LookupFFN

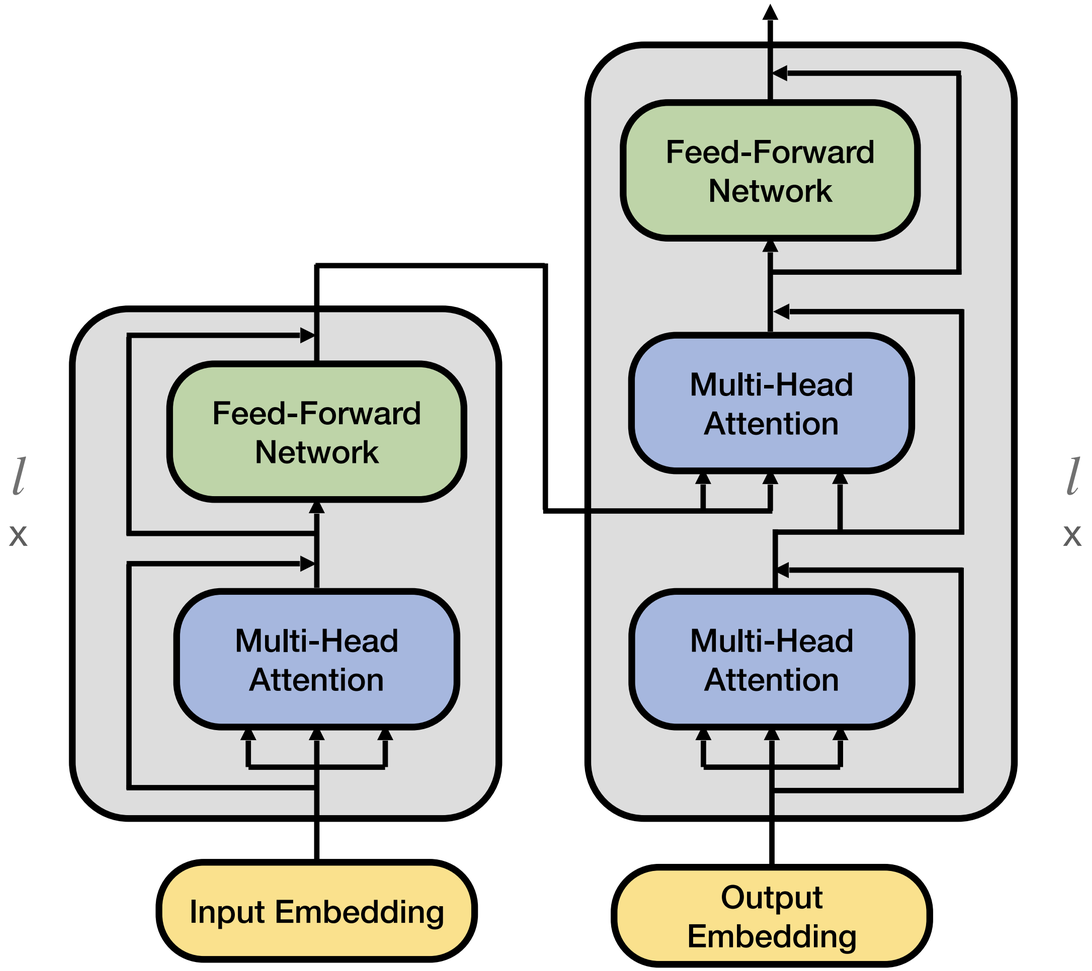
Making Transformers Compute-lite for CPU inference

Zhanpeng Zeng Michael Davies Pranav Pulijala Karthikeyan Sankaralingam Vikas Singh

Motivation

Transformer

Transformers are extremely effective on LLM and visions, but very compute intensive.



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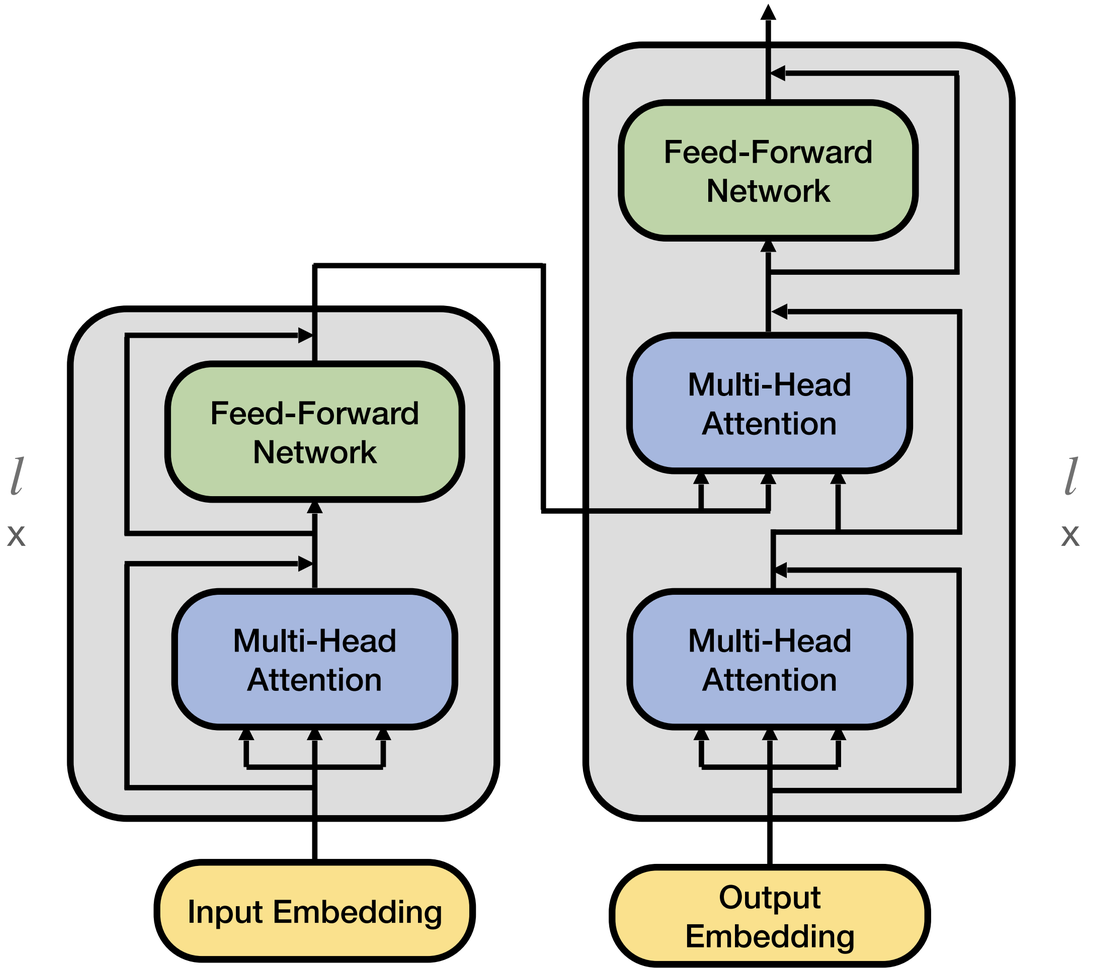
How can we make it compute-lite?



Motivation

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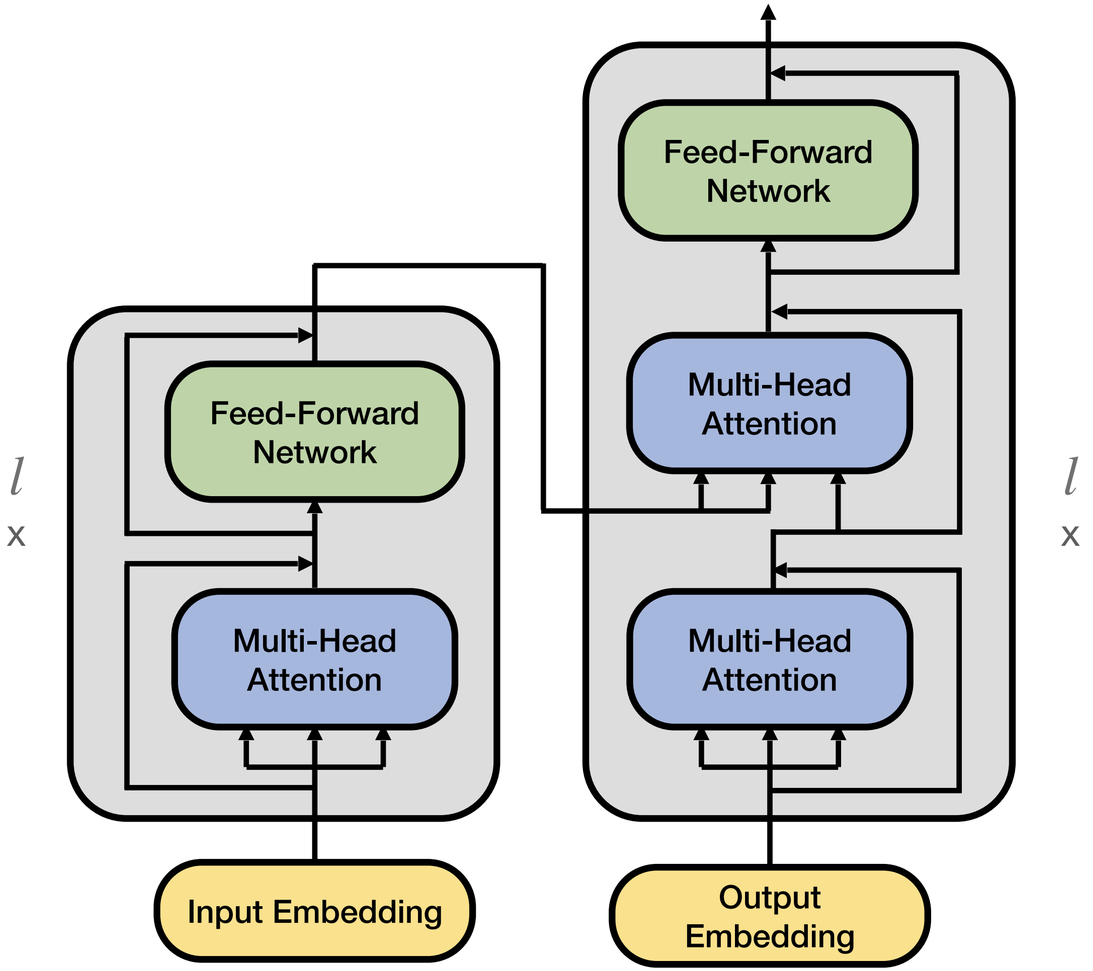
Transformers are extremely effective on LLM and visions, but very compute intensive.

How can we make it compute-lite?

Can we almost eliminate matrix multiplications?

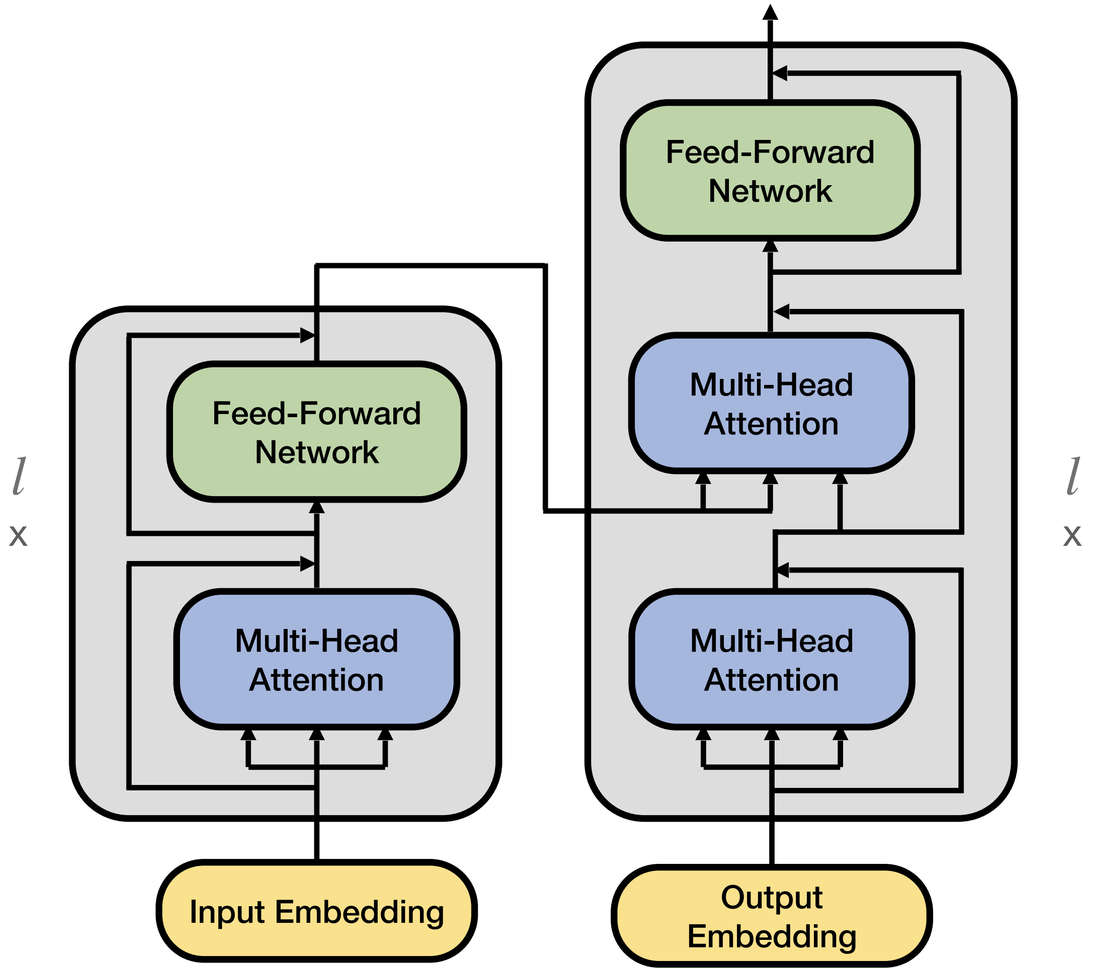


Motivation

Transformer

$$Y = \mathscr{F}(A) + A$$

$$A = \mathscr{G}(X) + X$$



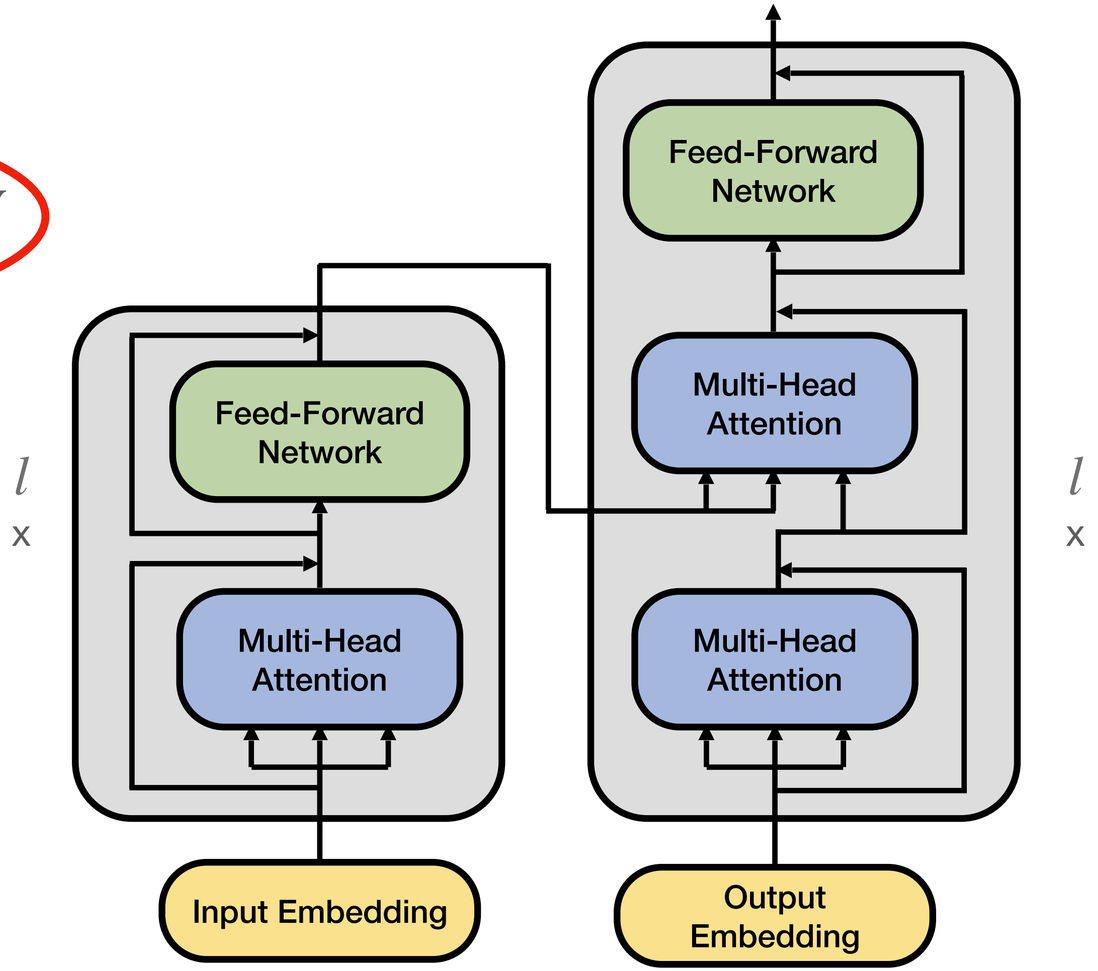
Motivation

Transformer

$$\mathscr{F}(x) = \sigma(xW^\top)V$$

$$Y = \mathscr{F}(A) + A$$

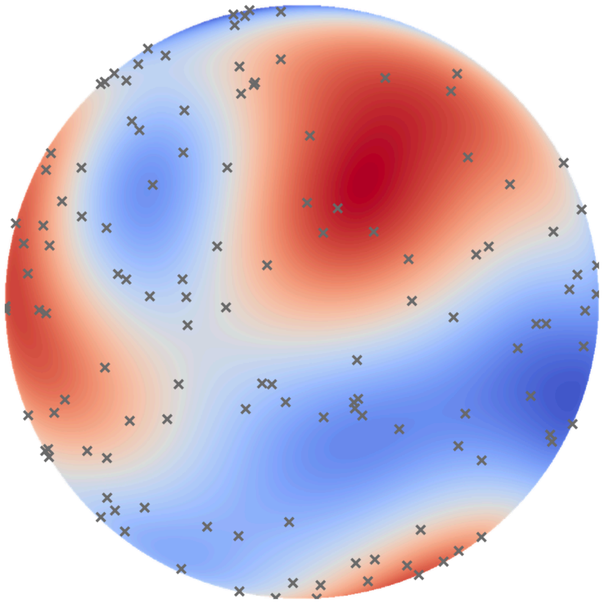
$$A = \mathscr{G}(X) + X$$



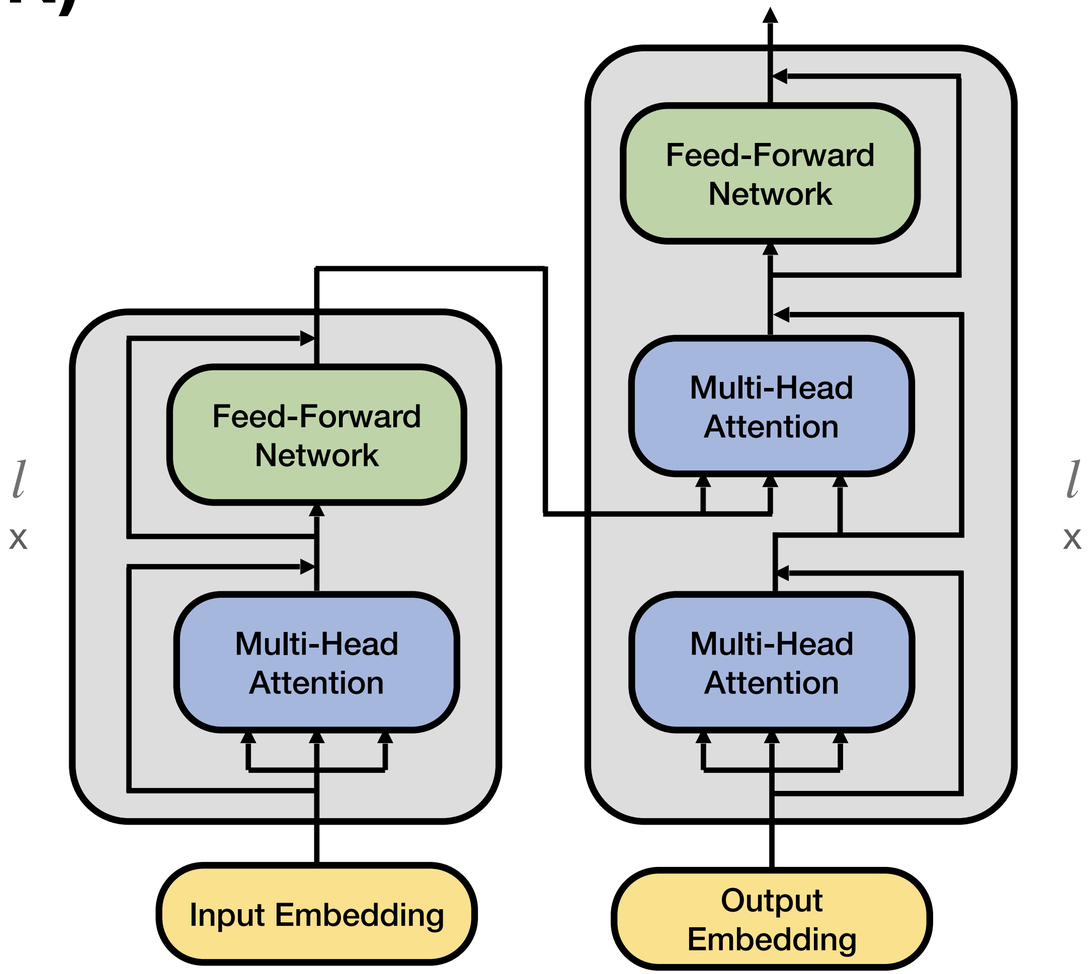
Motivation

Two Layer Feed-Forward Network (FFN)

$$\mathcal{F}(\cdot) = \sigma(\cdot W^\top)V = \sum_{i=1}^{t} \sigma(\langle \cdot, [W]_i \rangle)[V]_i$$



True $\mathcal{F}$



Motivation

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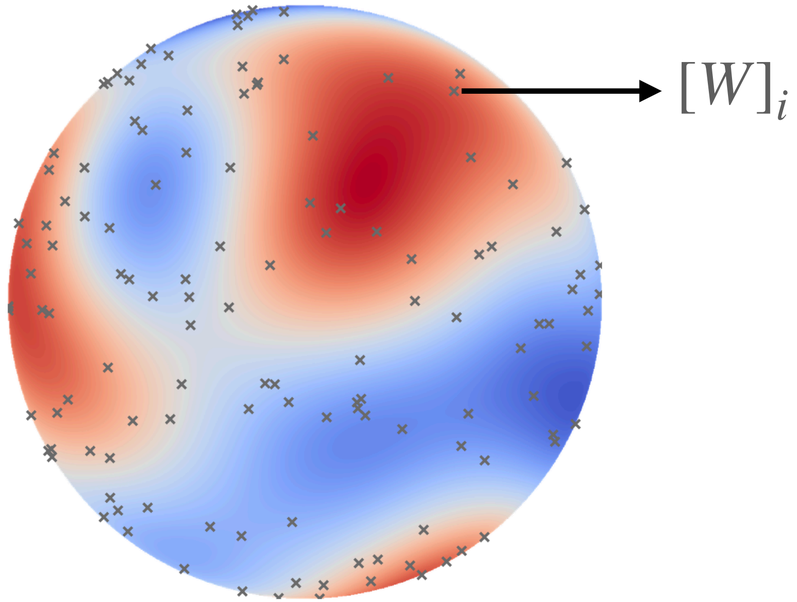

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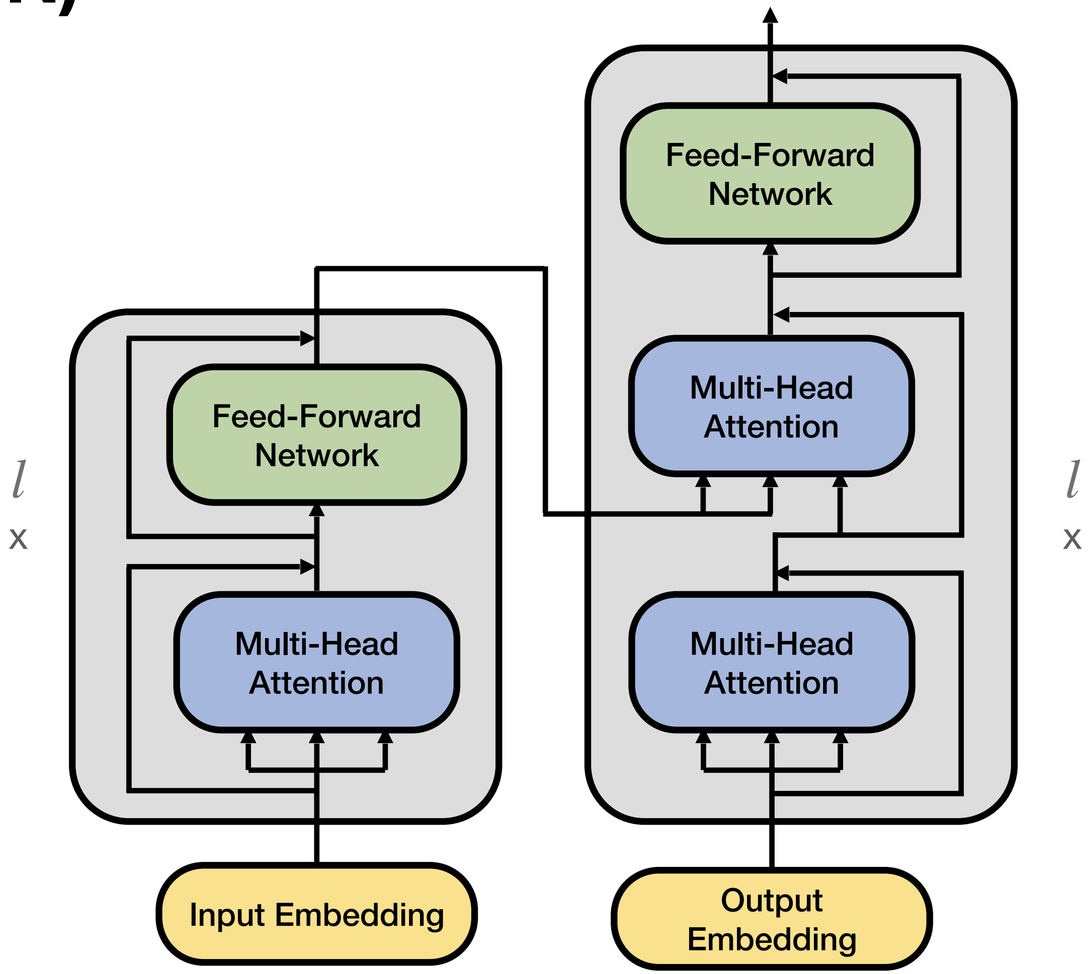
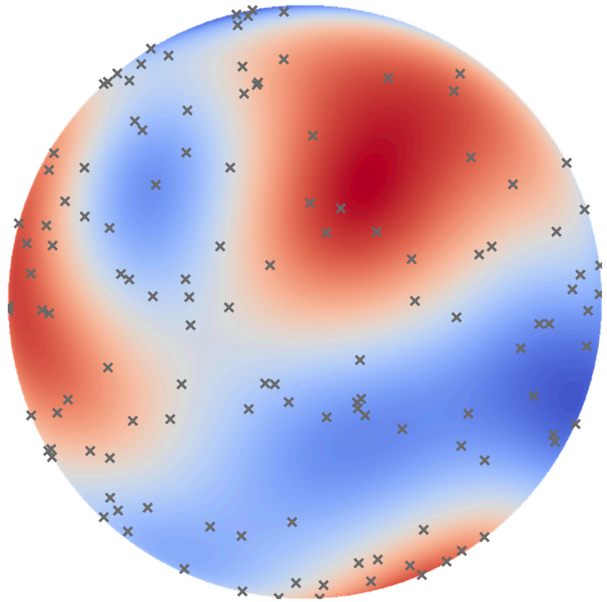


Making FFN Efficient

Revisiting LSH for approximating FFN

$$\mathcal{F}(\cdot) = \sum_{i=1}^{t} \sigma(\langle \cdot, [W]_i \rangle)[V]_i$$



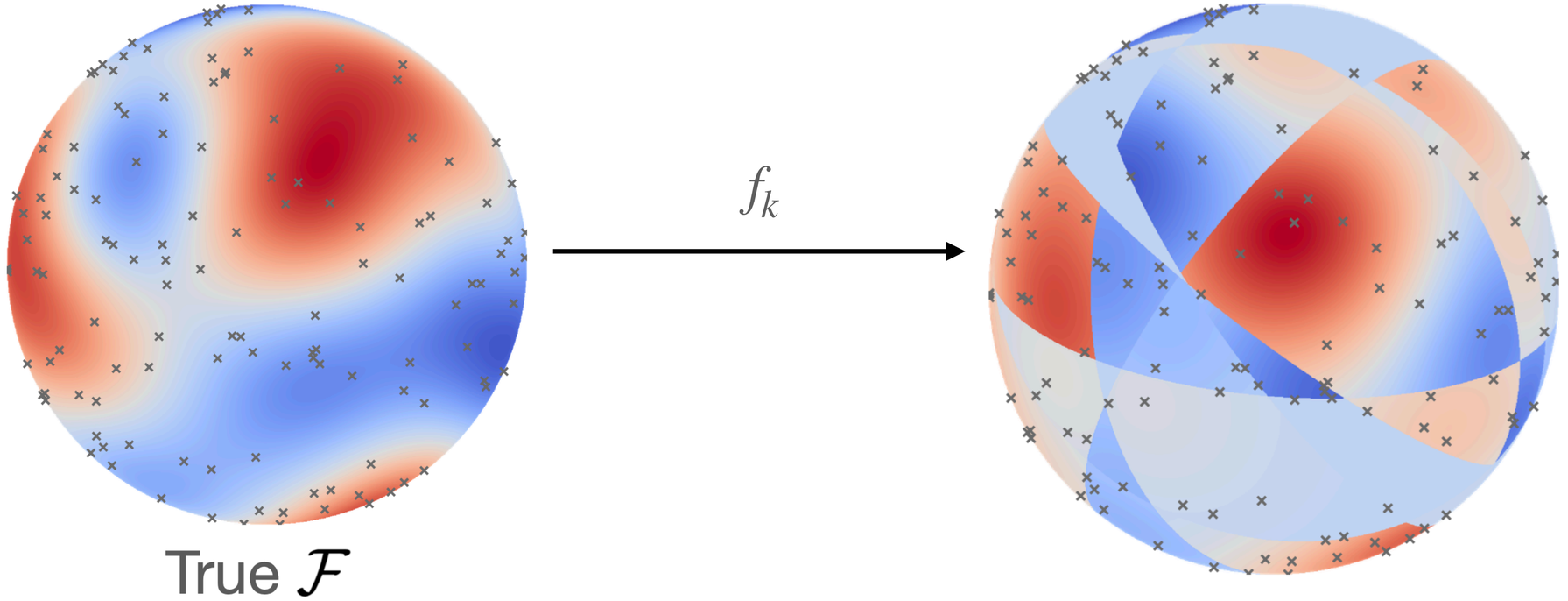
True $\mathcal{F}$

Making FFN Efficient

Revisiting LSH for approximating FFN

$$\mathcal{F}(\cdot) = \sum_{i=1}^{t} \sigma(\langle \cdot, [W]_i \rangle)[V]_i$$

$$\mathcal{F}(\cdot) = \sum_{i \in S(\cdot)} \sigma(\langle \cdot, [W]_i \rangle)[V]_i$$

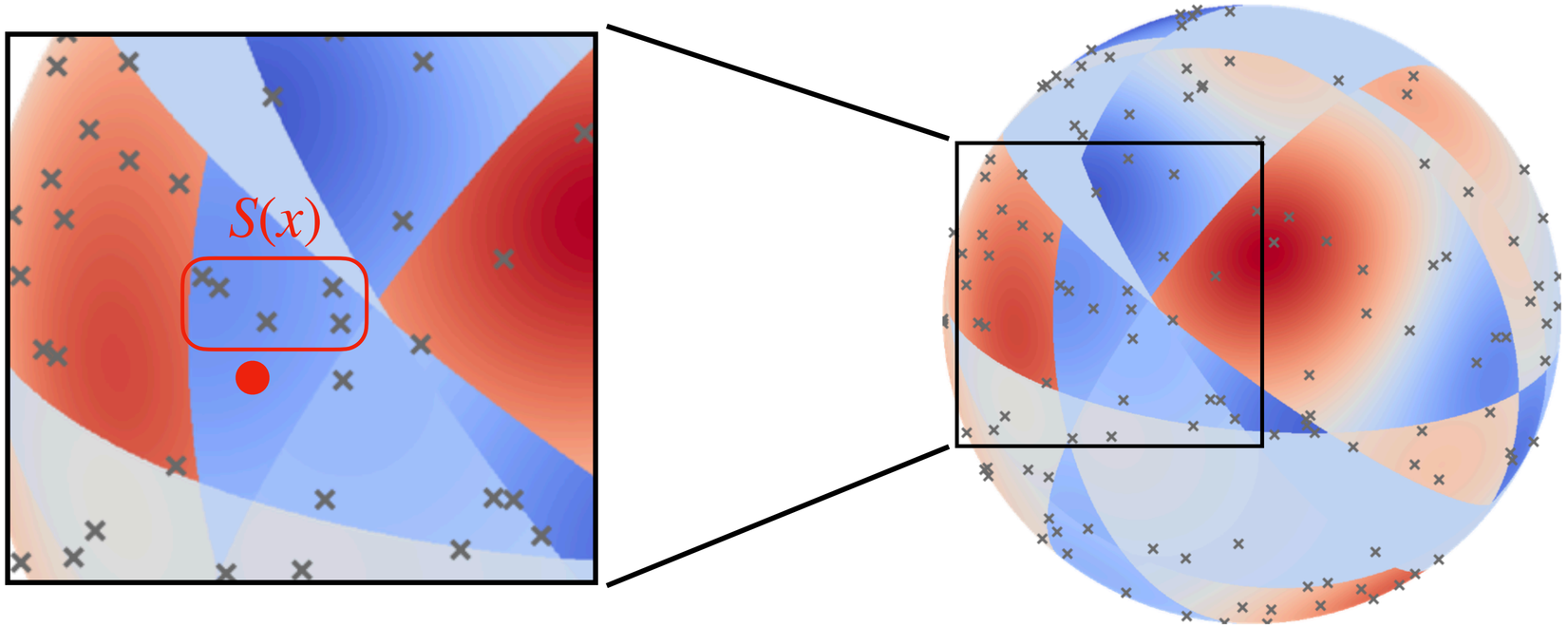


Chen, B., Medini, T., Farwell, J., gobriel, s., Tai, C., and Shrivastava, A. Slide: In defense of smart algorithms over hardware acceleration for large-scale deep learning systems. MLSys 2020

Making FFN Efficient

Revisiting LSH for approximating FFN

$$\mathcal{F}(x) = \sum_{i \in S(x)} \sigma(\langle x, [W]_i \rangle)[V]_i \qquad \mathcal{F}(\cdot) = \sum_{i \in S(\cdot)} \sigma(\langle \cdot, [W]_i \rangle)[V]_i$$



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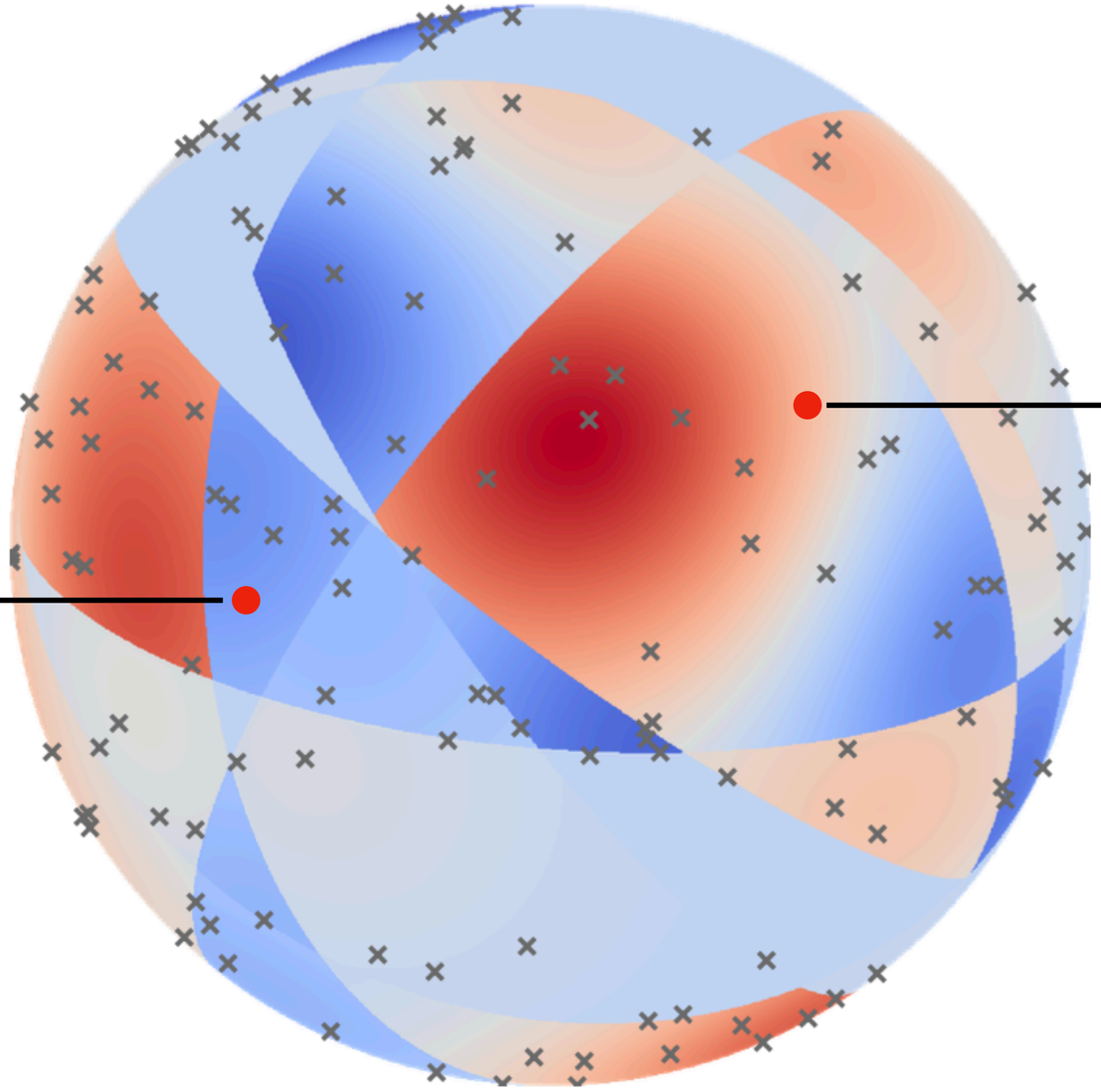
Making FFN Efficient

Revisiting LSH for approximating FFN: Challenge

$$\mathcal{F}(x) = \sum_{i \in S(x)} \sigma(\langle x, [W]_i \rangle)[V]_i$$

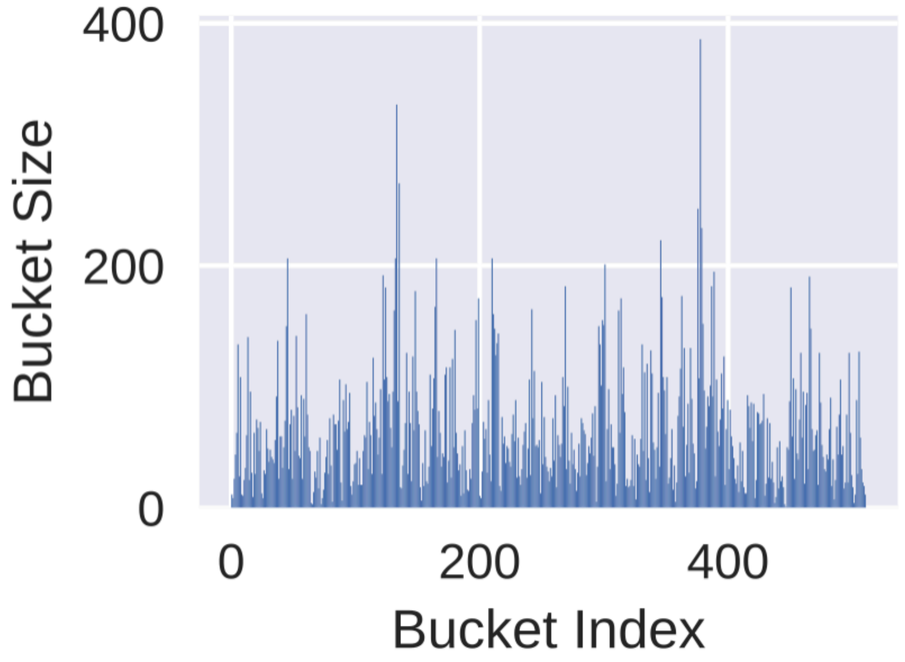


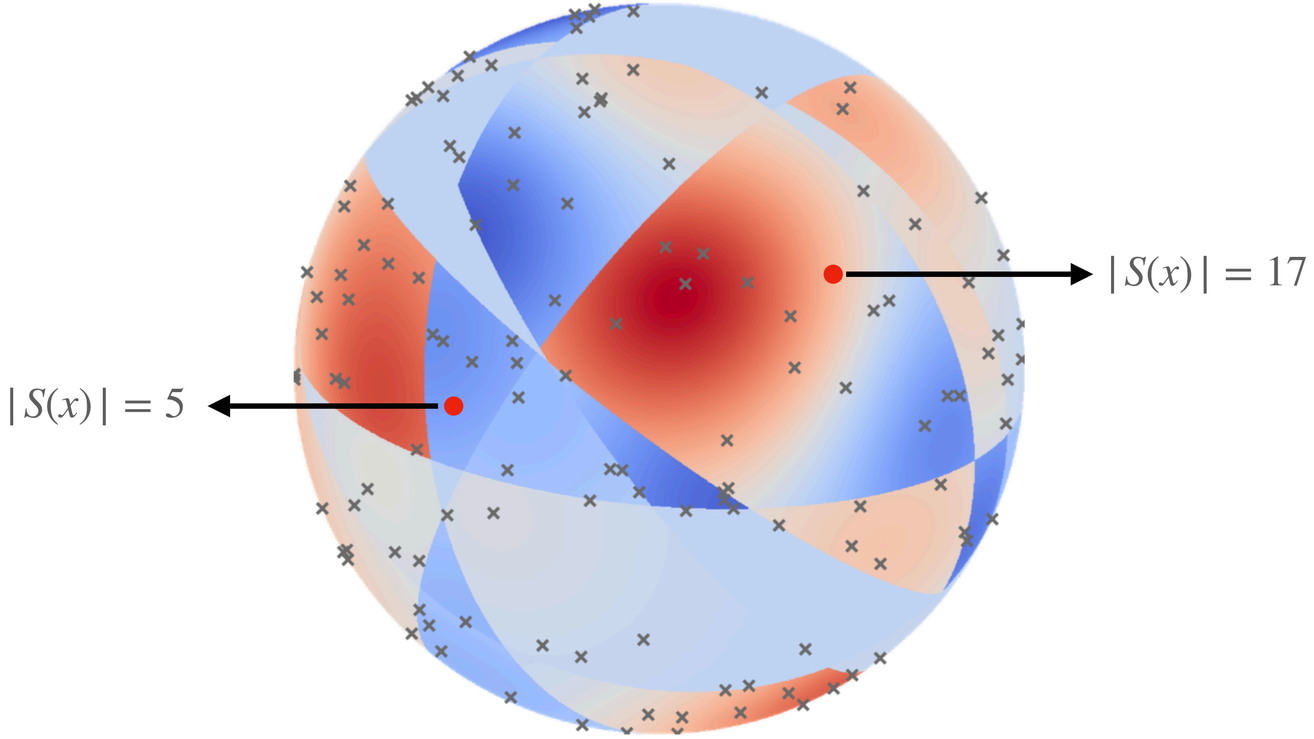
$|S(x)| = 17$

$|S(x)| = 5$

Making FFN Efficient

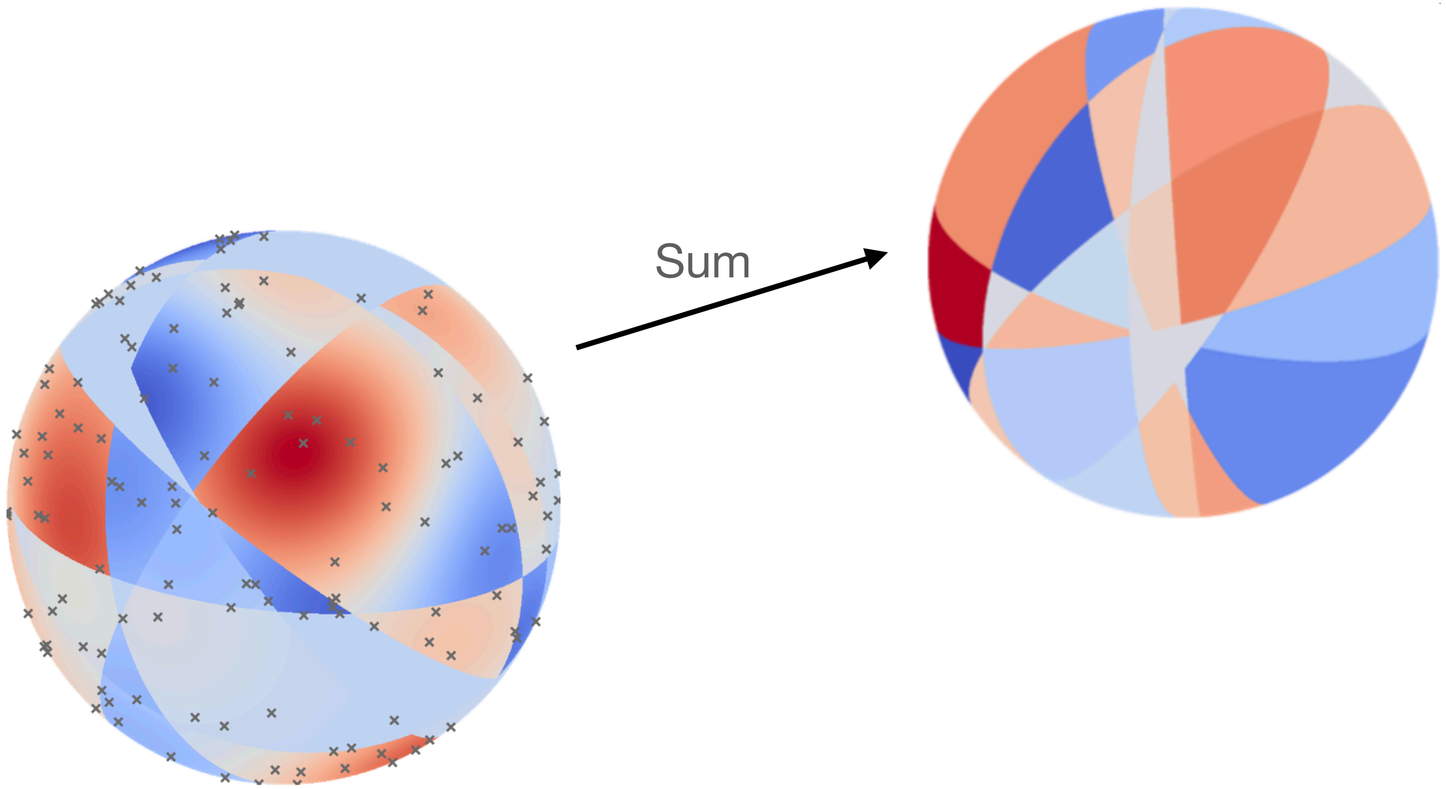
Revisiting LSH for approximating FFN: Challenge





Making FFN Efficient

Revisiting LSH for approximating FFN: Potential Solution



Zeng, Z., Xiong, Y., Ravi, S., Acharya, S., Fung, G. M., and Singh, V. You only sample (almost) once: Linear cost self-attention via bernoulli sampling. ICML 2021.

Making FFN Efficient

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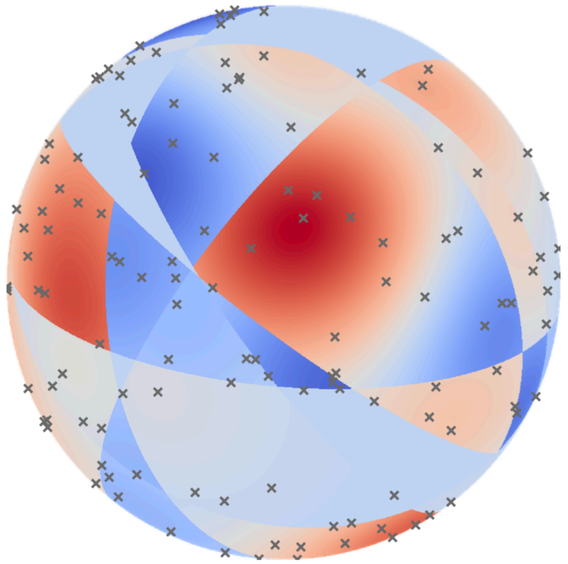

Sum

$$[T_k]_j = \sum_{f_k([W]_i)=j} [V]_i$$

Code	Hash Table
0	v4 + v6
1	v2 + v7
2	v3
3	v0 + v1 + v5

V	W	Code
v0	w0	3
v1	w1	3
v2	w2	1
v3	w3	2
v4	w4	0
v5	w5	3
v6	w6	0
v7	w7	1

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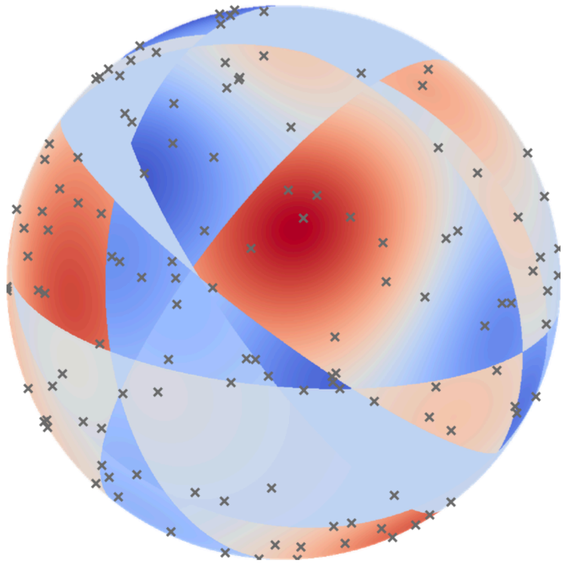

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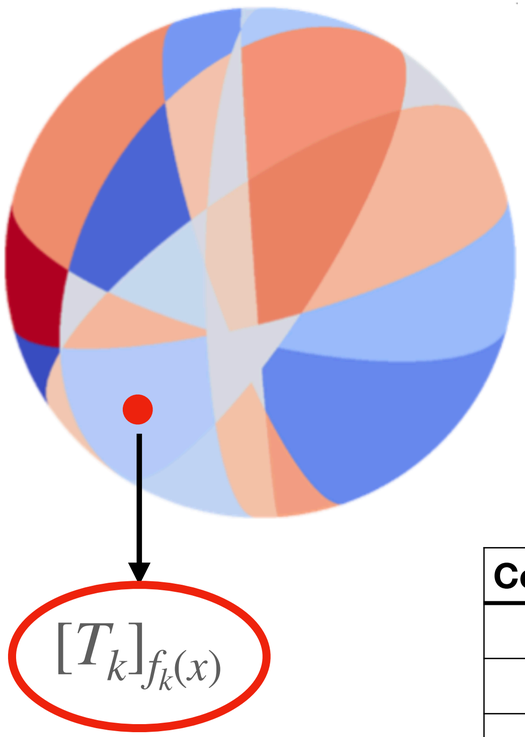

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Making FFN Efficient

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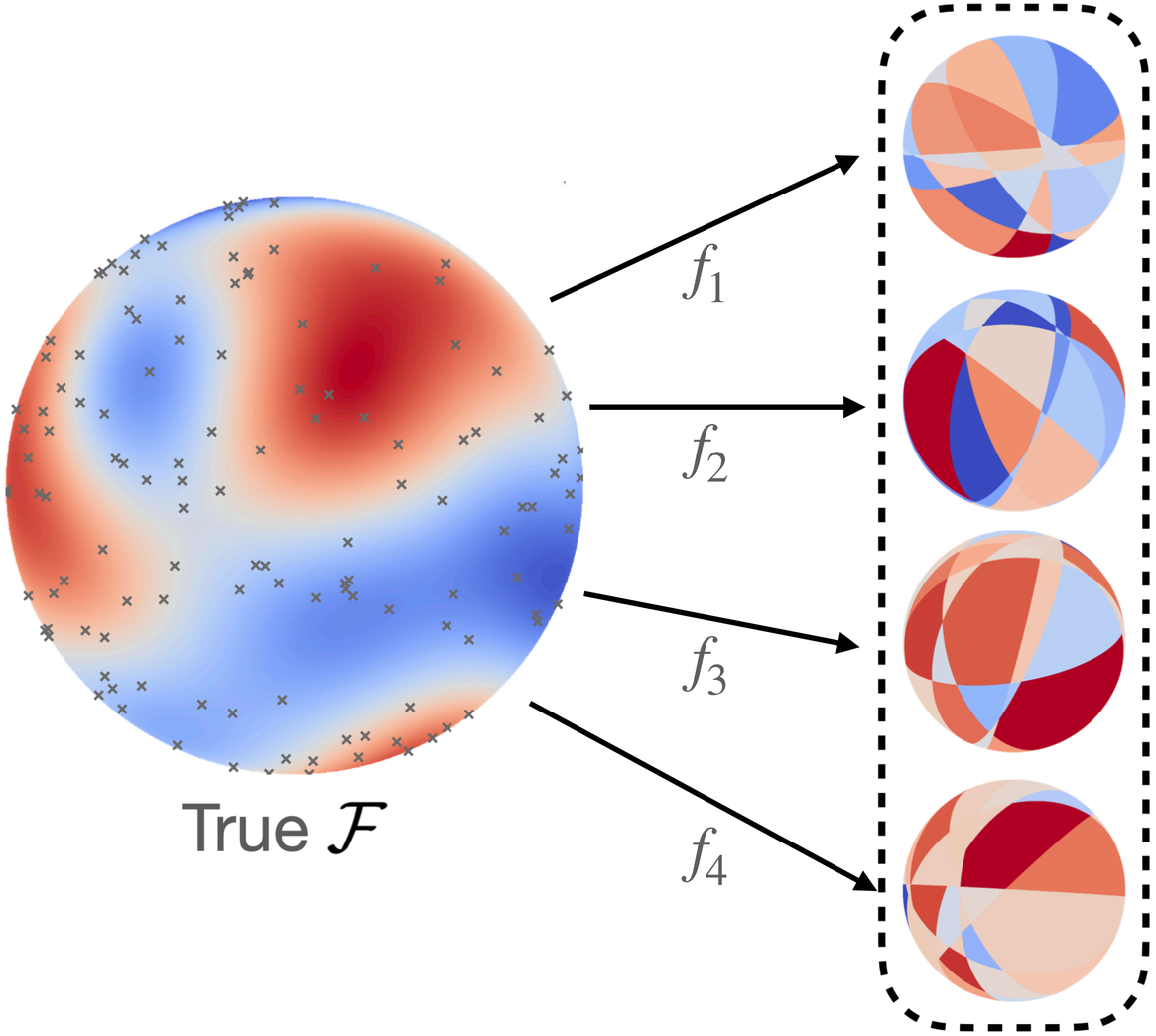

Making FFN Efficient

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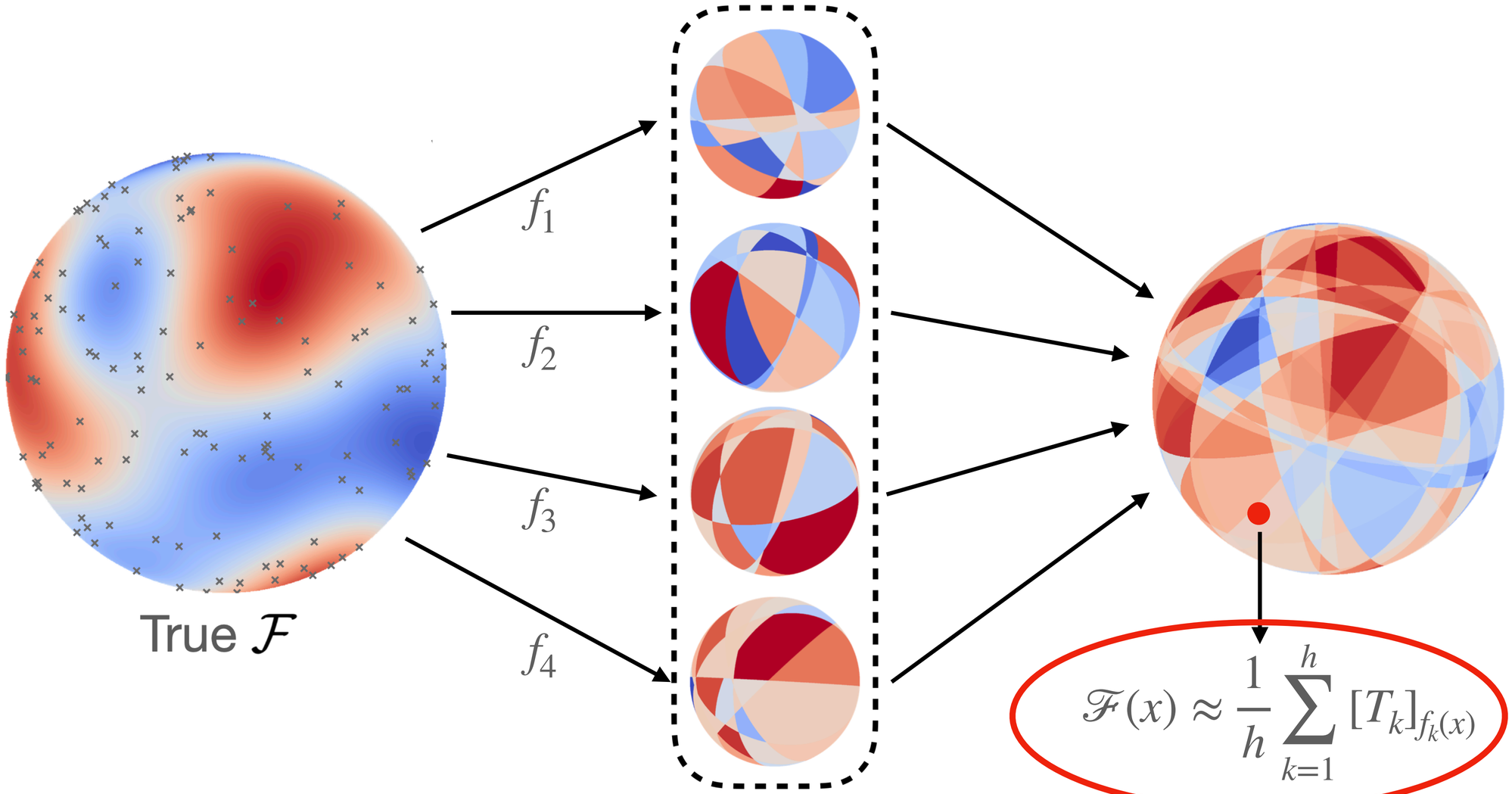
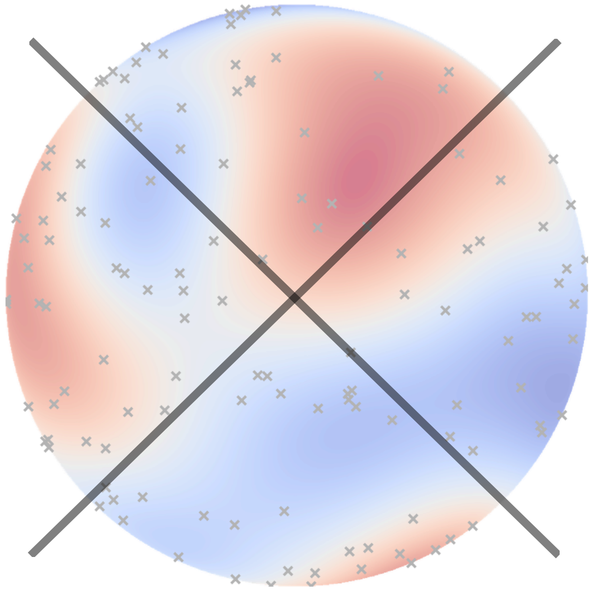
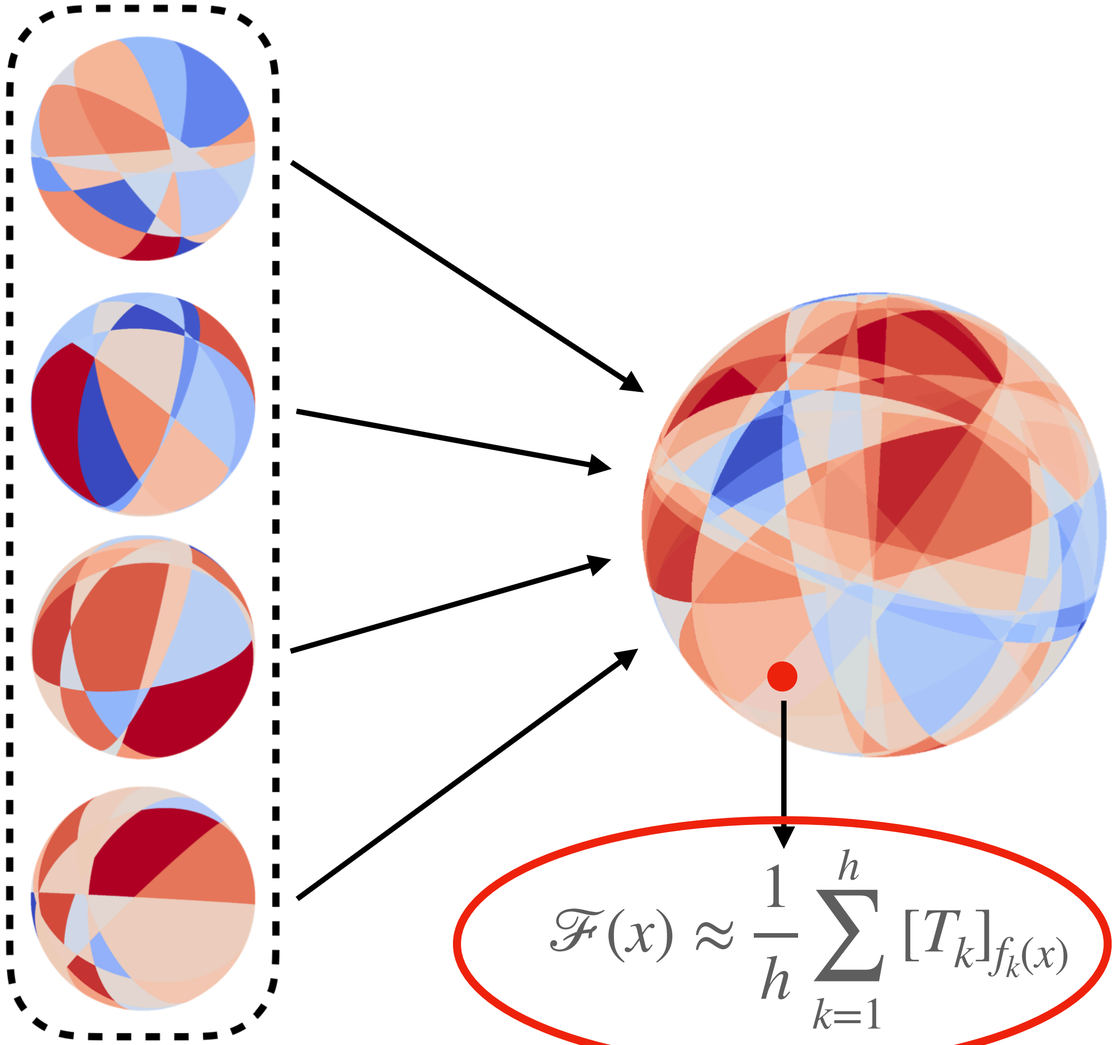


Making FFN Efficient

LookupFFN: Breaking dependency on True F



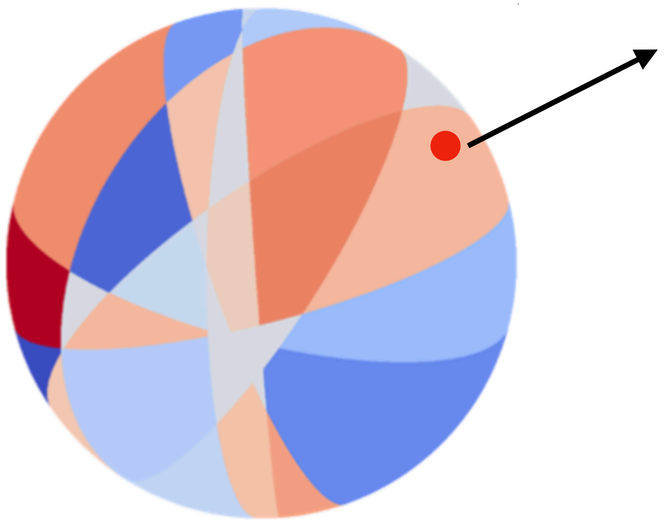
True $\mathcal{F}$



$$\mathcal{F}(x) \approx \frac{1}{h} \sum_{k=1}^{h} [T_k]_{f_k(x)}$$

Making FFN Efficient

LookupFFN: Make it differentiable



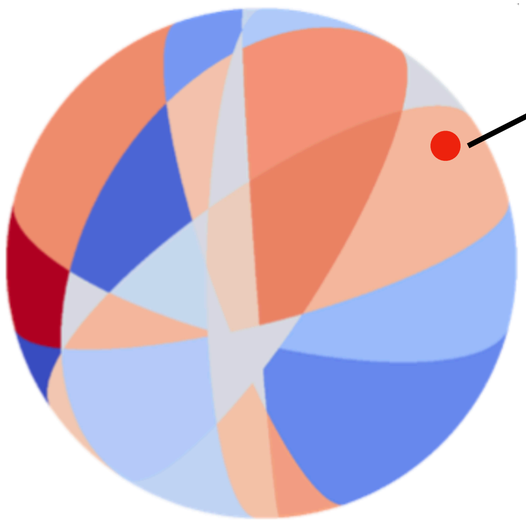
$[T_k]_{f_k(x)}$

$z_k = xR_k$

$f_k(x) = \text{decimal}(\text{sign}(z_k))$

Making FFN Efficient

LookupFFN: Make it differentiable



$[T_k]_{f_k(x)}$

$z_k = xR_k$

$$f_k(x) = \text{decimal}(\text{sign}(z_k))$$

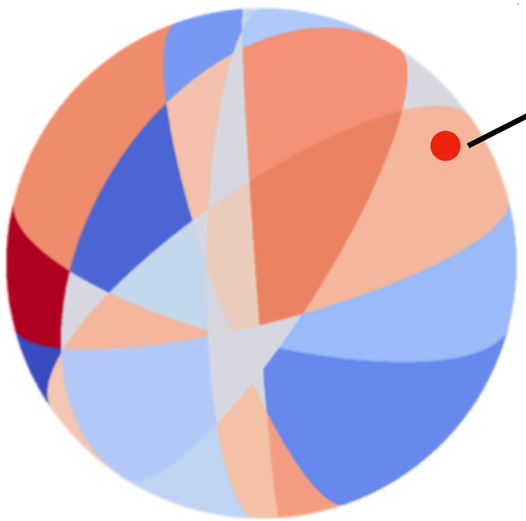
Equivalent

$$f_k(x) = \arg\max_i(\langle z_k, [S]_i \rangle)$$

-1	-1	-1
-1	-1	1
-1	1	-1
-1	1	1
1	-1	-1
1	-1	1
1	1	-1
1	1	1

Making FFN Efficient

LookupFFN: Make it differentiable



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Differentiable Relaxation

$$\frac{\sum_{i=1}^{2^\tau} \exp(\langle z_k, [S]_i \rangle)[T_k]_i}{\sum_{j=1}^{2^\tau} \exp(\langle z_k, [S]_j \rangle)}$$

-1	-1	-1
-1	-1	1
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-1	1	1
1	-1	-1
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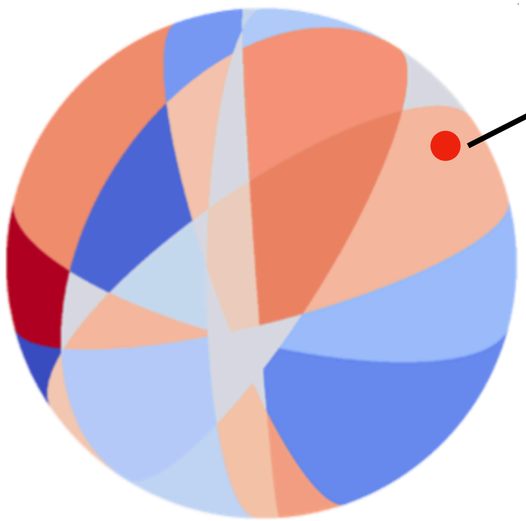
1	1	-1
1	1	1

Making FFN Efficient

LookupFFN: Make it differentiable



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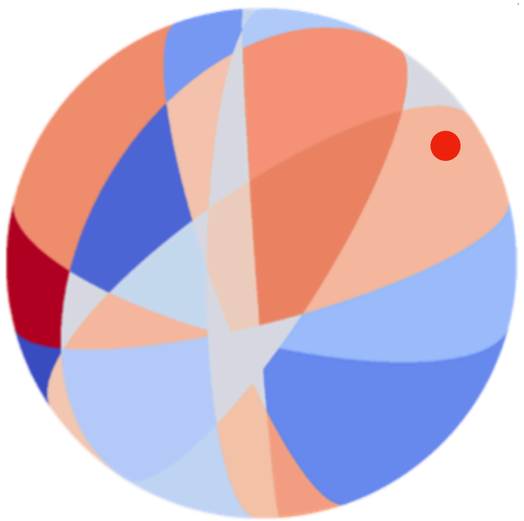
-1	-1	1
-1	1	-1
-1	1	1
1	-1	-1
1	-1	1
1	1	-1
1	1	1

Making FFN Efficient

LookupFFN: Efficient Approximation



$$z_k = xR_k$$

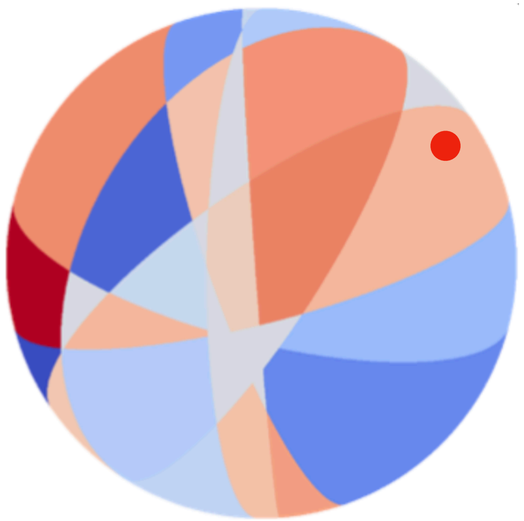
$$\frac{\sum_{i=1}^{2^\tau} \exp(\langle z_k, [S]_i \rangle)[T_k]_i}{\sum_{j=1}^{2^\tau} \exp(\langle z_k, [S]_j \rangle)}$$

Equivalent

$$\prod_{j=1}^{\tau} (\exp([z_k]_j) + \exp(-[z_k]_j))$$

Making FFN Efficient

LookupFFN: Efficient Approximation



$$z_k = xR_k$$

Approximate

$$\sum_{i \in N(xR_k)} \exp(\langle z_k, [S]_i \rangle)[T_k]_i$$

$$\frac{\sum_{i=1}^{2^\tau} \exp(\langle z_k, [S]_i \rangle)[T_k]_i}{\sum_{j=1}^{2^\tau} \exp(\langle z_k, [S]_j \rangle)}$$

Equivalent

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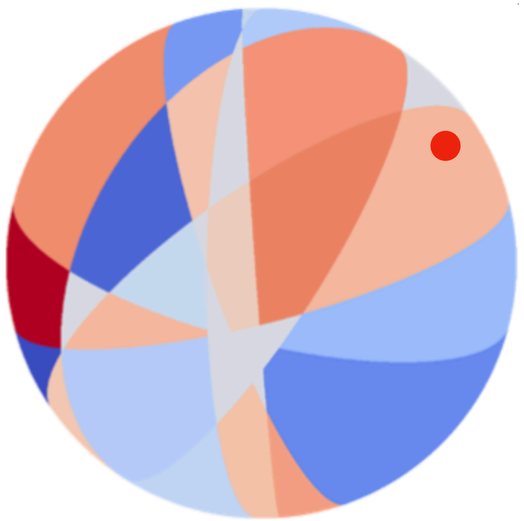
Making FFN Efficient

LookupFFN: Efficient Approximation



$$z_k = xR_k$$

$$\arg\max_i \langle z_k, [S]_i \rangle = \text{decimal}(\text{sign}(z_k))$$

Approximate

$$\sum_{i \in N(xR_k)} \exp(\langle z_k, [S]_i \rangle)[T_k]_i$$

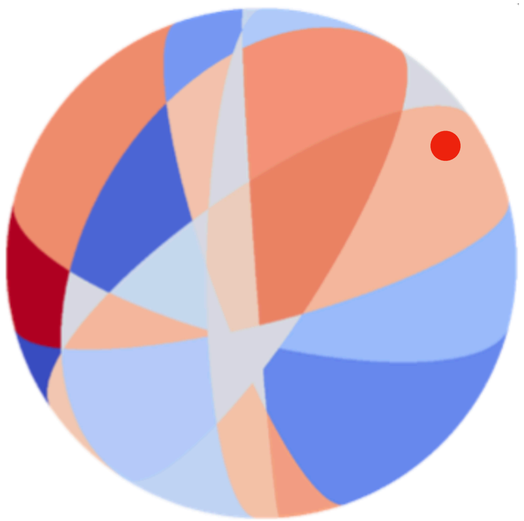
$$\frac{\sum_{i=1}^{2^\tau} \exp(\langle z_k, [S]_i \rangle)[T_k]_i}{\sum_{j=1}^{2^\tau} \exp(\langle z_k, [S]_j \rangle)}$$

Equivalent

$$\prod_{j=1}^{\tau} (\exp([z_k]_j) + \exp(-[z_k]_j))$$

Making FFN Efficient

LookupFFN: Put it together



$$\frac{\sum_{i=1}^{2^\tau} \exp(\langle z_k, [S]_i \rangle)[T_k]_i}{\sum_{j=1}^{2^\tau} \exp(\langle z_k, [S]_j \rangle)}$$

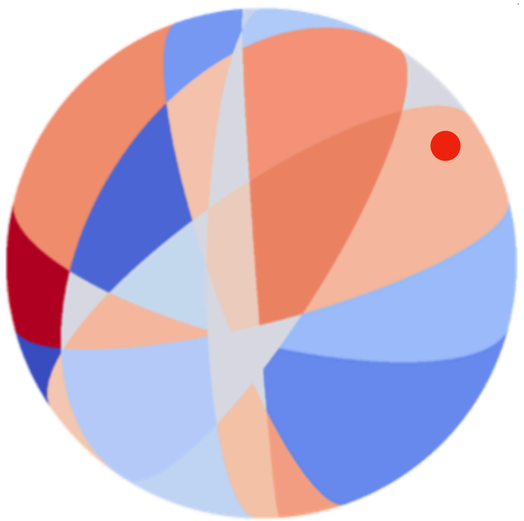
$$\downarrow$$

$$\frac{\exp(\langle z_k, [S]_{f_k(x)} \rangle)[T_k]_{f_k(x)}}{\prod_{j=1}^{\tau} (\exp([z_k]_j) + \exp(-[z_k]_j))}$$

$$z_k = xR_k$$

Making FFN Efficient

LookupFFN: Put it together



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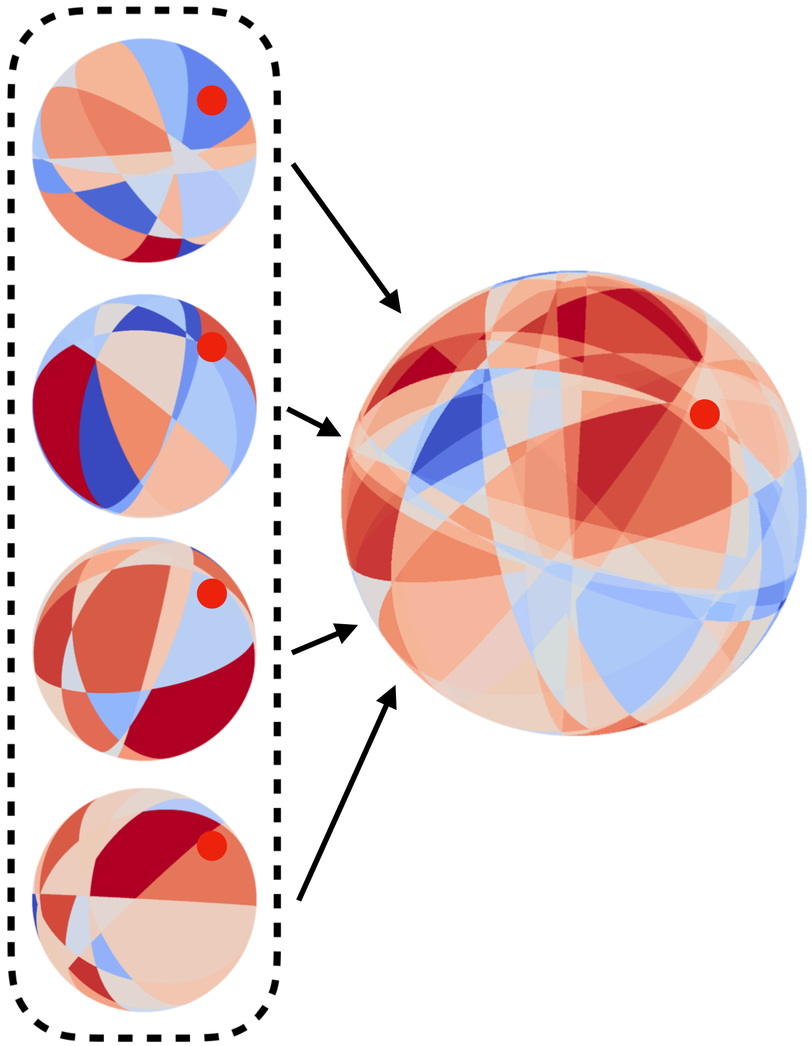
$$z_k = xR_k$$

$$\frac{\exp(\langle z_k, [S]_{f_k(x)} \rangle)[T_k]_{f_k(x)}}{\prod_{j=1}^{\tau} (\exp([z_k]_j) + \exp(-[z_k]_j))} \longrightarrow f_k(x) = \text{decimal}(\text{sign}(z_k))$$

Making FFN Efficient

LookupFFN: Put it together



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$$z_k = xR_k$$

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$$\frac{1}{h} \sum_{k=1}^{h} \frac{\exp(\langle z_k, [S]_{f_k(x)} \rangle)[T_k]_{f_k(x)}}{\prod_{j=1}^{\tau} (\exp([z_k]_j) + \exp(-[z_k]_j))}$$

Making FFN Efficient

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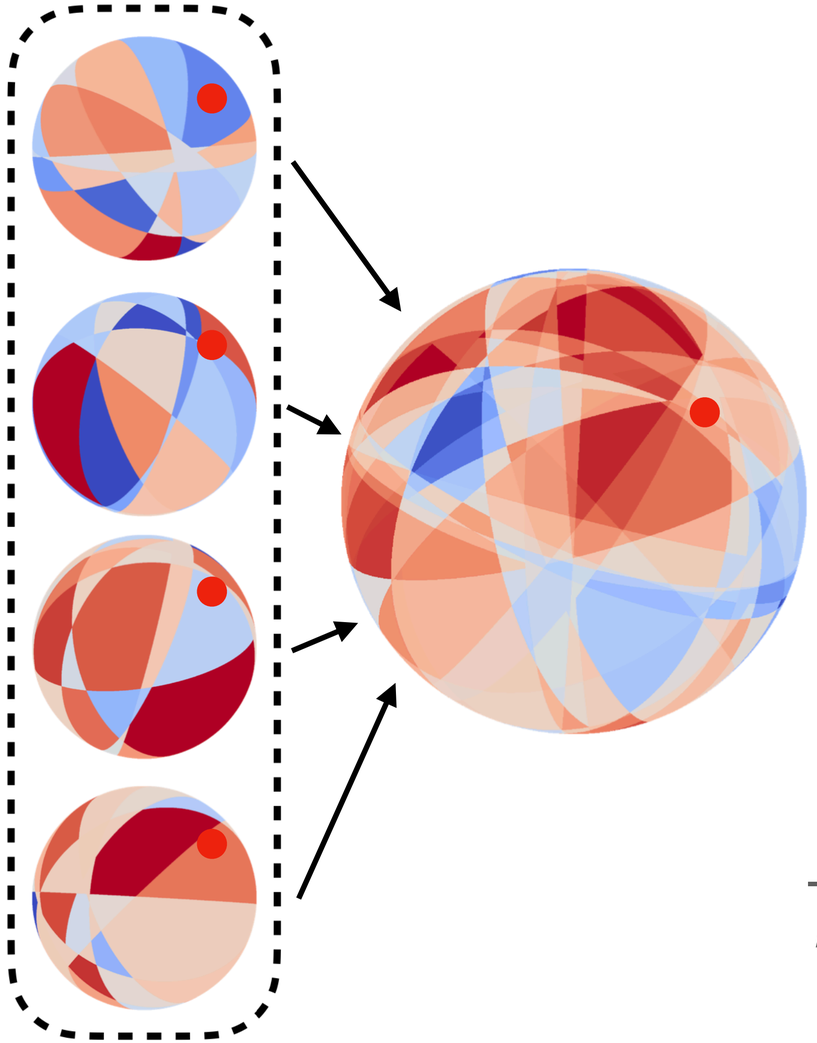
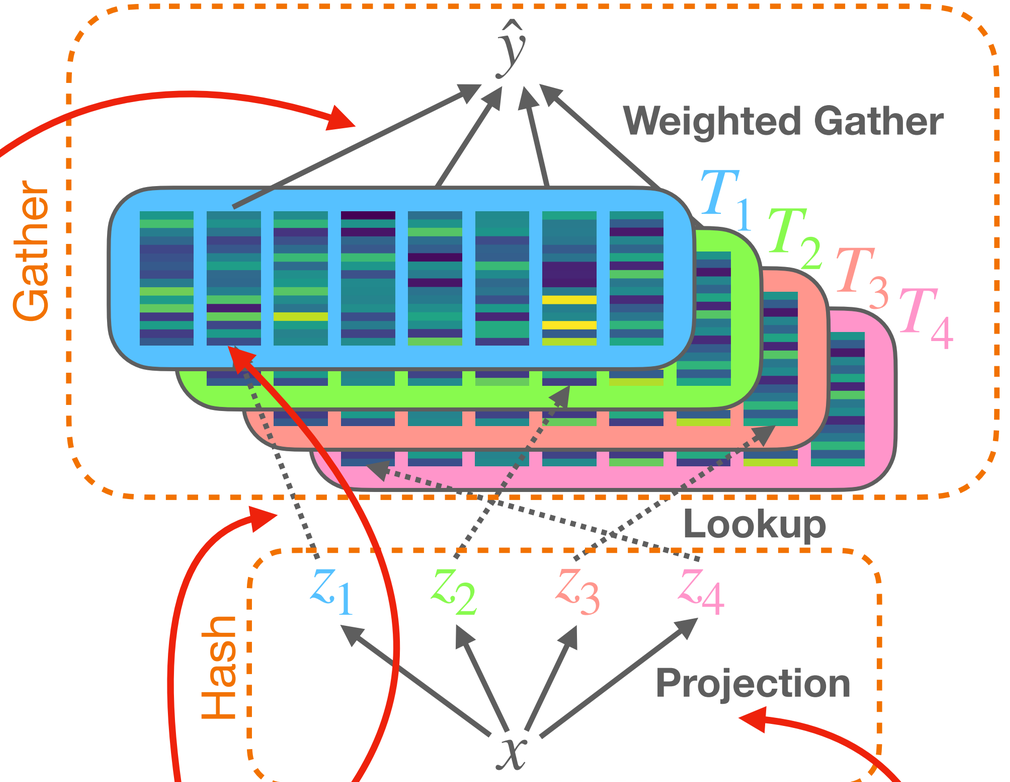


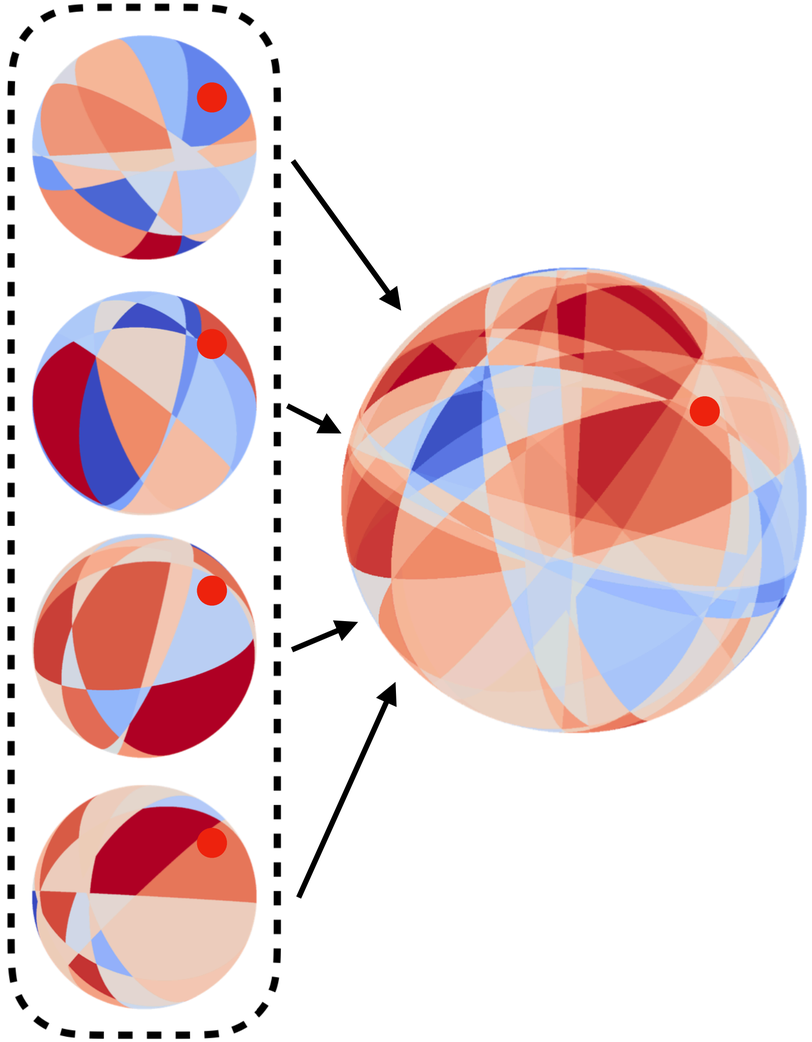


$$\frac{1}{h}\sum_{k=1}^{h}\frac{\exp(\langle z_k, [S]_{f_k(x)}\rangle)[T_k]_{f_k(x)}}{\prod_{j=1}^{\tau}(\exp([z_k]_j)+\exp(-[z_k]_j))}$$

$$z_k = xR_k$$

Making FFN Efficient

LookupFFN: Connection to Vanilla FFN



$$\sum_{k=1}^{h} \frac{\exp(\langle z_k, [S]_{f_k(x)} \rangle)[T_k]_{f_k(x)}}{\prod_{j=1}^{\tau} (\exp([z_k]_j) + \exp(-[z_k]_j))} \qquad z_k = xR_k$$

Sigmoid Activation:

$$\sum_{k=1}^{h} \frac{\exp(z_k)[V]_k}{\exp(z_k) + \exp(-z_k)} \qquad z_k = 0.5x[W]_k$$

Fast Approx. of GELU:

$$\sum_{k=1}^{h} \frac{1.175 z_k \exp(z_k)[V]_k}{\exp(z_k) + \exp(-z_k} \qquad z_k = 0.851x[W]_k$$

Making FFN Efficient

Remaining Challenge: Efficient Projection

GEMM with complexity $\mathcal{O}(dh\tau)$ for all k $\quad z_k = xR_k$

$$\sum_{k=1}^{h} \frac{\exp(\langle z_k, [S]_{f_k(x)} \rangle)[T_k]_{f_k(x)}}{\prod_{j=1}^{\tau} (\exp([z_k]_j) + \exp(-[z_k]_j))}$$

Making FFN Efficient

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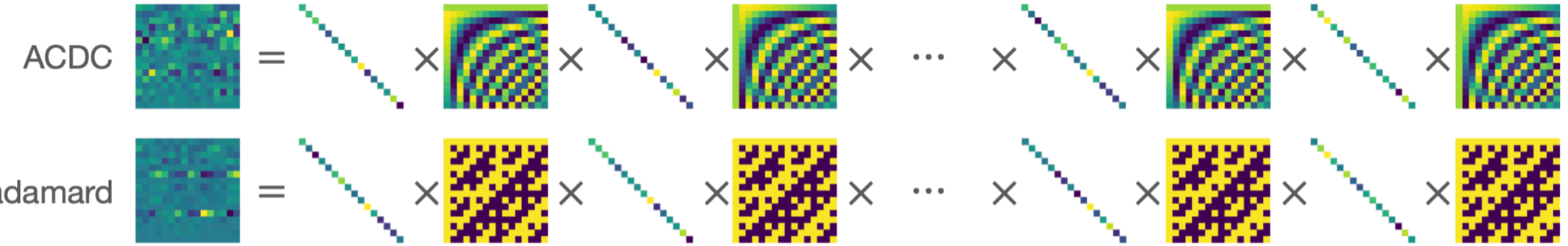
$$xD_1HD_2H \cdots D_{k-1}HD_kH$$



Moczulski, M., Denil, M., Appleyard, J., and de Freitas, N. ACDC: A structured efficient linear layer. In Ben- gio, Y. and LeCun, Y. (eds.), ICLR 2016

Making FFN Efficient

Remaining Challenge: Efficient Projection

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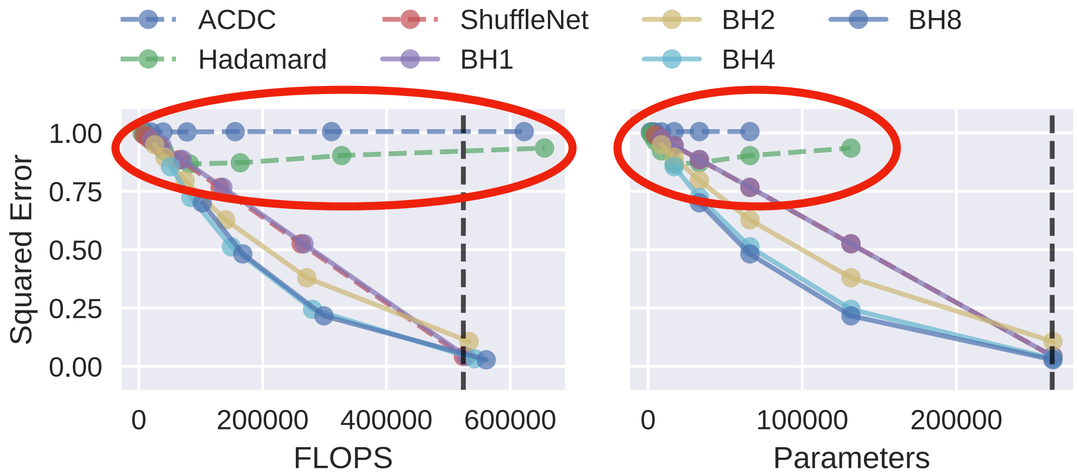
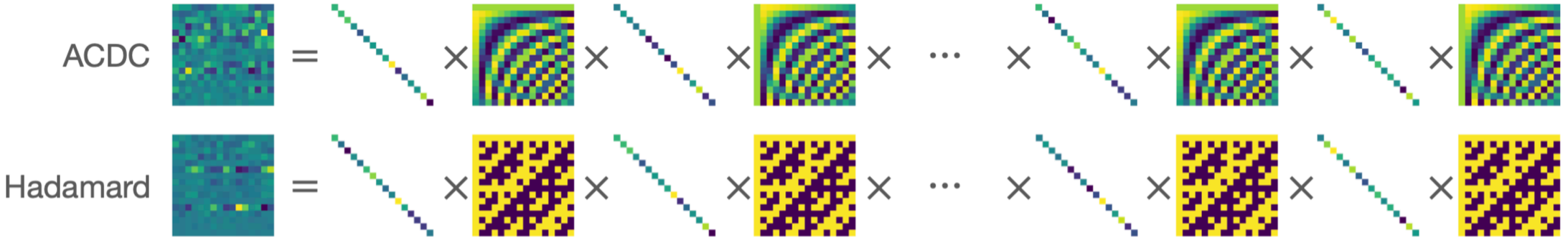




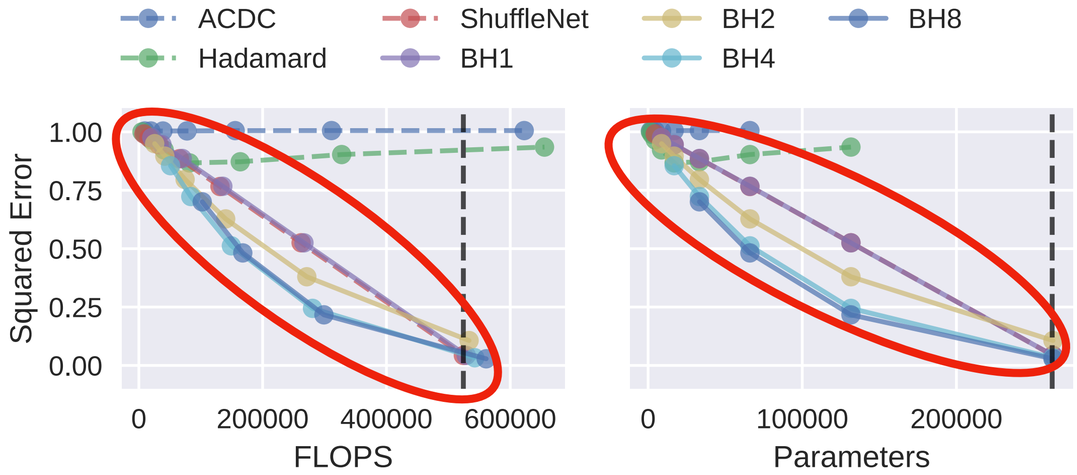
Making FFN Efficient

Efficient Projection: BH4

GEMM with complexity $\mathcal{O}(dh\tau)$ for all k $\quad z_k = xR_k$

$$xD_1HD_2H\cdots D_{k-1}HD_kH$$

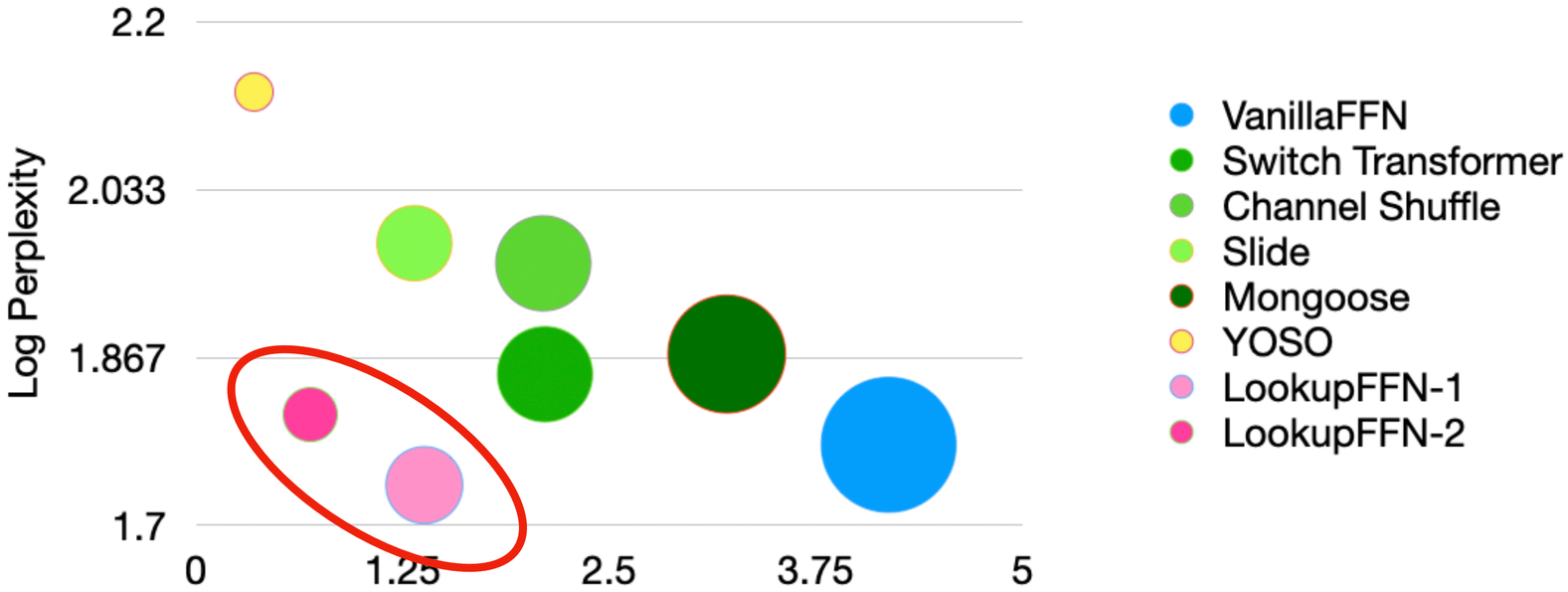
$$xB_1HB_2HB_3HB_4H$$



Evaluation

Baseline Comparison

RoBERTa pretraining (small size and base size)

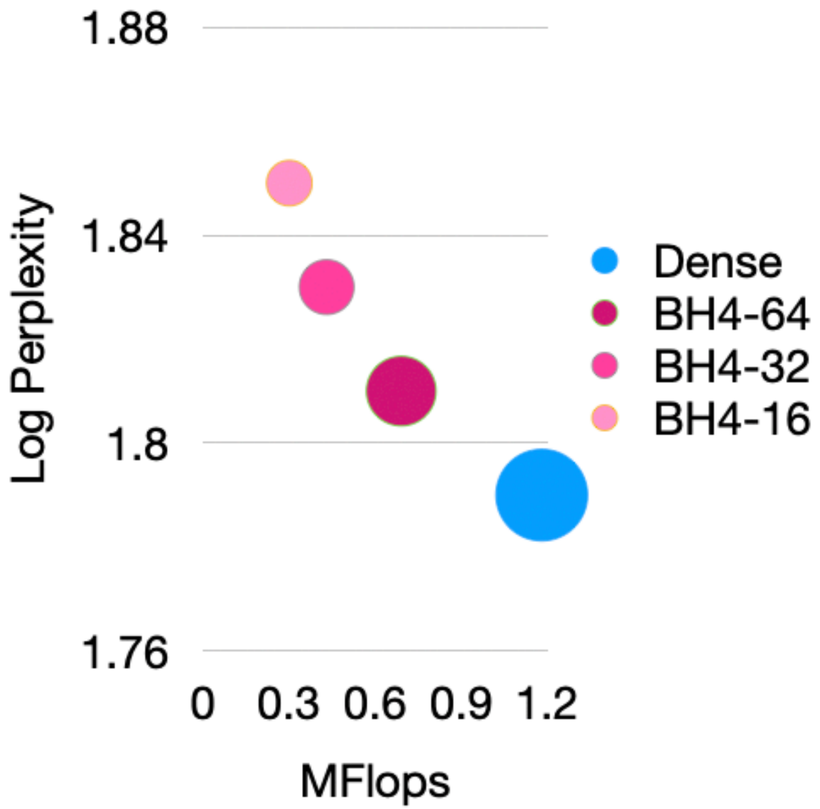


Method	h	τ	MFLOP	Log Perplexity
VanillaFFN	-	-	9.44	1.37
LookupFFN	170	9	1.39	1.41

Table 2. Log perplexity when scaling to a RoBERTa-base model.

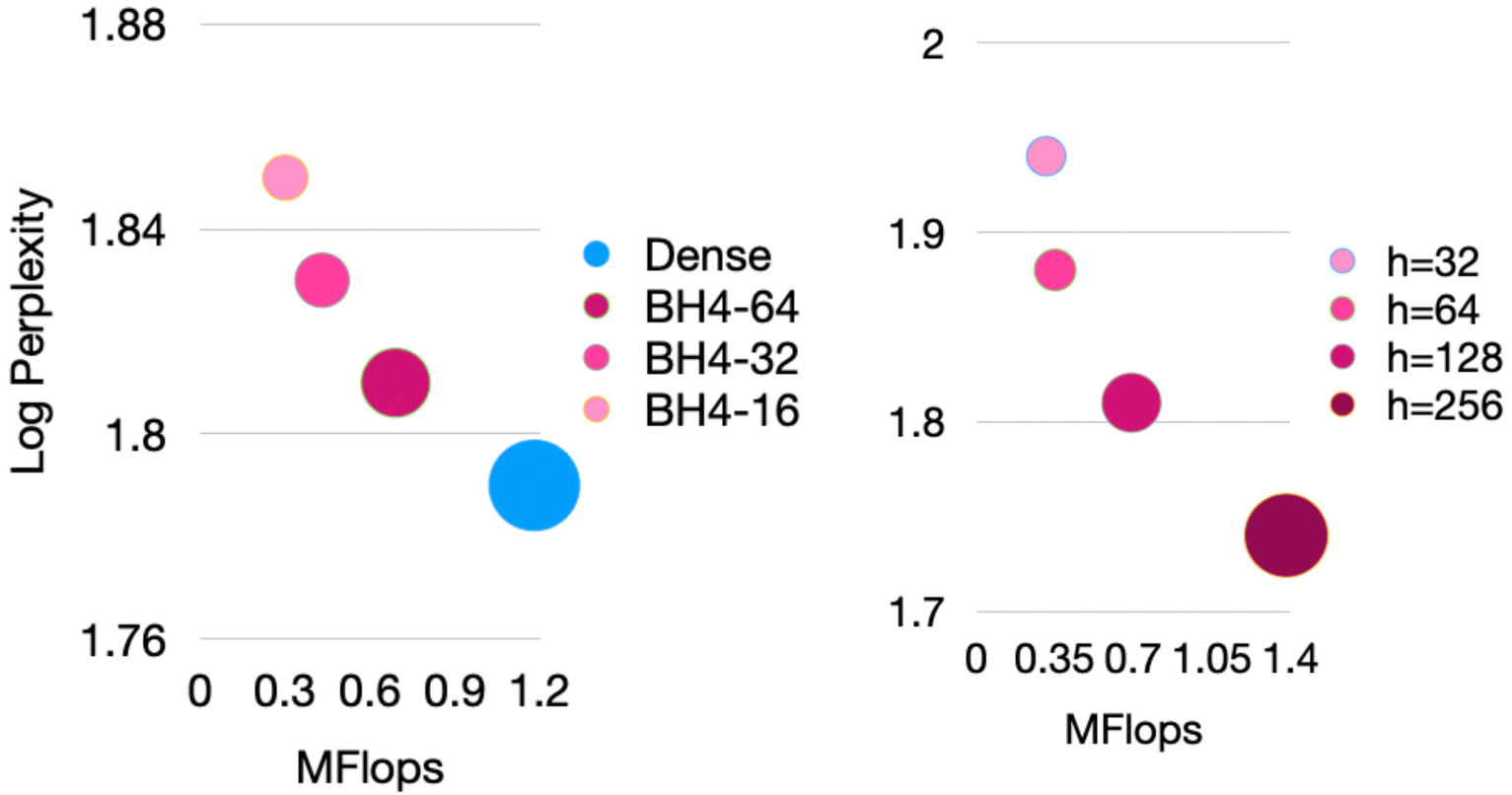
Comparing Different Configurations

RoBERTa pretraining (small size)



Comparing Different Configurations

RoBERTa pretraining (small size)



Comparing Different Configurations

RoBERTa pretraining (small size)

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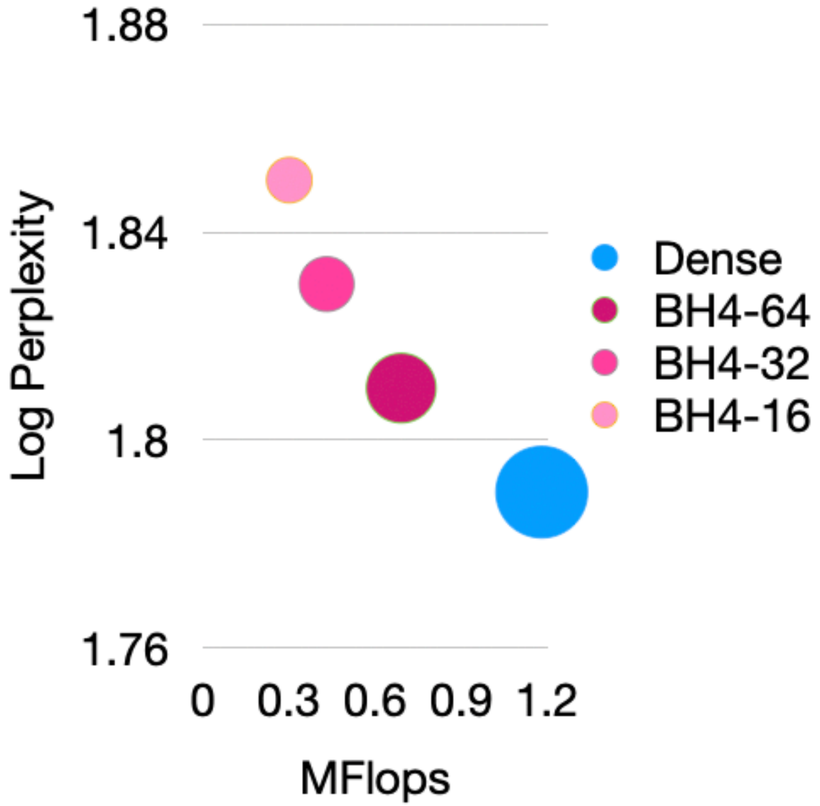

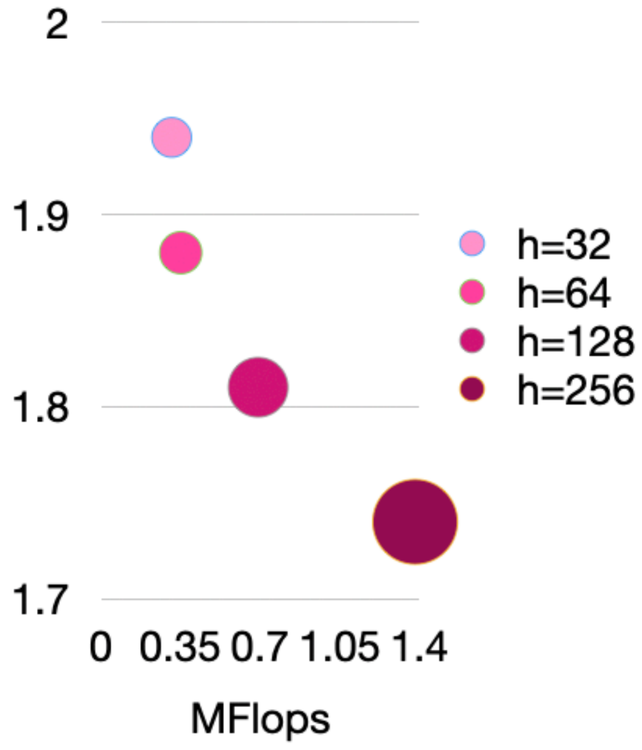

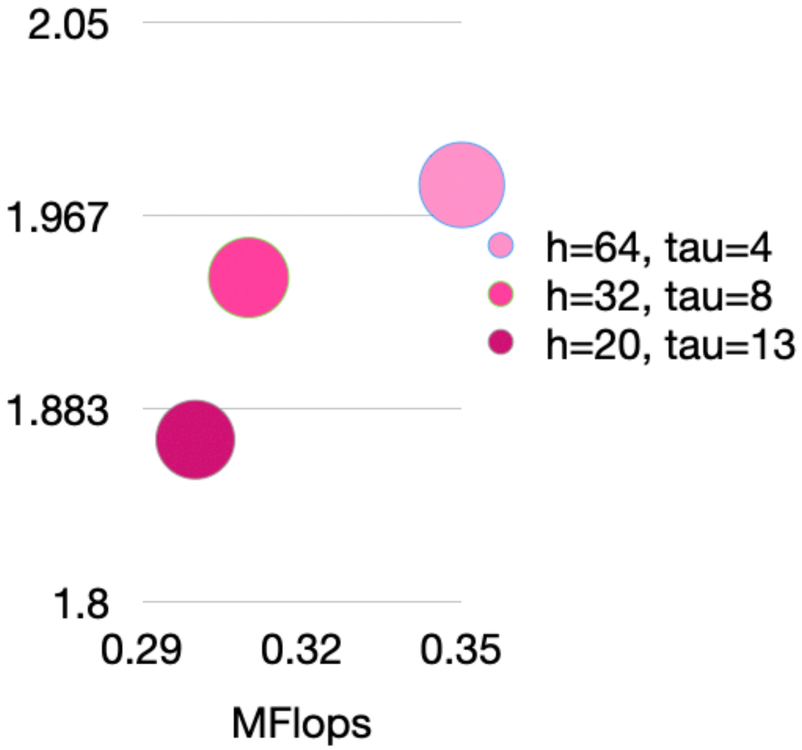

CPU Latency Comparison

RoBERTa pretraining (small size)

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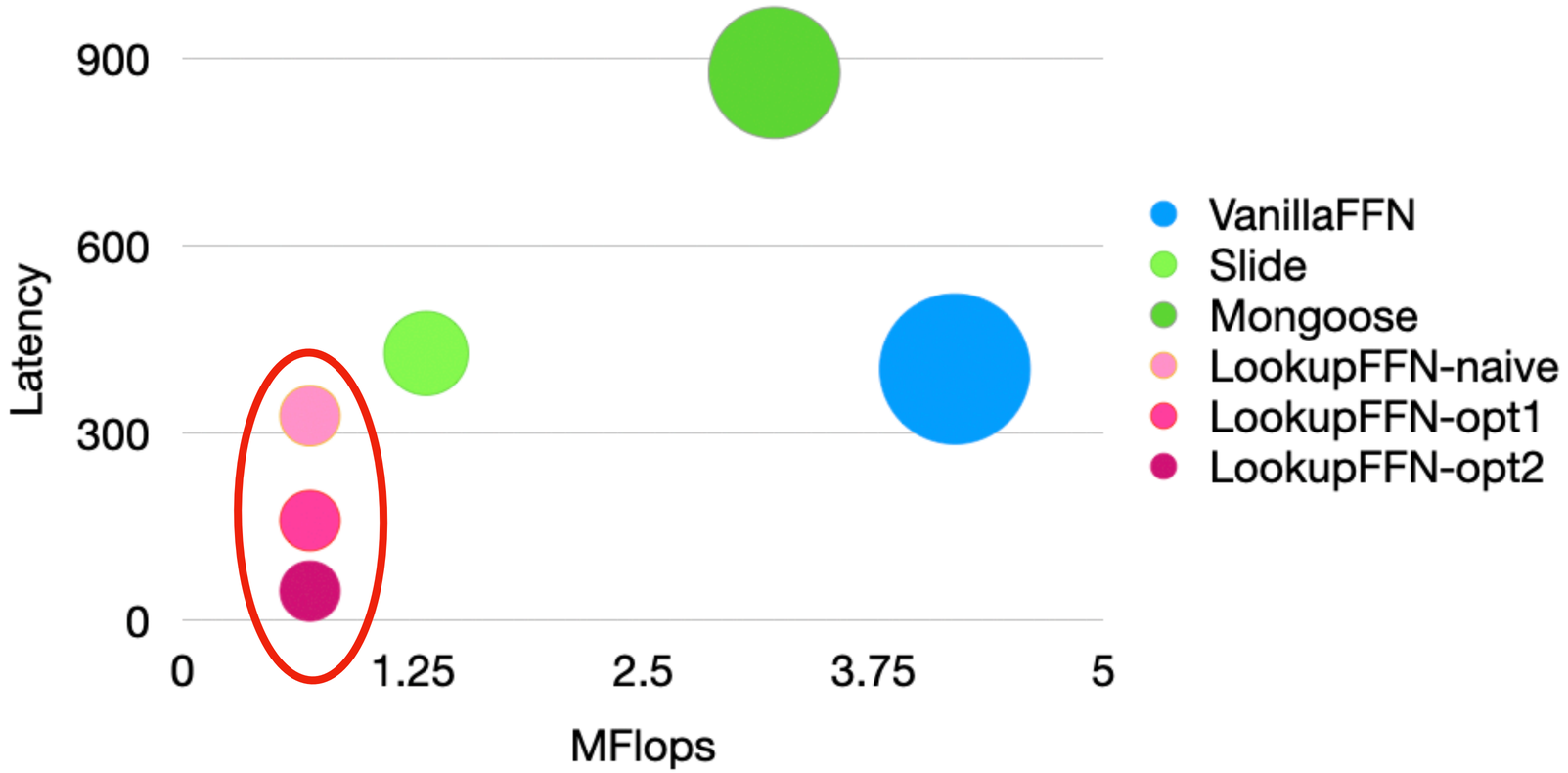


Takeaway

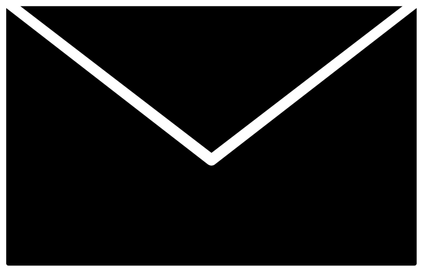
LookupFFN recasts FFNs using efficient LSH strategies.

When married with LSH based approximations of self-attention, GEMM operations within Transformer models can be nearly eliminated.

The resultant compute-lite model can allow deployment on CPU-based platforms.

Many other software/hardware opportunities exist: we have only scratched the surface.

mlpen/LookupFFN



zzeng38@wisc.edu

End