Counterfactual Analysis in Dynamic Latent-State Models

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Motivating Application: Surviving Cancer and Legal Reasoning

A person recently died from breast cancer

The exact progression of her disease is unknown (latent)

Her insurance company incorrectly denied her regular scans (mammograms)

Had the scans gone ahead, cancer may have been found earlier and patient's life saved

Now, a court wants to know:

What's the probability she would have survived had the scans been permitted?

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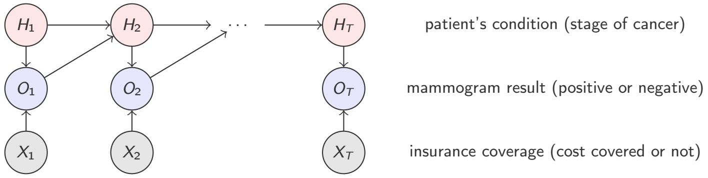
To formalize such a question, we need a dynamic latent-state model ...

A Dynamic Latent-State Model and Probability of Necessity



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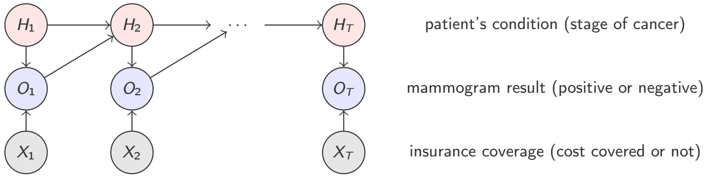

Observed data: suppose we observe following patient-level data:

$$\underbrace{o_1, \ldots, o_{\tau_s - 1}}_{-\text{ test result}}, \underbrace{o_{\tau_s}, \ldots, o_{\tau_e}}_{\text{no screening}}, \underbrace{o_{\tau_e + 1}, \ldots, o_{\tau_d - 1}}_{+\text{ test result}}, \underbrace{o_{\tau_d}, \ldots, o_T}_{\text{death}}$$

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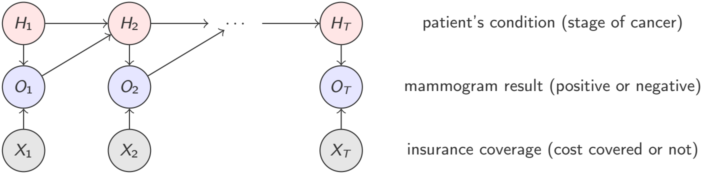

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Counterfactual query: probability of necessity (PN)

What is the probability she would have not died had the screening been covered in every period *given* the observed data?

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Principled framework for counterfactual analysis in dynamic latent-state models

- compute lower- and upper-bounds via polynomial optimization
- bring together ideas from causality, state-space models, simulation, and opt.

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- extend core idea of Balke & Pearl (94) to dynamic latent-state models
- show how domain-specific knowledge (e.g., CS) can be encoded as constraints
  - and that such knowledge can lead to (much) tighter bounds
- answer the open question of Oberst & Sontag (19)
  - is Gumbel-max the unique solution to counterfactual stability? [No]

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Application: show the application of our framework on a breast cancer case study

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See our paper for more details!
https://arxiv.org/abs/2205.13832

References

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