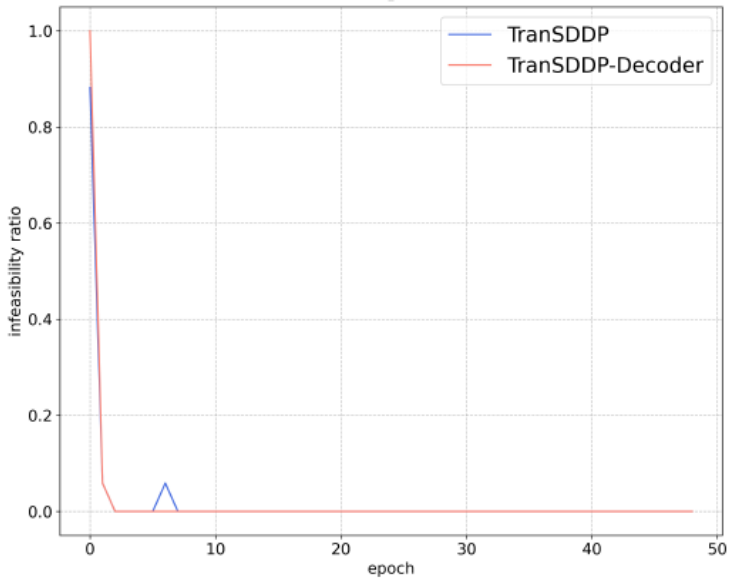
Task	Algorithm	Error ratio	Evaluation time (s)	Training time
	MSP	-	486.046	-
	SDDP	$0.072 \pm 0.115\%$	98.637	-
	L1	$0.239 \pm 0.762\%$	49.486	-
$T = 7$	VFGL	$0.692 \pm 0.687\%$	266.514	-
	ν-SDDP	$7.112 \pm 2.648\%$	0.017	10h 58m
	TranSDDP	$3.628 \pm 3.341\%$	2.410	2h 16m
	TranSDDP-Decoder	$0.838 \pm 0.831\%$	2.018	1h 56m
	MSP	-	1103.122	-
	SDDP	$0.076 \pm 0.110\%$	151.908	-
	L1	$0.096 \pm 0.259\%$	78.630	-
$T = 10$	VFGL	$0.440 \pm 0.430\%$	282.234	-
	ν-SDDP	$2.770 \pm 2.030\%$	0.02	19h 3m
	TranSDDP	$3.580 \pm 3.510\%$	2.594	2h 18m
	TranSDDP-Decoder	$0.967 \pm 0.182\%$	2.235	1h 57m



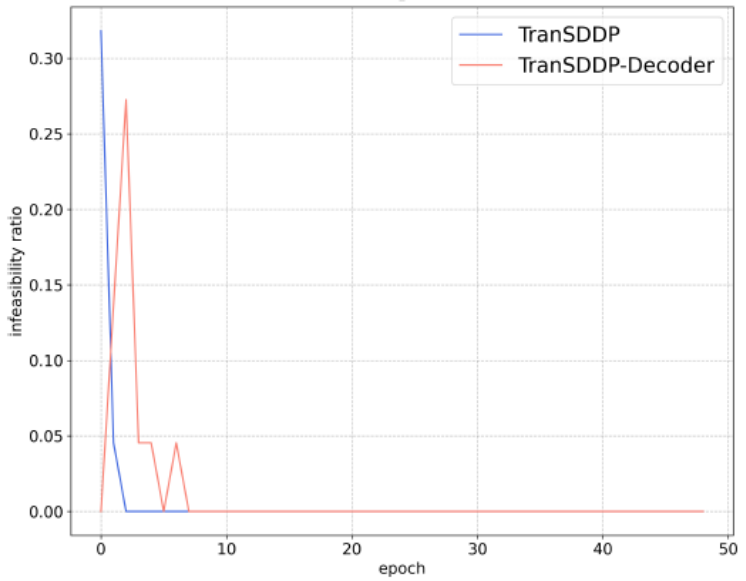
(c) Production Planning

THANK YOU FOR LISTENING

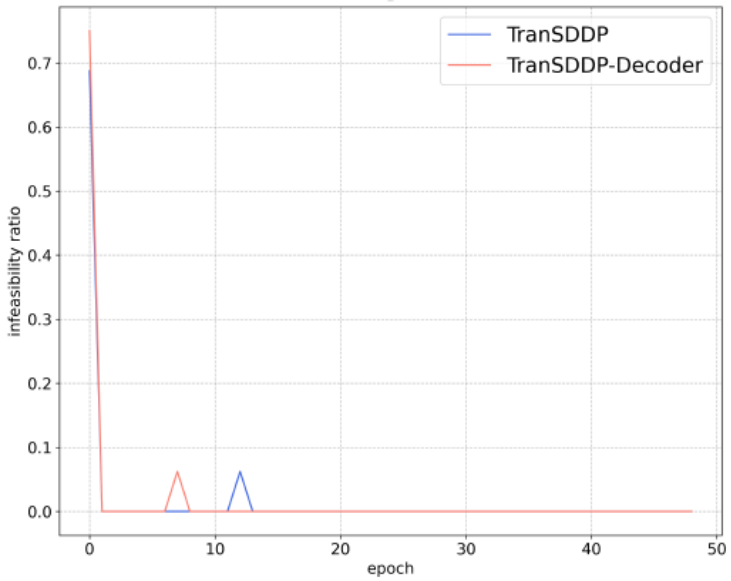
Feasibility



(a) Energy Planning



(b) Financial Planning



(c) Production Planning

Figure 2. Infeasibility ratio per epoch

Value Function Comparison: Energy Planning

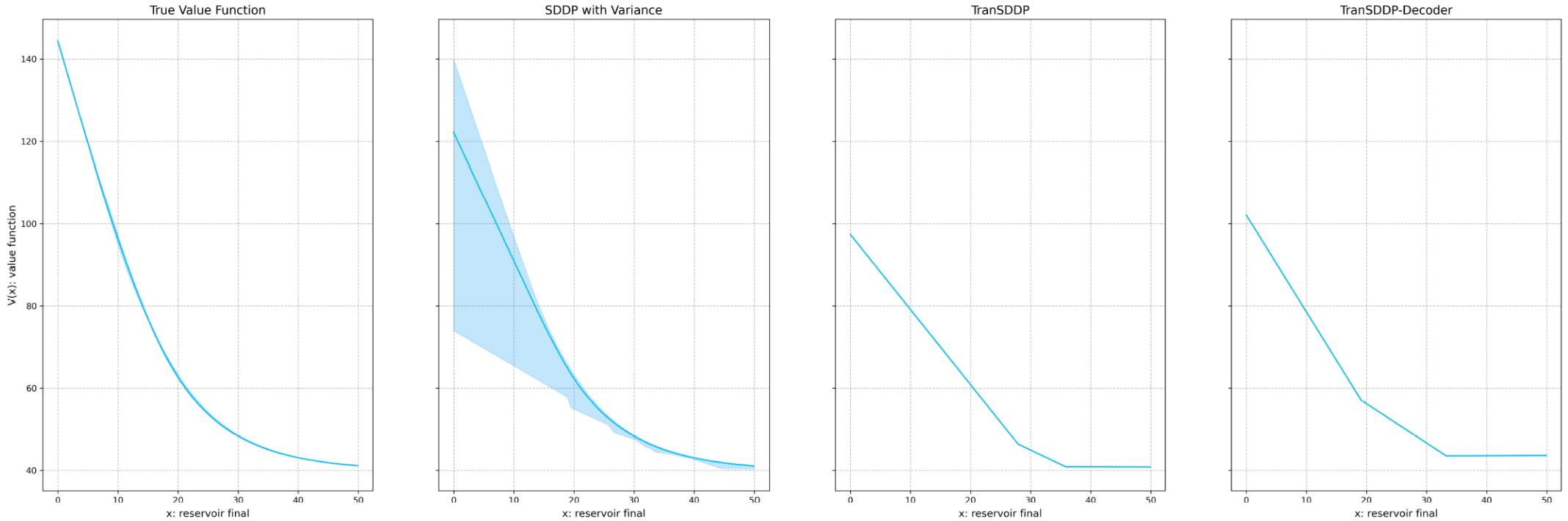


Figure 8. Comparison of the value function and its approximations for energy planning problem

Value Function Comparison: Financial Planning

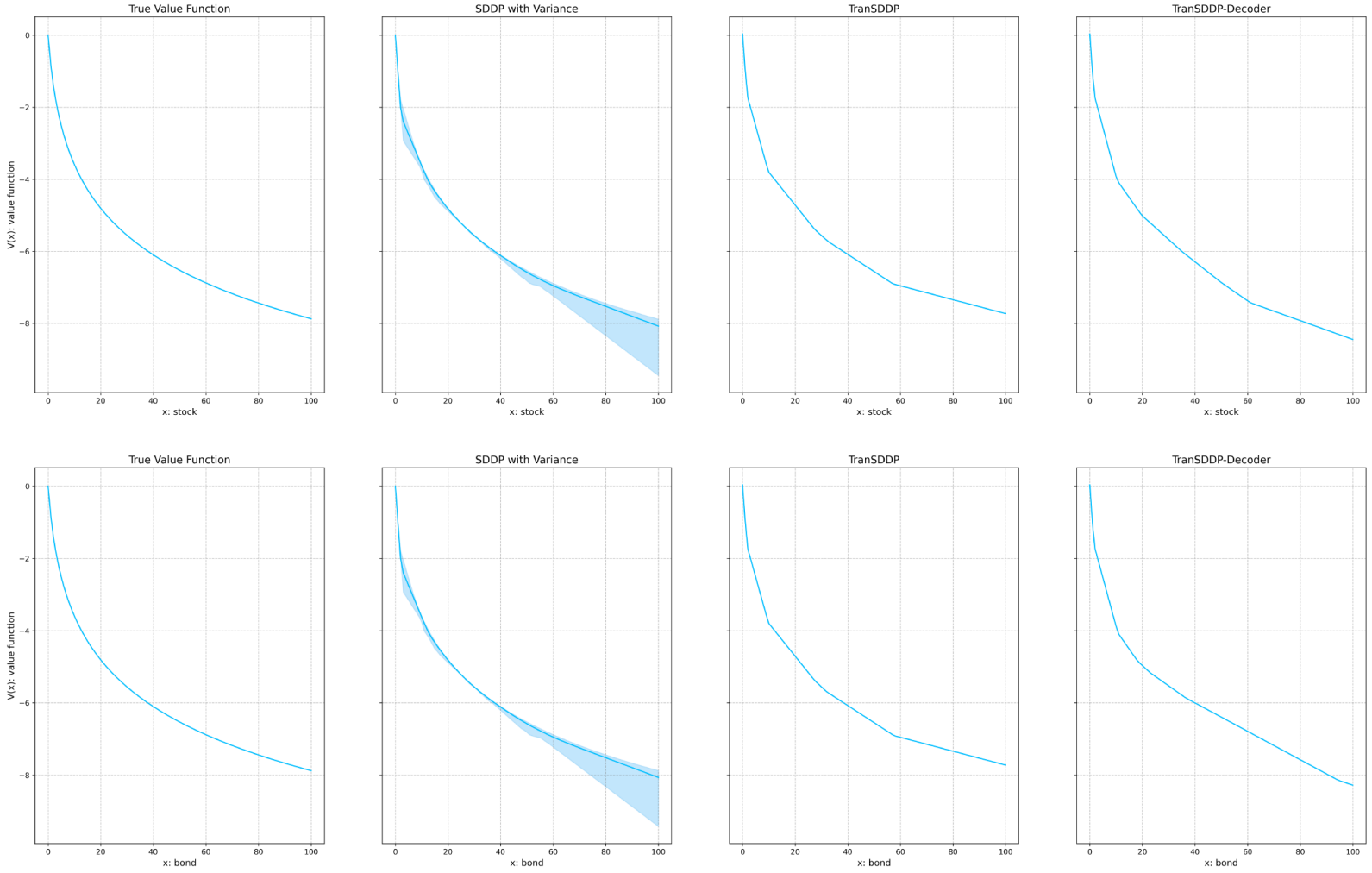


Figure 11. Comparison of the value function and its approximations for financial planning problem

Value Function Comparison: Production Planning

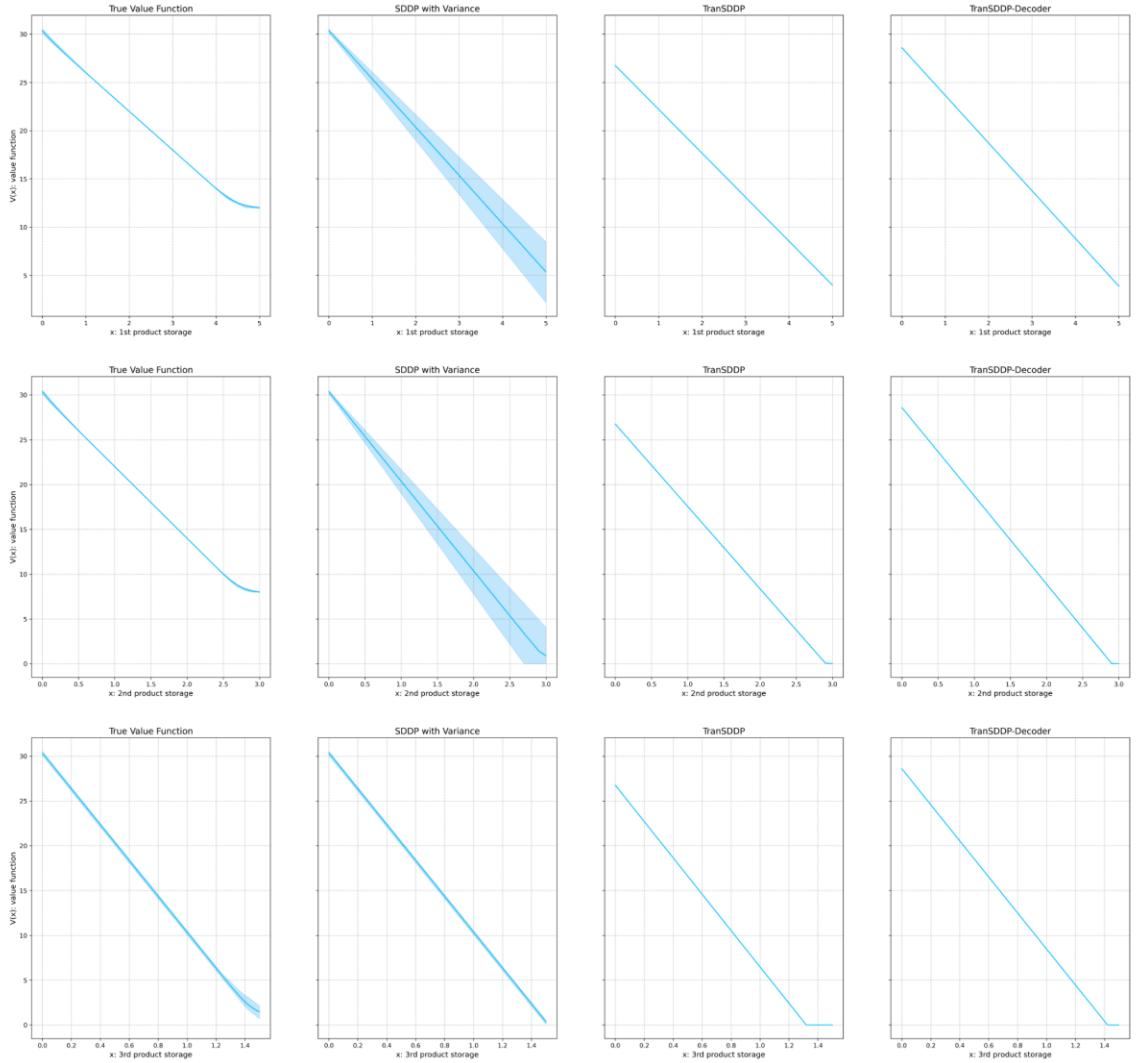


Figure 14. Comparison of the value function and its approximations for production planning problem