ICML
International Conference
On Machine Learning

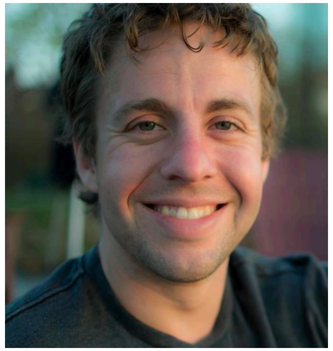
Stanford University

TEL AVIV UNIVERSITY

RECAPP: *Crafting a More Efficient Catalyst for Convex Optimization*

Yair Carmon, Arun Jambulapati, Yujia Jin, Aaron Sidford

(presenting)



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aka. Approximate Proximal Point Acceleration (APPA) [G92, SS12, FGKS15]

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step-size independent of iterates

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Reduce to regularized sub-problems

Usually require high-accuracy solutions

When accurate enough, converge at rate $O(\lambda R^2/T^2)$.

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$O\left(\sqrt{\frac{\lambda}{\epsilon}} \cdot R\right)$ solves of subproblem $x_{t+1} \leftarrow \arg\min_x F(x) + \frac{\lambda}{2}\|x - s_t\|^2$

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Bregman divergence w.r.t previous iter x_t

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separating two x based on their different use in updates

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Relaxed Error Condition

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① $$F^{\lambda}_{s_t}(x) - F^{\lambda}_{s_t}(x_\star) \le \frac{\frac{\lambda}{2}\|x_\star - s_t\|^2 + V^F_{x_\star}(x_t)}{8}$$

② Unbiased-ness: $\mathbb{E}x = x_\star$

$$\mathbb{E}\|x - x_\star\|^2 \le \frac{\frac{\lambda}{2}\|x_\star - s_t\|^2 + V^F_{x_\star}(x_t)}{4\lambda}$$

[BP15] Blanchet, Jose H., and Peter W. Glynn. "Unbiased Monte Carlo for optimization and functions of expectations via multi-level randomization." *2015 Winter Simulation Conference (WSC)*. IEEE, 2015.
[ACJJS21] Asi, Hilal, et al. "Stochastic bias-reduced gradient methods." *Advances in Neural Information Processing Systems* 34 (2021): 10810-10822.

RECAPP: full algorithm

$$s_t \leftarrow (1-\alpha_{t+1})x_t + \alpha_{t+1} v_t$$

$$x_{t+1} \overset{①}{\approx} \arg\min_x F(x) + \frac{\lambda}{2}\|x - s_t\|^2$$

$$\tilde{x}_{t+1} \overset{②}{\approx} \arg\min_x F(x) + \frac{\lambda}{2}\|x - s_t\|^2$$

$$v_{t+1} \leftarrow v_t - \frac{1}{\alpha_{t+1}}(s_t - \tilde{x}_{t+1})$$

Relaxed Error Condition

(original condition)

① $F^\lambda_{s_t}(x) - F^\lambda_{s_t}(x_\star) \le \frac{\frac{\lambda}{2}\|x_\star - s_t\|^2 + V^F_{x_\star}(x_t)}{8}$

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reduction via multi-level Monte-Carlo [BP15, CJJS21]

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Key observation

Relaxed Error Condition

$$F_s^\lambda(x) - F_s^\lambda(x_\star) \leq \frac{\frac{\lambda}{2}\|x_\star - s\|^2 + V_{x_\star}^F(x_t)}{8}$$

can be satisfied **without extraneous log** term when F is **finite-sum** or **min-the-max** structure

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\+ full gradient query at x_t

$n + L/\lambda$ **gradient queries suffice**

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Max-structured minimization

classical mirror-prox [N04] on minimax objective
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+ initialize y at best response* of x_t

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Conclusion and Results

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a *general* acceleration framework with relaxed error condition

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