Towards Theoretical Understanding of Inverse Reinforcement Learning

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Research Question

How many samples are needed to accurately solve the Inverse Reinforcement Learning (IRL) problem with high probability?

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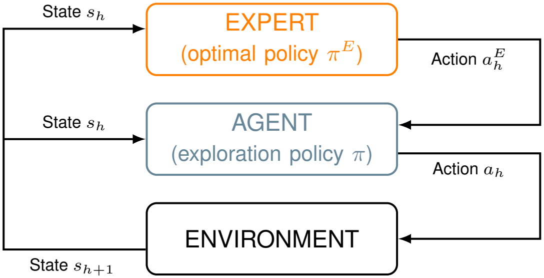
Sample Complexity **Lower Bound** for IRL

Inverse Reinforcement Learning (IRL)

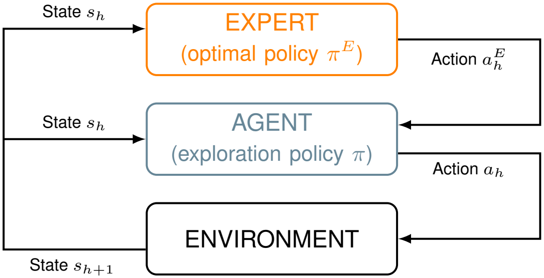


- At every stage $h \in [\![H]\!]$:
  - Observe state s_h
  - Observe **expert action** $a_h^E \sim \pi_h^E(\cdot|s_h)$
  - Play **exploratory action** $a_h \sim \pi_h(\cdot|s_h)$
  - Transition to next state $s_{h+1} \sim p_h(\cdot|s_h, a_h)$

- Traditional Goal of IRL (Arora and Doshi, 2021; Adams et al., 2022)

Find **one feasible reward function** r^* that makes the **expert's policy** π^E **optimal**, i.e.,

$$\pi^E \in \arg\max_{\pi} V^{\pi}(\cdot; r^*)$$



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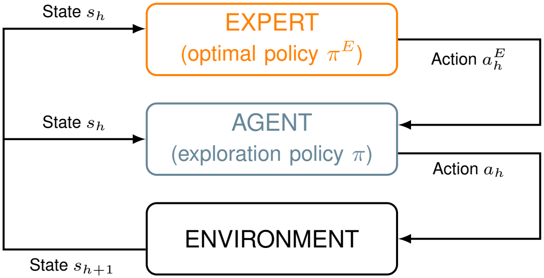

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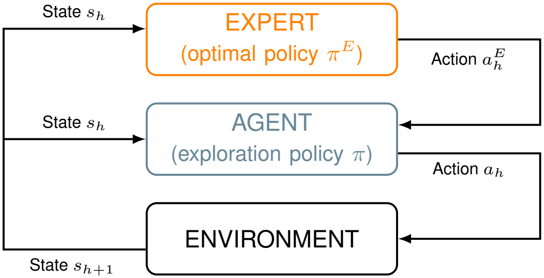

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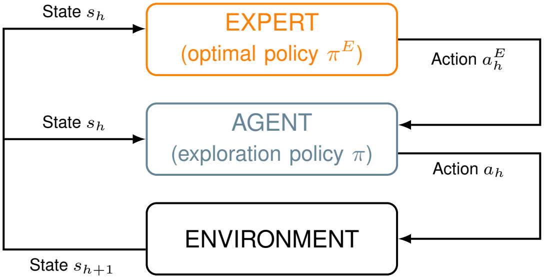

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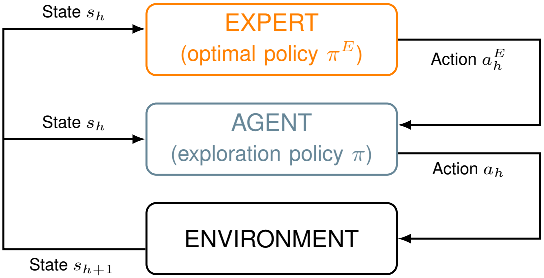

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Feasible Reward Set

- **Ambiguity** problem (Ng and Russell, 2000)
- Study the **full** set of feasible rewards r^* → **Feasible Reward Set** (Metelli et al., 2021)

Find **all feasible reward functions** $\mathcal{R}$ that make the **expert's policy** π^E **optimal**, i.e.,

$$\mathcal{R} := \left\{ \text{all rewards } r^* \ : \ \pi^E \in \arg\max_{\pi} V^{\pi}(\cdot; r^*) \right\}$$

- $\mathcal{R}$ defined through **linear constraints**
- If p and π^E are **known**, check if $\widehat{r}$ is feasible takes $O(HS^2A)$

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- Transition model p and expert's policy π^E **unknown**
- Estimate $\widehat{p}$ and $\widehat{\pi}^E$ with **samples** inducing $\widehat{\mathcal{R}}$
- **Hausdorff distance** between $\mathcal{R}$ (true) and $\widehat{\mathcal{R}}$ (estimated) feasible reward sets

$$\mathcal{H}_d\left(\mathcal{R}, \widehat{\mathcal{R}}\right) = \max\left\{ \sup_{r \in \mathcal{R}} \inf_{\widehat{r} \in \widehat{\mathcal{R}}} d(r, \widehat{r}), \sup_{\widehat{r} \in \widehat{\mathcal{R}}} \inf_{r \in \mathcal{R}} d(r, \widehat{r}), \right\}$$

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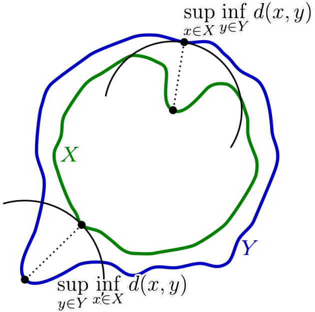

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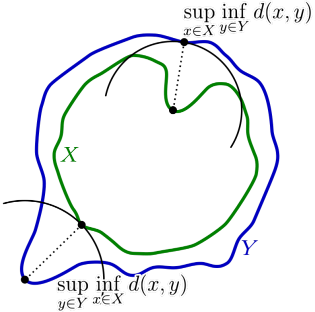

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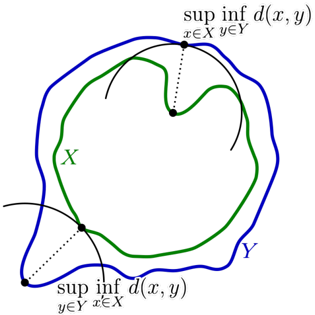

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- An algorithm $\mathfrak{A}$ that outputs $\widehat{\mathcal{R}}_\tau$ after τ calls to the environment is (ϵ, δ)-PAC if

$$\mathbb{P}\left(\mathcal{H}_d\left(\mathcal{R}, \widehat{\mathcal{R}}_\tau\right) > \epsilon\right) \leqslant \delta$$

$$\tau = \text{sample complexity}$$

- **Sample complexity lower bound**

$$\tau \geqslant \text{poly}\left(S, A, H, \frac{1}{\epsilon}, \log\left(\frac{1}{\delta}\right)\right)$$

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Main Result: Lower Bound

Theorem

For any (ϵ, δ)-PAC algorithm $\mathfrak{A}$, with ϵ and δ sufficiently small, there exists an IRL problem, with S, A and H sufficiently large, such that the **expected sample complexity** *is lower bounded by:*

- *if the transition model p is time-inhomogeneous (i.e., $p_h \neq p_{h+1}$):*

$$\mathbb{E}\left[\tau\right] \geqslant \Omega\left(\frac{H^3 SA}{\epsilon^2}\left(\log\left(\frac{1}{\delta}\right) + S\right)\right);$$

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Proof Sketch

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Two regimes of δ → **Small-δ regime**

- Expert's policy $\pi^E(s) = a_0$
- **Hard** to identify which action behaves like a_*
- Construct $\Theta(A)$ hard instances
- **Technical tool**: Bretagnolle-Huber inequality (Lattimore and Szepesvári, 2020)

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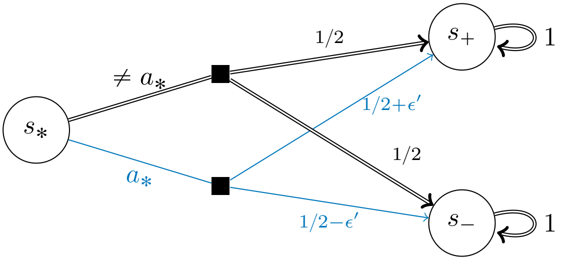

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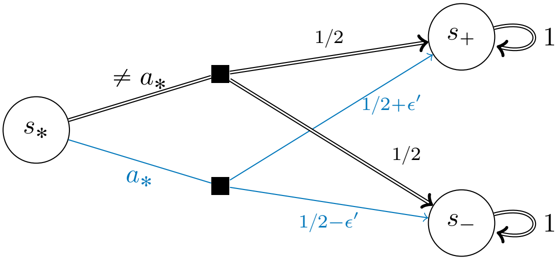

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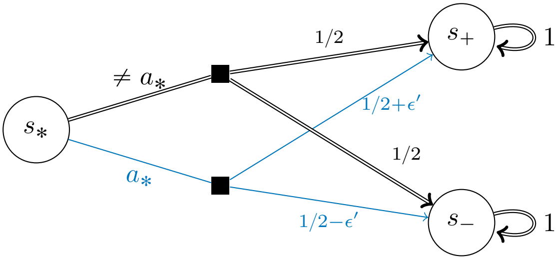

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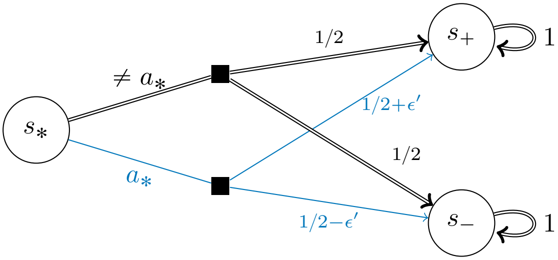

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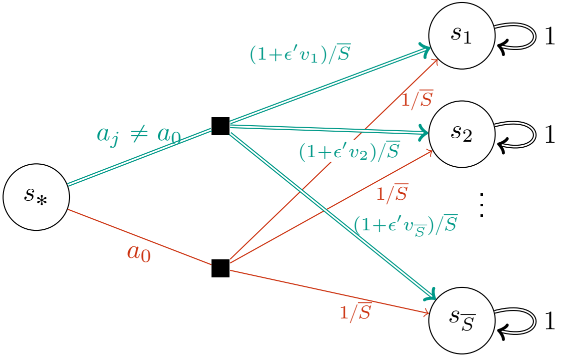

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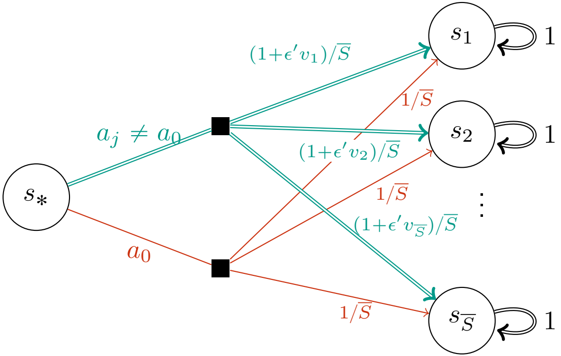

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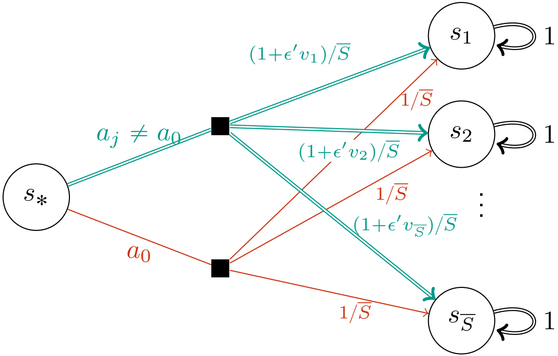

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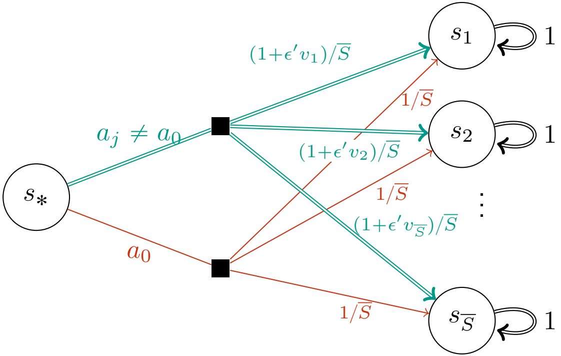

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Thank You for Your Attention!

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Link: https://icml.cc/virtual/2023/poster/24193

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