Counterfactual Identifiability of Bijective Causal Models

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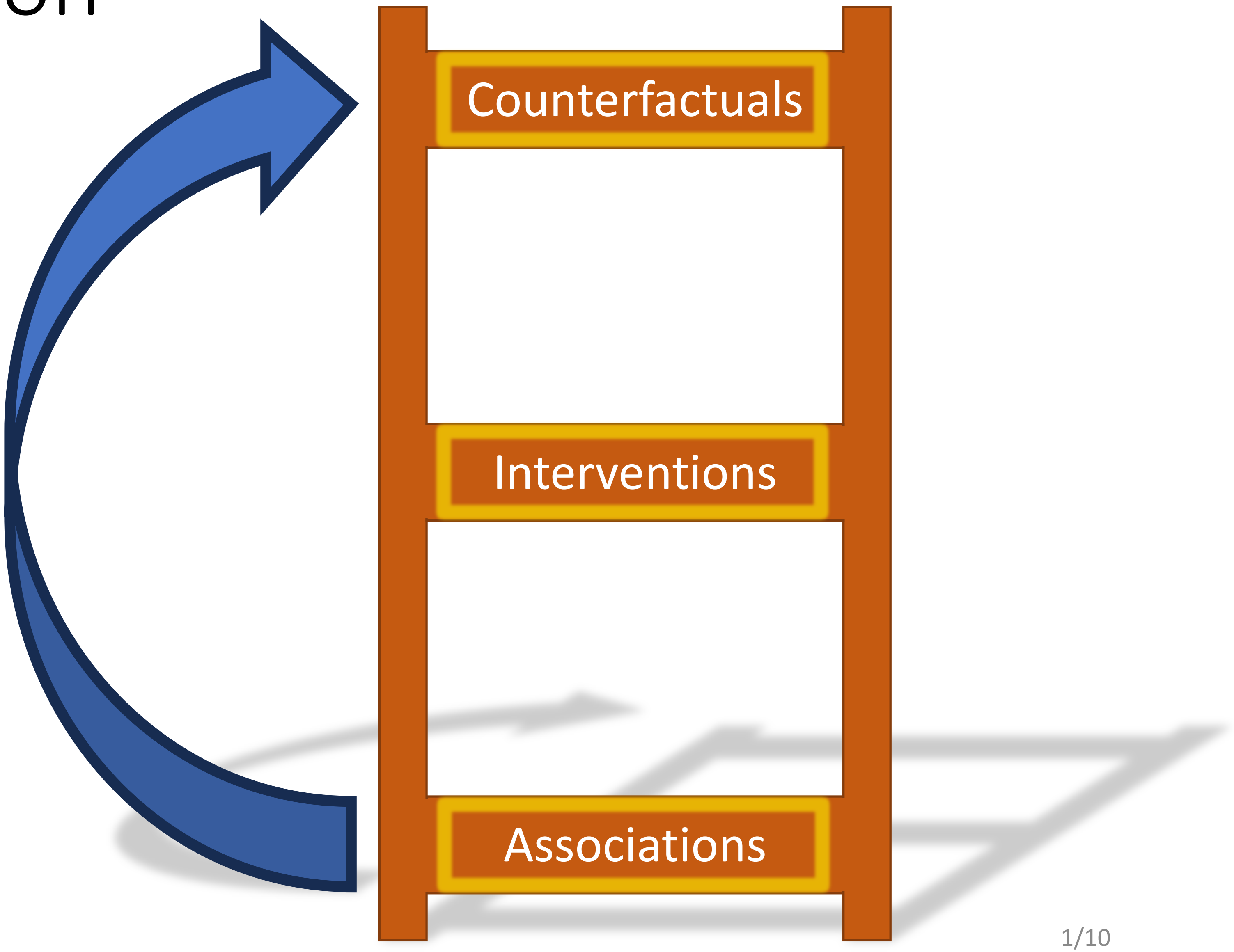
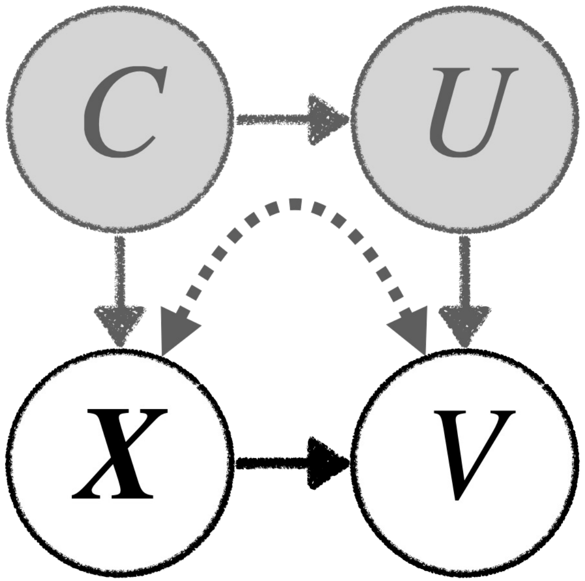
Ladder of Causation

- L_3: Imagining
- L_2: Doing
- L_1: Seeing



Background: Causal Models

Causal DAG



Structural Causal Model

Exogenous Noise

$$V = f(\boldsymbol{X}, U)$$

Causal Parents

Bijective Causal Models

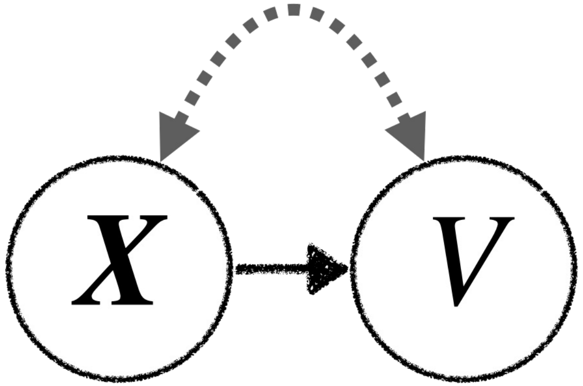
- Consist of **Bijective Generation Mechanism (BGM)**

BGM Definition

$$\forall \boldsymbol{x}:\ f(\boldsymbol{x},\ \cdot) \text{ is a bijection.}$$
$$\rightarrow \forall \boldsymbol{x}:\ v = f^{-1}(\boldsymbol{x}, u).$$

- Subsumes several causal models studied in the literature:
  - ✓ Nonlinear Additive Noise models (ANM) [Hoyer et. al., NeurIPS 2008]
  - ✓ Location Scale Noise models (LSNM) [Immer et. al., ICML 2023]
  - ✓ Post Nonlinear Causal Model (PNL) [Zhang et. al., UAI 2009]

Counterfactual Identifiability of BGMs



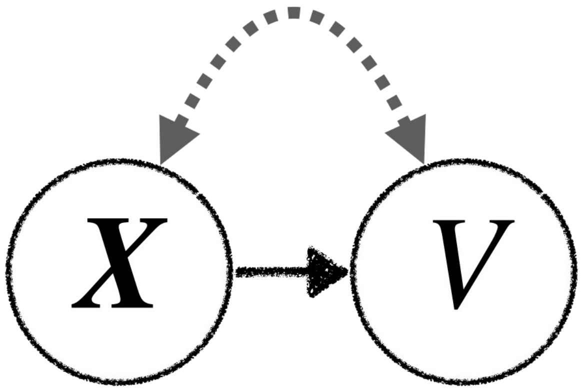
$$\boldsymbol{x}_i, v_i \sim P_{\boldsymbol{X},V}$$

Observational Data

$$\xrightarrow{?} \quad f(\cdot,\cdot)$$

Identifiability Theorem:
The Backdoor Criterion (BC)

- $V = f(\boldsymbol{X}, U)$
- $U \not\perp\!\!\!\perp X$



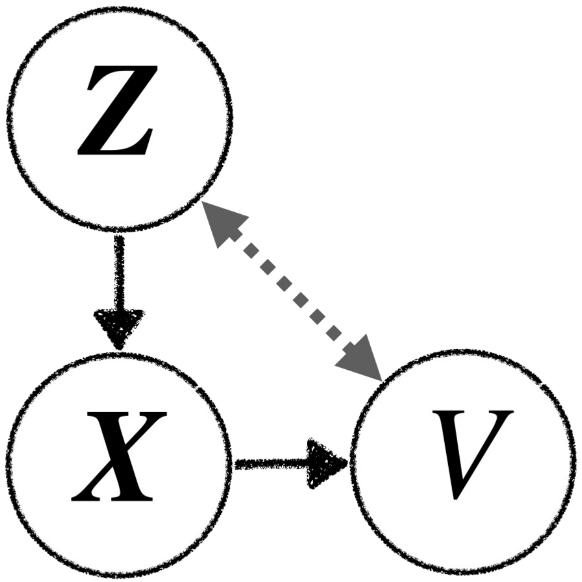
Identifiability Theorem:
The Backdoor Criterion (BC)

- $U, V \in \mathbb{R}^d$.
- X and Z can be discrete or continuous.

Identifiable from $P_{\mathbf{X},V,\mathbf{Z}}$ if:

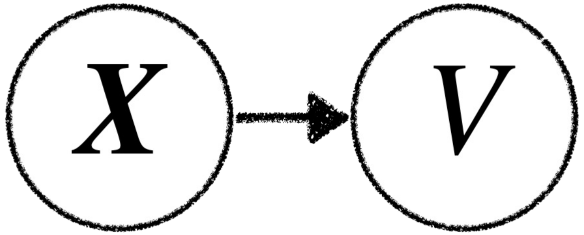
- $\forall \boldsymbol{x}$: $\nabla_{\boldsymbol{x}} \left|\det J_{f(\boldsymbol{x},\cdot)}\right|$ and $\nabla_{\boldsymbol{x}} \left|\det J_{f^{-1}(\boldsymbol{x},\cdot)}\right|$ exist.
- **(Variability)** $\exists \{\boldsymbol{z}_1, \ldots, \boldsymbol{z}_{d+1}\}$: $P_{U|\mathbf{Z}}(\cdot \mid \boldsymbol{z}_i)$ are distinct.
  - ! Please see the paper for precise definition of this condition.

- $U \perp\!\!\!\perp \mathbf{X}|\mathbf{Z}$

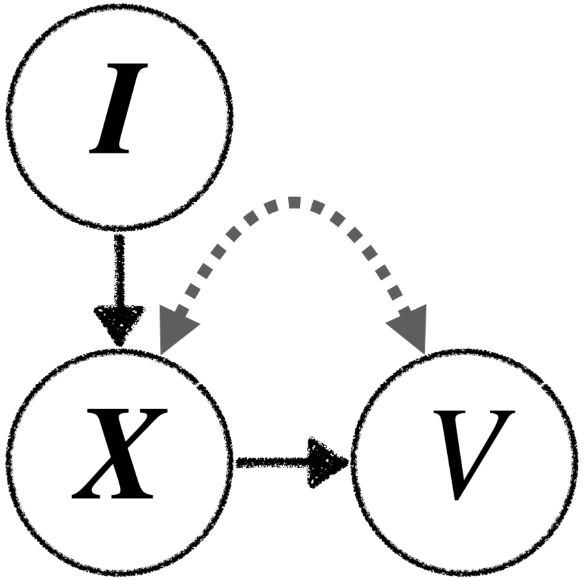


Other Identifiability Theorems

The Markovian Case

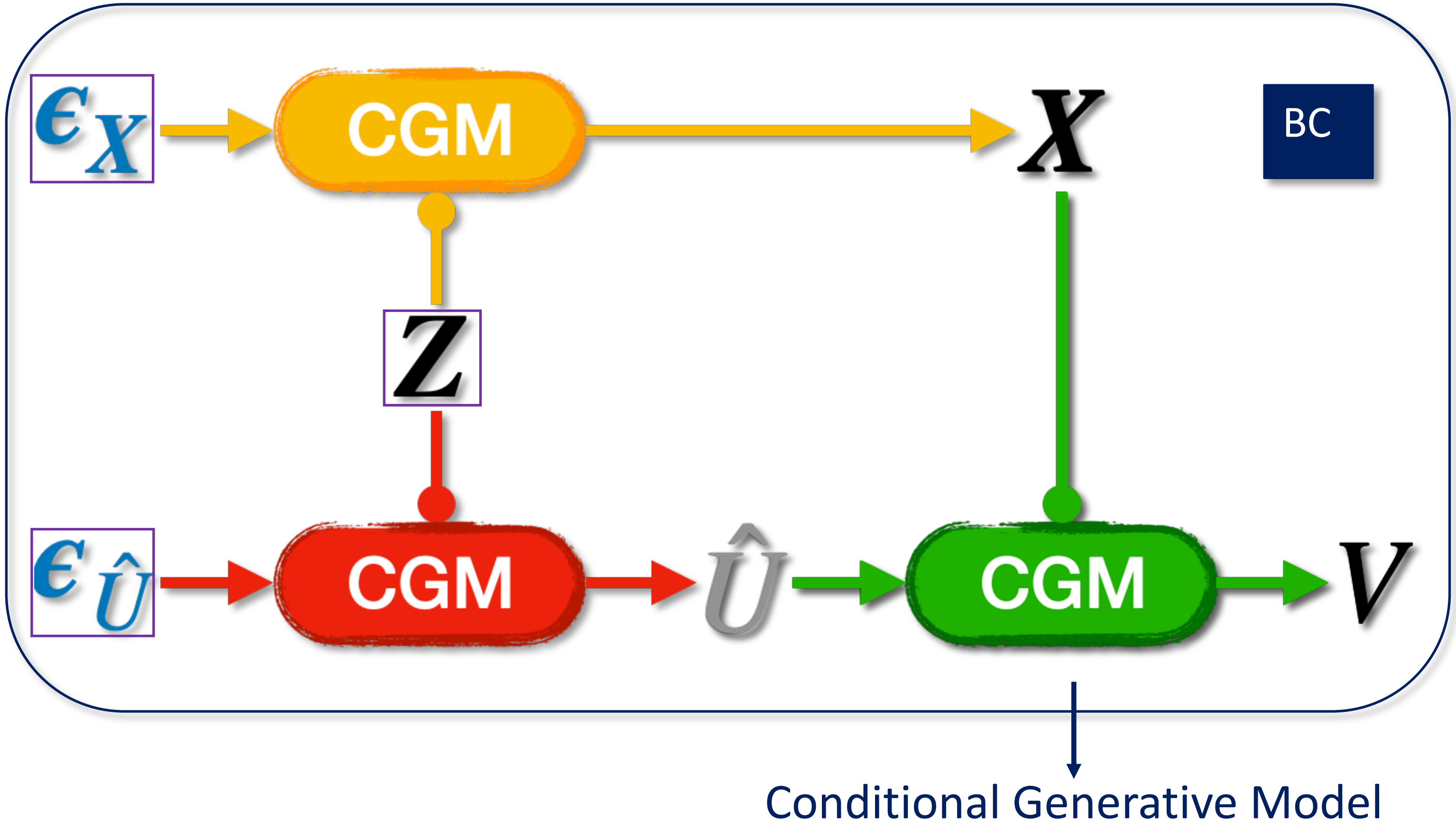


Instrumental Variable (IV)



Practical Algorithm

- $U \perp\!\!\!\perp \mathbf{X}|\mathbf{Z}$



Video Streaming Simulation

Scheme	Normalized MSE (%)
Markovian	4.1
IV	10.0
BC	10.4
CausalSim	9.3

CausalSim: A Causal Framework for Unbiased Trace-Driven Simulation [Alomar et. al., NSDI 2023]

Contributions

- Introduced Bijective Causal Models.

- Established their counterfactual identifiability in three well-known causal structures.

- Proposed a practical method for learning them.

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