A Hybrid Quantum-Classical Approach Based on the Hadamard Transform for the Convolutional Layer

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Motivation and Background

- We want to reduce the parameters and the computational cost for the deep neural networks, with the accuracy is maintained or improved.
- The convolutional layer, which is the critical component in the deep neural networks, is our target to be revised.
- Transforming the input data into the frequency domain can help capture the most important features while reducing the redundancy in the input data.

Background: Hadamard Transform

Hadamard Transform (HT) and its inverse (IHT):

- HT: $\mathbf{X} = \mathbf{H}_N\mathbf{x}$, IHT: $\mathbf{X} = \frac{1}{N}\mathbf{H}_N\mathbf{x}$.
- HT can be considered a binary version of DFT.

$$\mathbf{H}_N = \begin{cases} 1, N = 1 \\ \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, N = 2 \\ \begin{bmatrix} \mathbf{H}_{\frac{N}{2}} & \mathbf{H}_{\frac{N}{2}} \\ \mathbf{H}_{\frac{N}{2}} & -\mathbf{H}_{\frac{N}{2}} \end{bmatrix}, N \geq 4 \end{cases}.$$

$$\mathbf{H}_N = \mathbf{H}_2 \otimes \mathbf{H}_{\frac{N}{2}}, N \geq 4.$$

Hybrid Quantum-Classical HT

Method	1D Complexity	2D Complexity
Classical HT (Matrix-Vector Product)	$O(N^2)$	$O(N^3)$
Classical Fast HT	$O(N \log N)$	$O(N^2 \log N)$
Hybrid Quantum-Classical HT	$O(N)$	$O(N^2)$

Algorithm 1 The hybrid quantum-classical HT algorithm.

Input: The input vector $\mathbf{x} = [x_0\ x_1\ \ldots\ x_{N-1}]^T \in \mathbb{R}^N$ where $N = 2^M$, $M \in \mathbb{N}$.

Output: The output vector $\mathbf{X} = [x_0\ x_1\ \ldots\ x_{N-1}]^T$ which is the HT of the $\mathbf{x}$.

$b = \epsilon + \sum_{k=0}^{N-1} |x_k|$, where ϵ is any positive number;

$\tilde{\mathbf{x}} = [b\ x_1\ x_2\ \ldots\ x_{N-1}]^T$;

$c = ||\tilde{\mathbf{x}}|| = \sqrt{b^2 + \sum_{k=1}^{N-1} x_k^2}$.

$\bar{\mathbf{x}} = [\tilde{x}_0\ \tilde{x}_1\ \ldots\ \tilde{x}_{N-1}]^T = \tilde{\mathbf{x}}/c$;

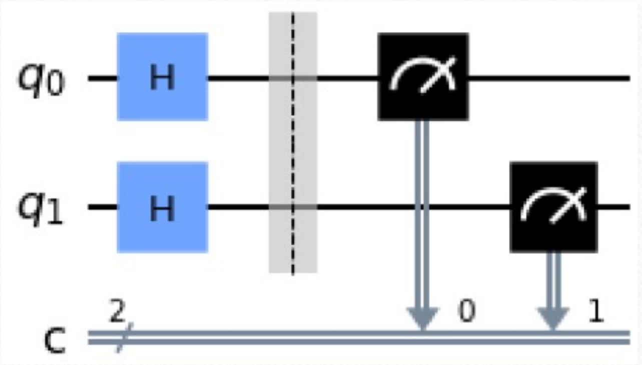
Prepare the state $|\Psi\rangle = \sum_{k=0}^{N-1} \bar{x}_k |k\rangle$ using M qubits;

Apply $\mathbf{H}^{\otimes M}$ on $|\Psi\rangle$;

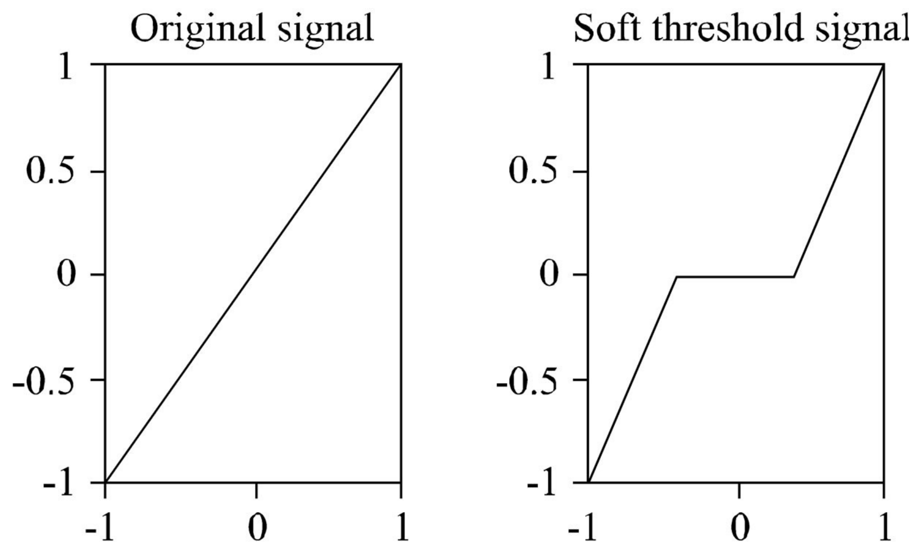
Measure all the M qubits to compute the probability p_k of obtaining the state $|k\rangle$ for $k = 0$ to $N-1$;

$\delta = \sqrt{\frac{1}{N}}(b - \mathbf{x}[0])$;

$\mathbf{X} = [c\sqrt{p_0} - \delta\ c\sqrt{p_1} - \delta\ \ldots\ c\sqrt{p_{N-1}} - \delta]^T$.



[1] Shukla, Alok, and Prakash Vedula. "A hybrid classical-quantum algorithm for solution of nonlinear ordinary differential equations." *Applied Mathematics and Computation* 442 (2023): 127708.

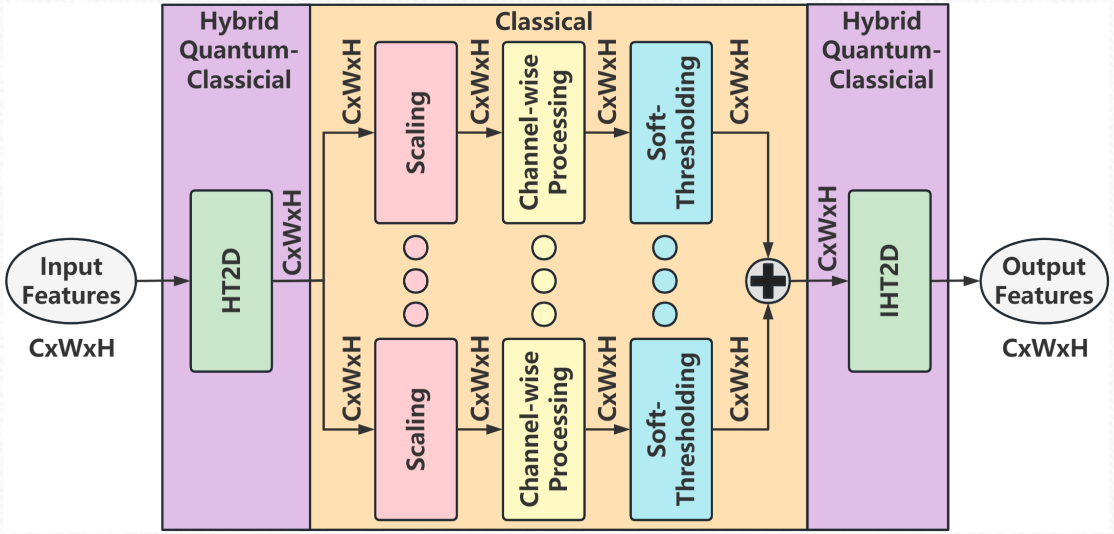


Soft-thresholding

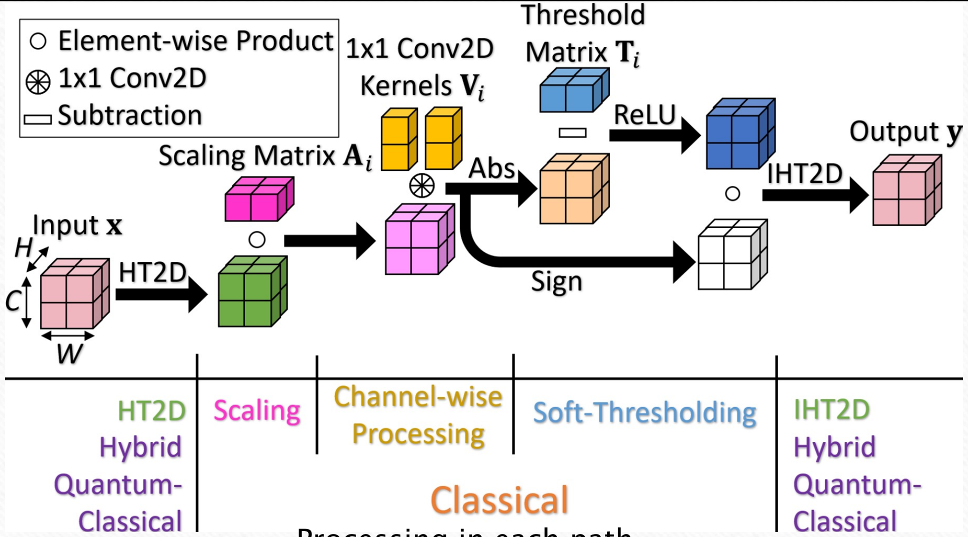
- $Y = S_T(X) = \mathrm{sign}(X)(|X| - T)_+$
- Soft-threshold is widely used in the wavelet to remove small noises in the transform domain.
- We don't use ReLU with the bias term because the positive and negative frequency components are equally important.

Hybrid Quantum-Classical HT-Perceptron Layer

- Layer are based on the convolution theorem: Convolution in the times/space domain is equivalent to the element-wise product in the transform domain.
- We apply width-wise and height-wise element-wise multiplication in the transform domain.
- We use the multi-path structure to utilize more trainable parameters to improve the accuracy.



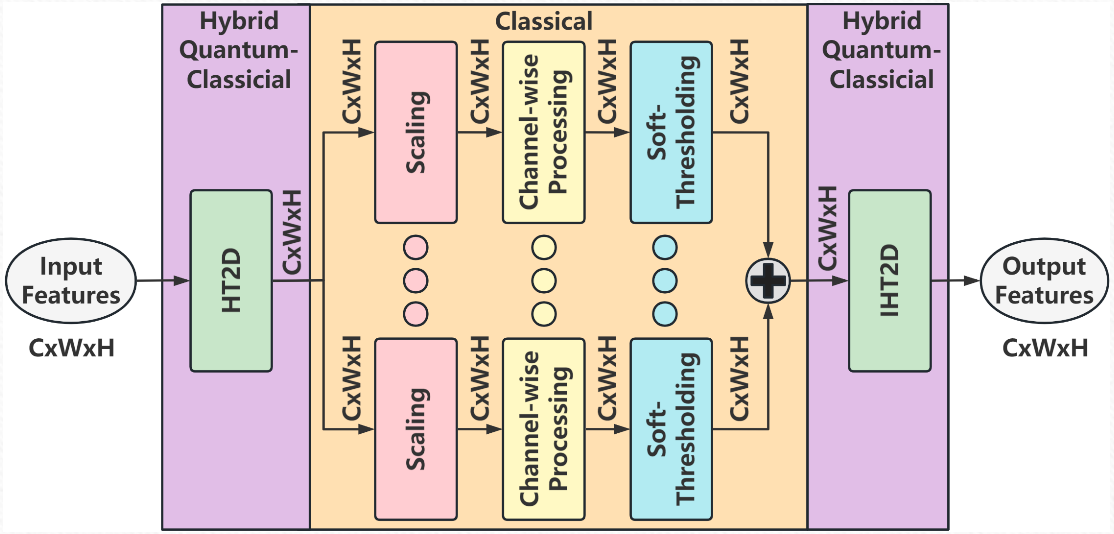
Structure of the proposed HT–perceptron layer



Processing in each path

Hybrid Quantum-Classical HT-Perceptron Layer

- We can use the hybrid quantum-classical HT algorithm to in the HT-perceptron layer to design a quantum-computer-friendly approach for the Conv2D layer.
- The layer is still trained using the classical backpropagation algorithm, because there is not quantum trainable variable. The quantum part is only to compute the HT and IHT.



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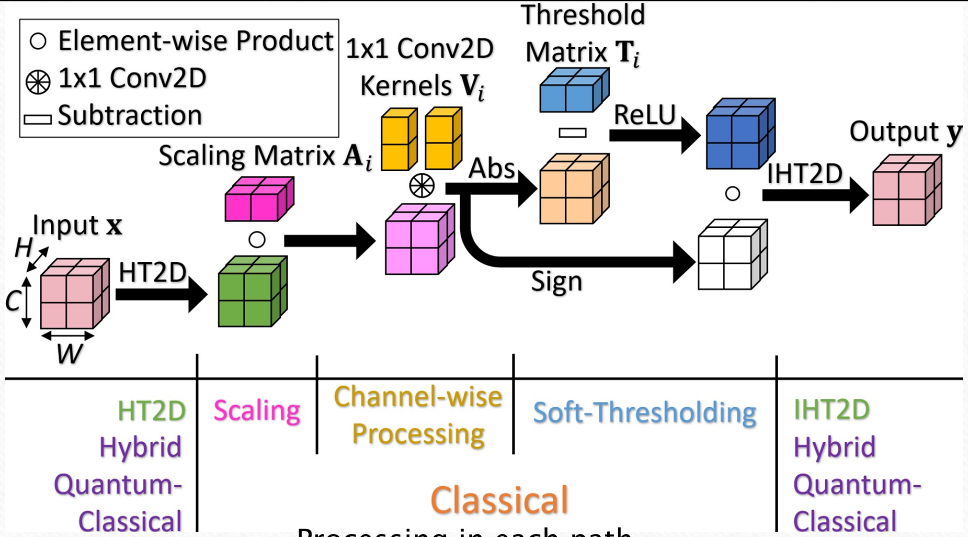

Processing in each path

Comparison with the Conv2D layer: Parameters and MACs on a $C \times N \times N$ tensor

Layer (Operation)	Parameters	Multiply-Accumulate (MACs)
$K \times K$ Conv2D	$K^2 C^2$	$K^2 N^2 C^2$
3×3 Conv2D	$9C^2$	$9N^2 C^2$
Scaling, Soft-thresholding	$2N^2$	$N^2 C$
1×1 Conv2D	C^2	$N^2 C^2$
$\boldsymbol{P}$-Path HT-Perceptron	$\boldsymbol{PC^2 + 2PN^2}$	$\boldsymbol{PN^2C^2 + PN^2C}$
3-Path HT-Perceptron	$\boldsymbol{6N^2 + 3C^2}$	$\boldsymbol{3N^2C^2 + 3N^2C}$

- In the modern neural networks' most hidden layers, $C \gg N$.
- The HT-perceptron layers have a smaller coefficient on C^2 term than the 3×3 Conv2D layer.

Experimental Results: MNIST

Method	Parameters	MACs (M)	Accuracy
CNN	1,059,562	10.85	99.26%
HT-CNN (3-Path)	1,059,562	4.66 (57.1%↓)	99.31%

Experimental Results: CIFAR-10

Method	Parameters	MACs (M)	Accuracy
ResNet-20 (Official [2])	0.27M	-	91.25%
HT-Based ResNet-20 (Our early work [3])	133,082 (51.16%↓)	-	90.12%
ResNet-20 (Our trial, baseline)	272,474	41.32	91.66%
HT-ResNet-20 (1-Path)	199,898 (26.64%↓)	22.53 (45.5%↓)	91.25%
HT-ResNet-20 (3-Path)	199,898 (26.64%↓)	27.42 (33.6%↓)	91.29%
HT-ResNet-20 (5-Path)	248,282 (8.88%↓)	32.31 (21.8%↓)	91.58%

[2] He, Kaiming, Xiangyu Zhang, Shaoqing Ren, and Jian Sun. "Deep residual learning for image recognition." In *Proceedings of the IEEE conference on computer vision and pattern recognition*, pp. 770-778. 2016.
[3] Pan, Hongyi, Diaa Badawi, and Ahmet Enis Cetin. "Block Walsh–Hadamard Transform-based Binary Layers in Deep Neural Networks." *ACM Transactions on Embedded Computing Systems* 21, no. 6 (2022): 1-25.

Experimental Results: Comparison with HT-based works on ImageNet-1K

Method	Publication	Params (M)	MACs (G)	Top-1 Acc	Top-5 Acc
ResNet-50 (Torchvision, Input 224^2)	CVPR 2015	25.56	4.12	76.13%	92.86%
ResNet-50 (AugSkip) Zero Init	ICLR 2022	25.56	4.12	76.37%	-
HT-ResNet-50 (3-Path, Input 224^2)	**This work**	**22.63 (11.5%↓)**	**3.60 (12.6%↓)**	**76.36%**	**93.02%**
ResNet-50 (Our trial, Input 256^2)	-	25.56	5.38	76.18%	92.94%
HT-ResNet-50 (3-Path, Input 256^2)	**This work**	**22.63 (11.5%↓)**	**4.59 (14.9%↓)**	**76.77%**	**93.26%**

Other HT-based work including [4] [5] contain no ResNet-based result, but their networks produce worse accuracy results than their baseline models according to Table 4 in each paper.

[4] Deveci, T. Ceren, Serdar Cakir, and A. Enis Cetin. "Energy efficient hadamard neural networks." *arXiv preprint arXiv:1805.05421* (2018).

[5] Akhauri, Yash. "Hadanets: Flexible quantization strategies for neural networks." In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition Workshops*, pp. 0-0. 2019.

Conclusion

- We presented a HT-based neural network layer to replace the Conv2D layer, which obtained comparable or higher accuracy results using fewer parameters and less computational cost on CIFAR-10 and ImageNet-1K compared to the vanilla ResNet models.
- We investigated the HT-based neural network layer in the quantum computation field to design a quantum-computer-friendly approach. We employ quantum Hadamard Gates to reduce the computational cost for the 2D HT.

Thank you!

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