

Inferring Cause and Effect

in the presence of heteroscedastic noise



Sascha Xu 

Osman Mian 

Alexander Marx 

Jilles Vreeken 

ETH zürich

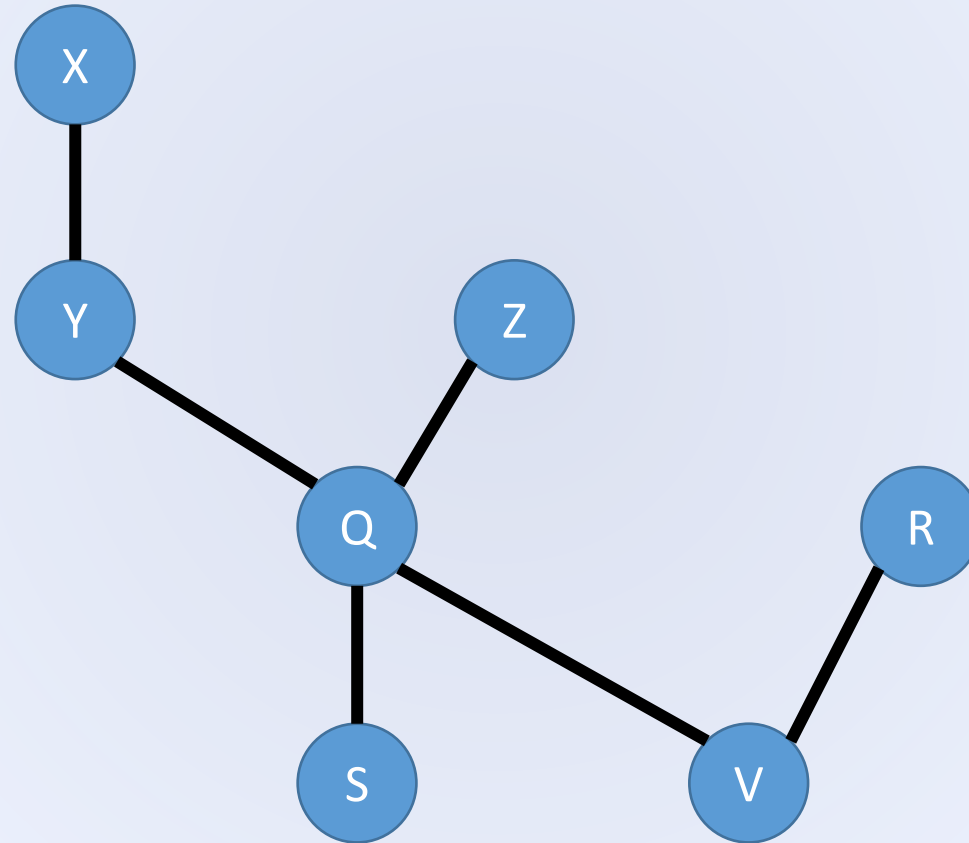


Question of the day

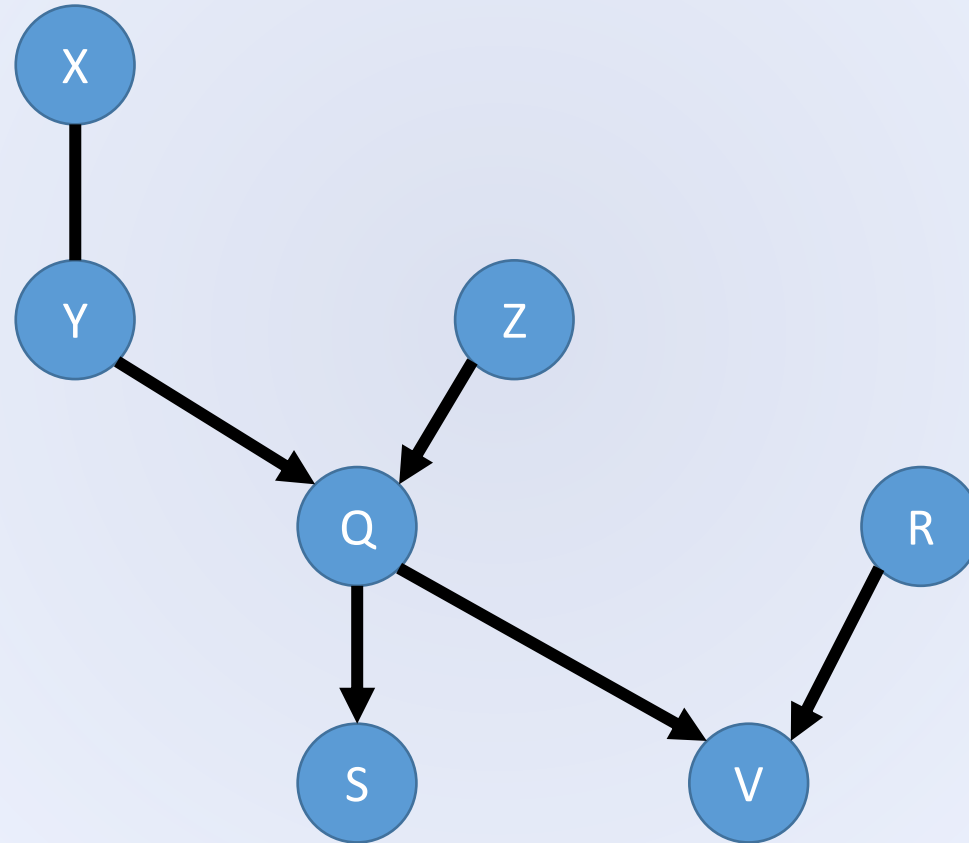


How can we
distinguish **cause** from **effect**
under **realistic** noise assumptions?

Inferring Cause and Effect

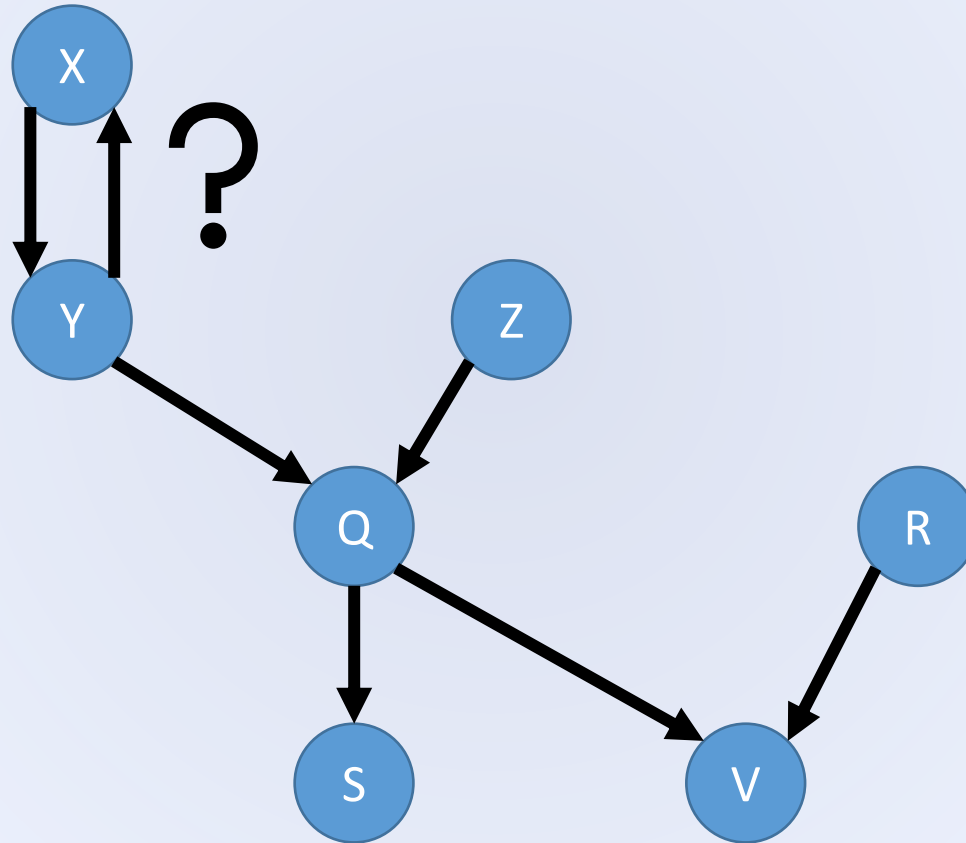


Inferring Cause and Effect



Inferring Cause and Effect

Bi-variate causal
inference



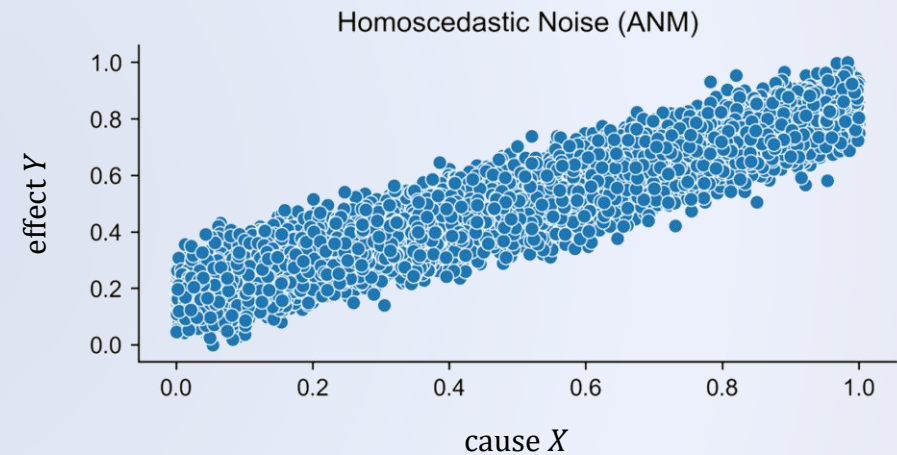
Causal Model

We consider **continuous valued** cause X , Gaussian noise N and effect Y

$$Y = f(X) + s(X) * N$$

where we use a **scaling function** $s(x)$ to model **heteroscedastic** noise

Very general causal model



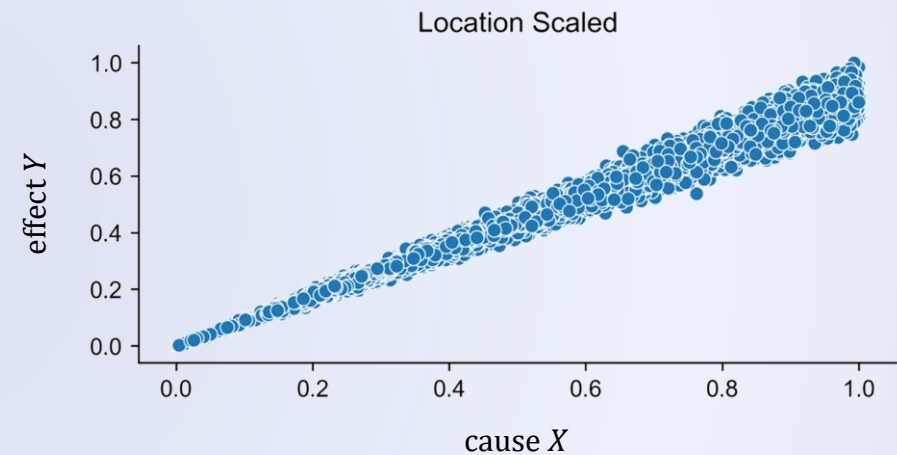
Causal Model

We consider **continuous valued** cause X , Gaussian noise N and effect Y

$$Y = f(X) + s(X) * N$$

where we use a **scaling function** $s(x)$ to model **heteroscedastic** noise

Very general causal model



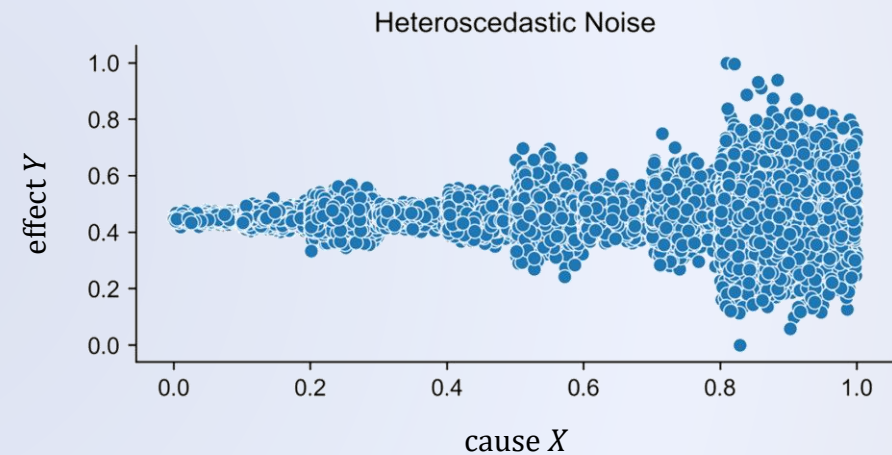
Causal Model

We consider **continuous valued** cause X , Gaussian noise N and effect Y

$$Y = f(X) + s(X) * N$$

where we use a **scaling function** $s(x)$ to model **heteroscedastic** noise

Very general causal model



Identifiability

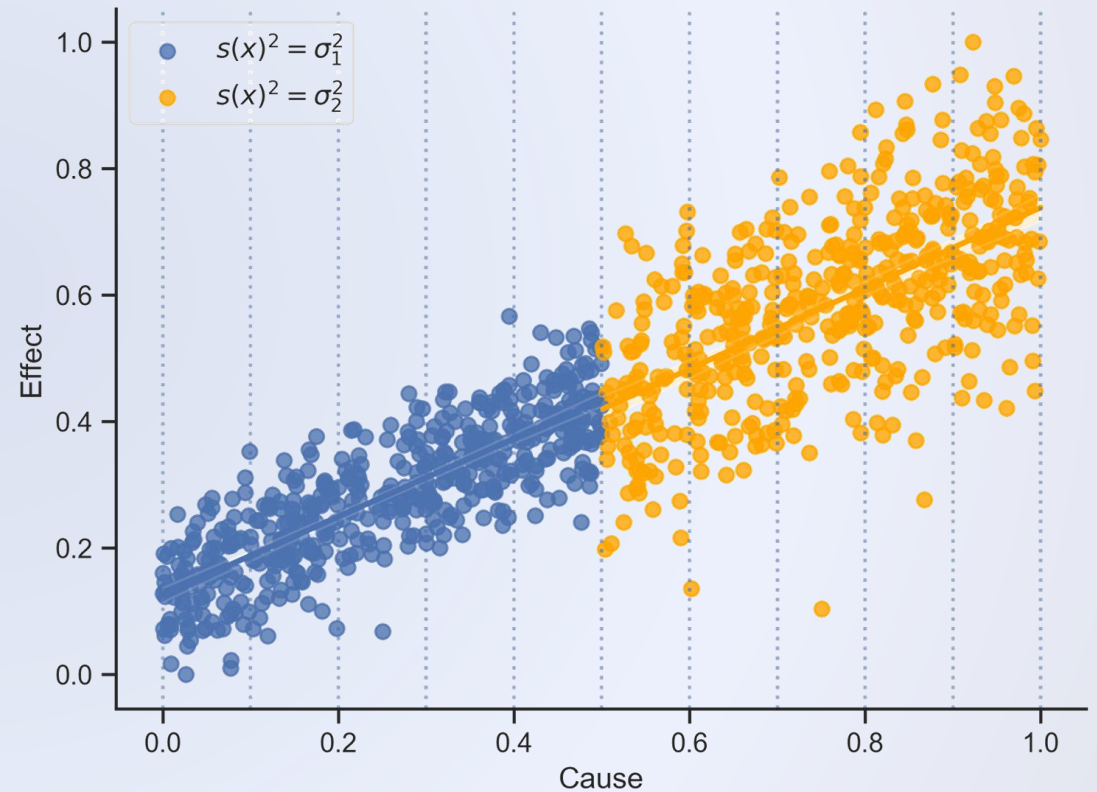
Linear functions, heteroscedastic noise

$$\int p_X(x) \log(\text{Var}(Y|X = x)) dx \\ \leq \int p_Y(y) \log(\text{Var}(X|Y = y)) dy$$

Non-linear, homoscedastic noise

$$E[\log(\text{Var}(Y|X))] < E[\log(\text{Var}(X|Y))]$$

We can **identify cause from effect**
by comparing log-likelihood!



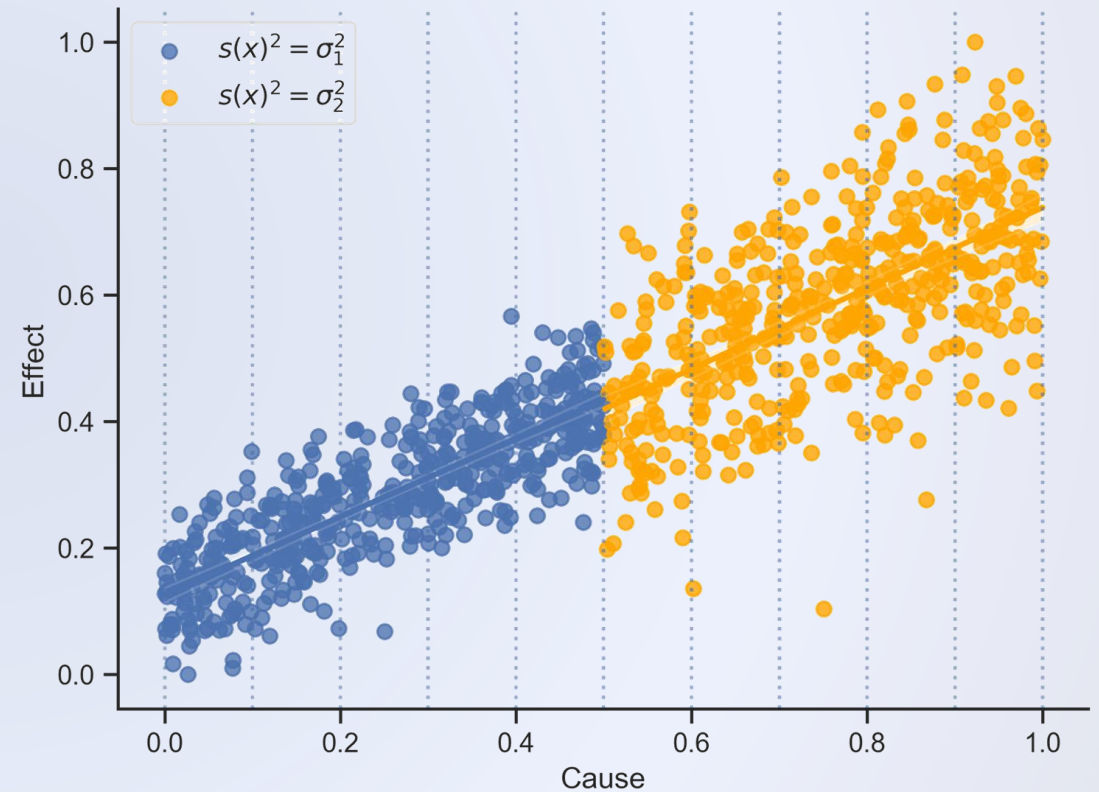
The Algorithm: HECI

Partition domain of the presumed cause into bins with constant noise variance

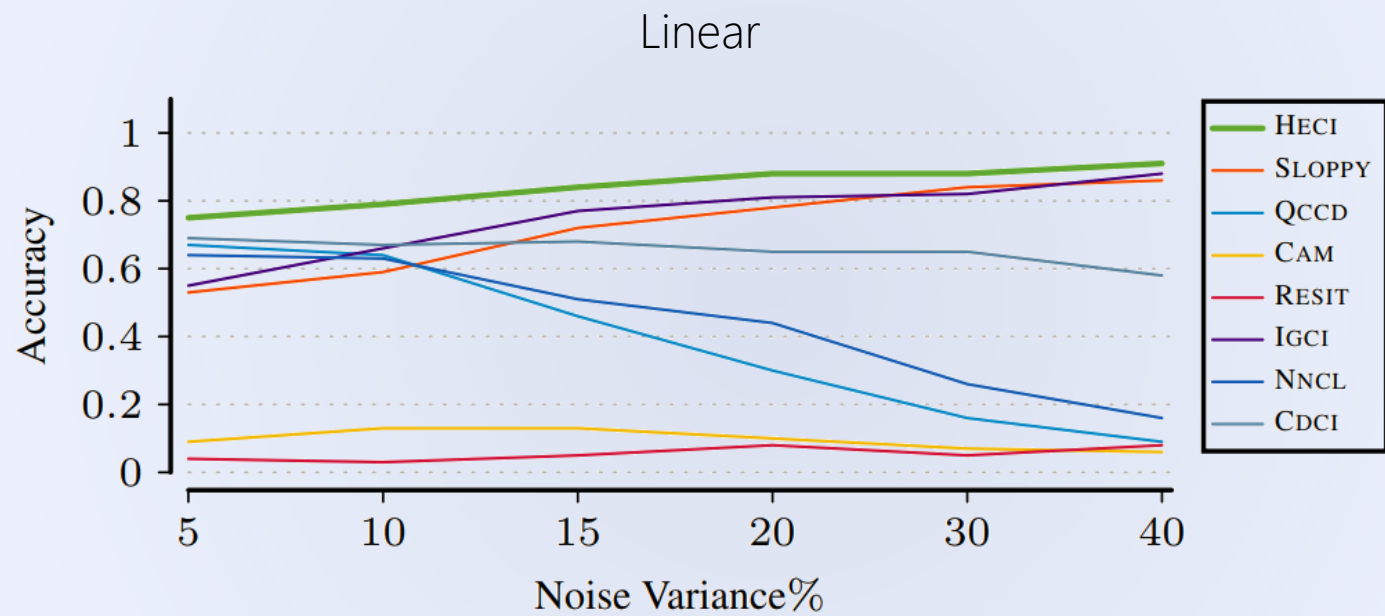
Determine number of bins using the Bayesian Information Criterion (*BIC*)

Determine the optimal binning using dynamic programming

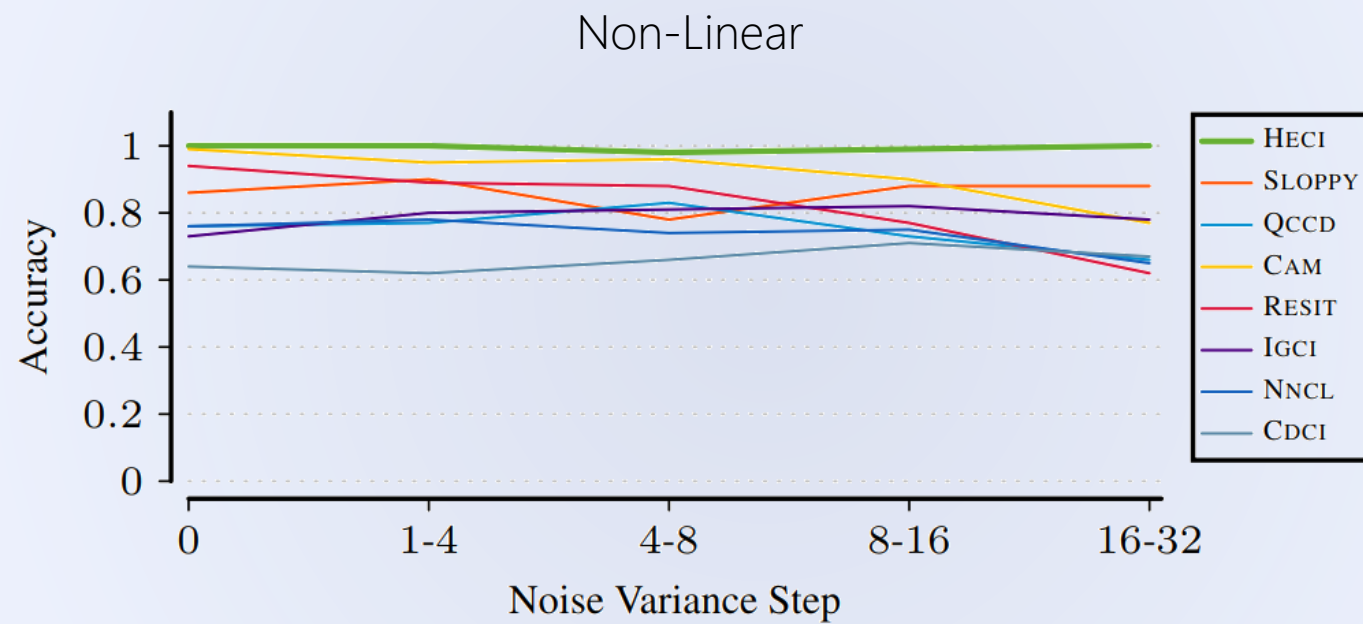
- optimal model is found in $\mathbf{O(b^2n)}$



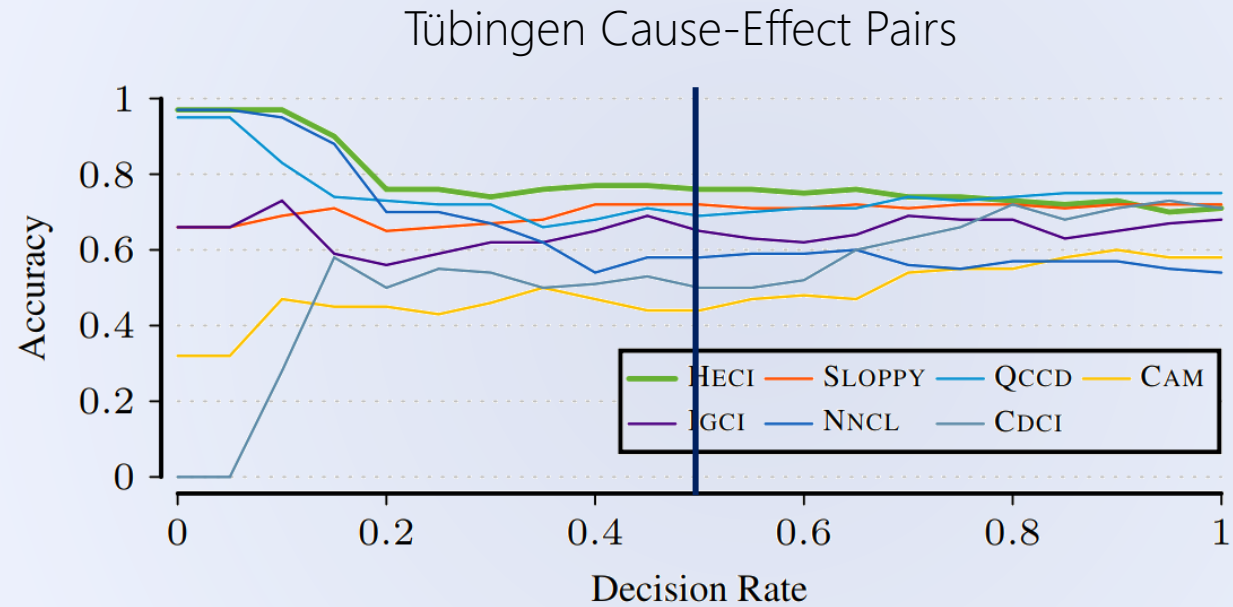
Experiments



Experiments



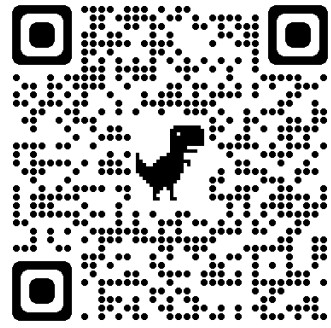
Experiments



Experiments

HECI performs **on par** with SOTA when
noise is stationary and **better** on
heteroscedastic noise

Conclusion



Causal model

$$Y = f(X) + s(X) * N$$

Identifiable

- linear with heteroscedastic noise
- non-linear with homoscedastic noise

Inference

- HECI for discovering optimal models
- **accurately** infers cause and effect in the presence of **heteroscedastic** noise

