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Israel Institute
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The Henry and Marilyn Taub
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Coin Flipping Neural Networks

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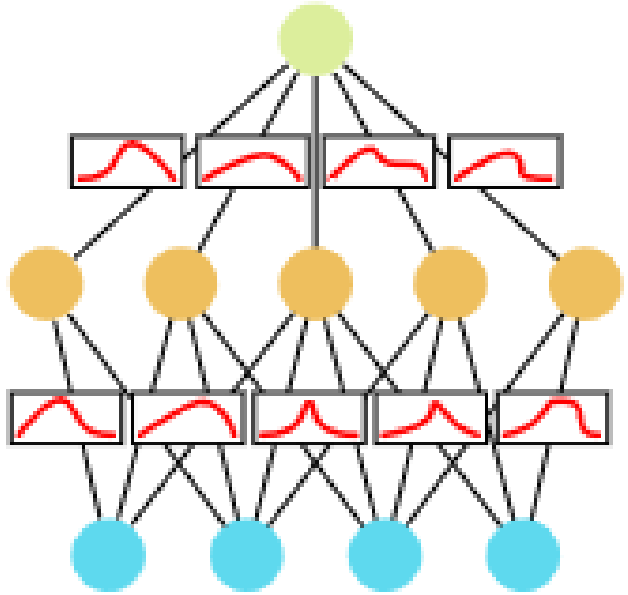
Definition

Coin-Flipping NNs are ***stochastic NNs***,

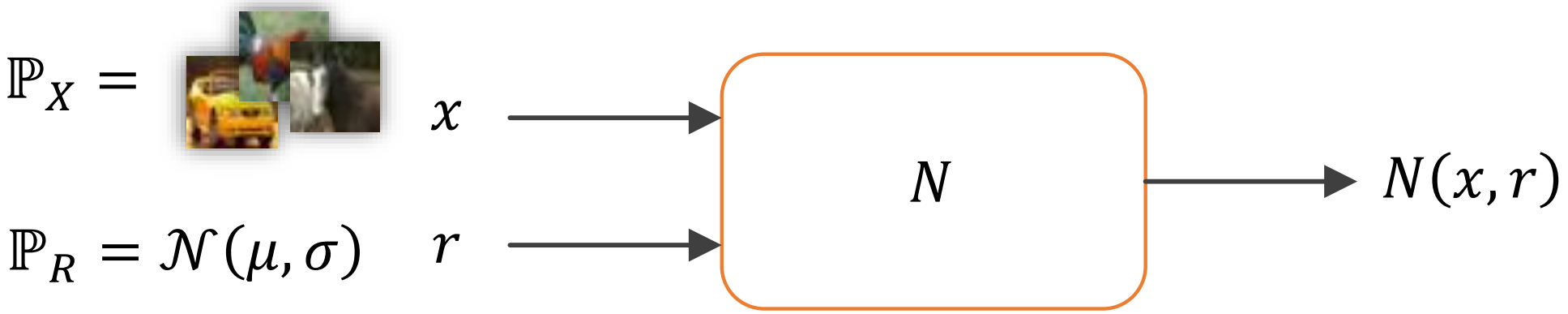
where we are allowed to ***amplify*** their outputs during inference.

Stochastic NNs

- Output is a Random Variable
  - Random weights, activations…
- Bayesian NN, Variational AE, etc.



- In general: $N(x, r), x \sim \mathbb{P}_X, r \sim \mathbb{P}_R$

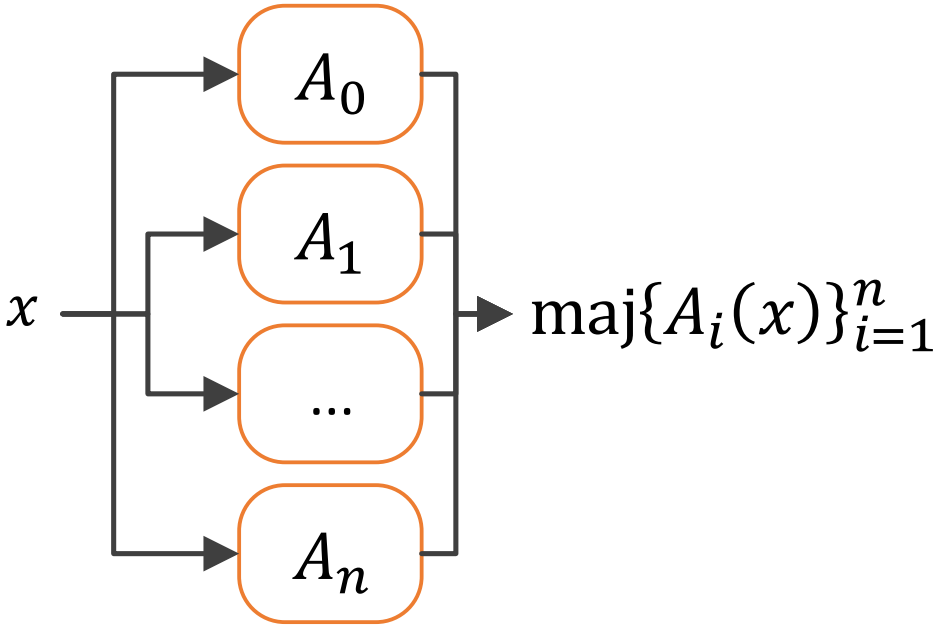


Amplification

- Randomized A solves decision problem f
  - $\Pr[A(x) = f(x)] = \frac{1}{2} + \varepsilon$

- Take multiple samples from $A(x)$
- Calculate majority

- $\Pr[\text{maj}\{A_i(x)\}_{i=1}^{n} = f(x)] \geq 1 - e^{-2\varepsilon^2 n}$



- Improved complexity in practice
  - Convex volume estimation, polynomial equality test…

CFNN

- *Can amplification add power to NNs? –* ***yes!***
- Reduce size, improve performance
- Our focus – classification

- Given $f(x): X \to \{1,2,\dots,C\}$, define:

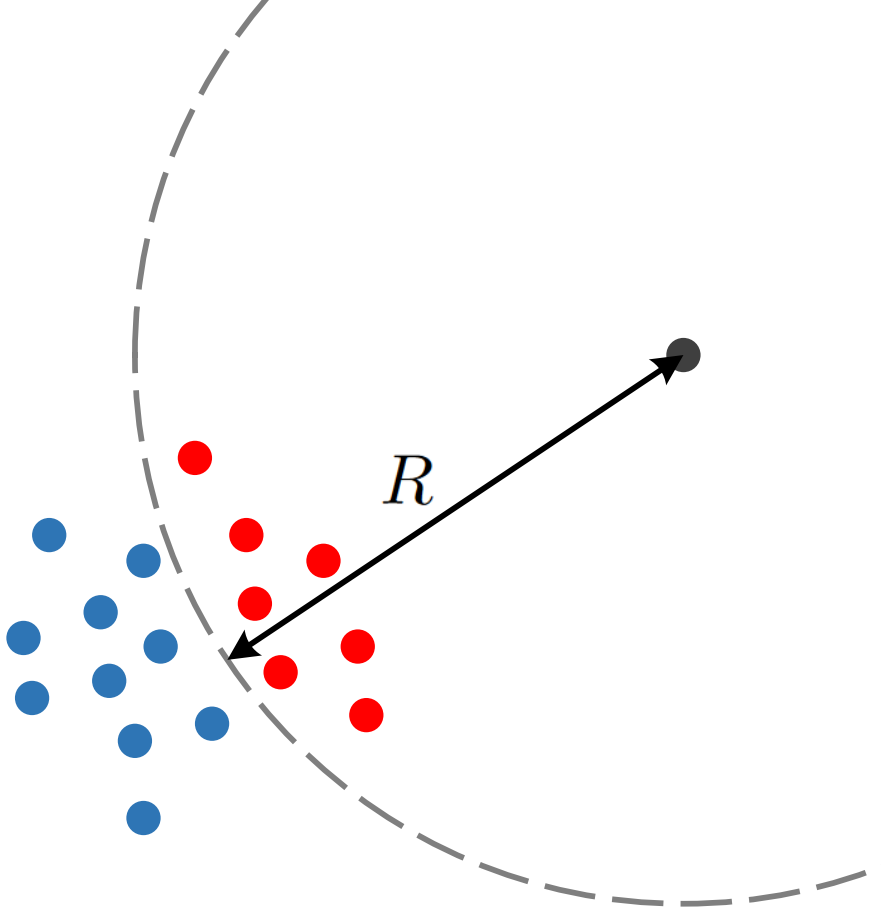
$$\textit{Random Accuracy} = \Pr\left[\lim_{n\to\infty} \text{maj}\{N(x,r_i)\}_{i=1}^{n} = f(x)\right]$$

- Probability that taking the maj of ∞ samples will give the right answer

- Lets see an example…

Ball Problem Example

- Given $x \in \mathbb{R}^d$, determine if $x \in B_R$



- Deterministic, 2-Layer NN[1]:
  - $\Omega(e^d)$ neurons
  - Otherwise, $MSE > \frac{c}{d^4}$

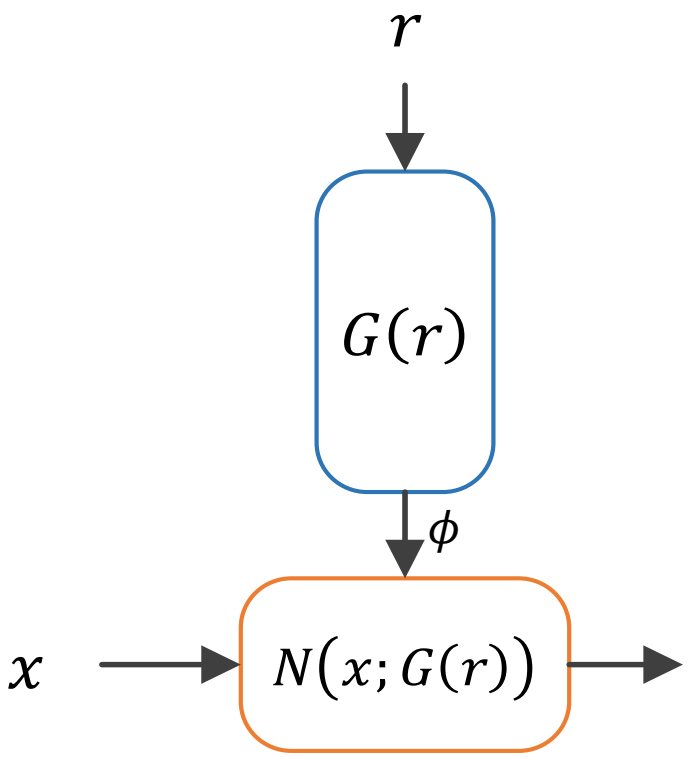
- We show that 2-layer CFNN only require $O(d)$ neurons!
  - Achieve 100% *Random Accuracy*
- Total complexity: $O(poly(d))$, an **exponential improvement!**

[1] Safran & Shamir (2016)

Hypernetwork CFNN

- Generator $\phi = G(r)$ of parameters for $N(x; \phi)$
- If N is sufficiently deep, $G(r) = Ar$ is enough to approx. any function[2]
- "Transposed" theorem:

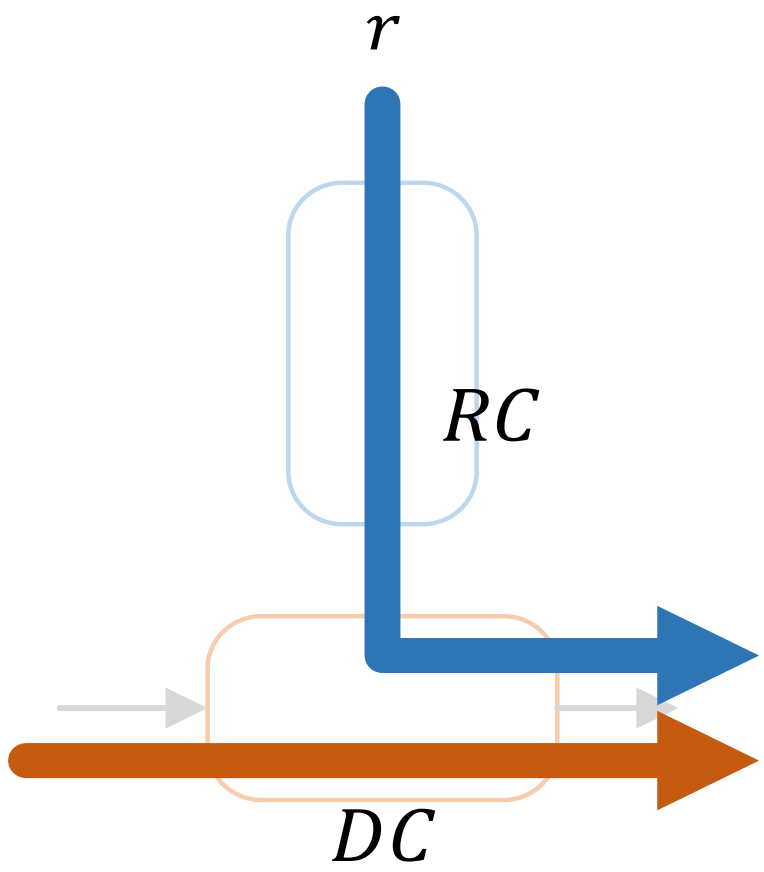
If G is sufficiently deep, $N(x; \phi) = \mathrm{sgn}(\phi x + b)$ is enough to approx. any function $f: \mathbb{R} \to \mathbb{R}$



[2] Dwaracherla et. al. (2020)

DC vs. RC tradeoff

- We suggest a new tradeoff:
- Data Complexity (DC)
  # neurons from x to output
- Random Complexity (RC)
  # neurons from r to output

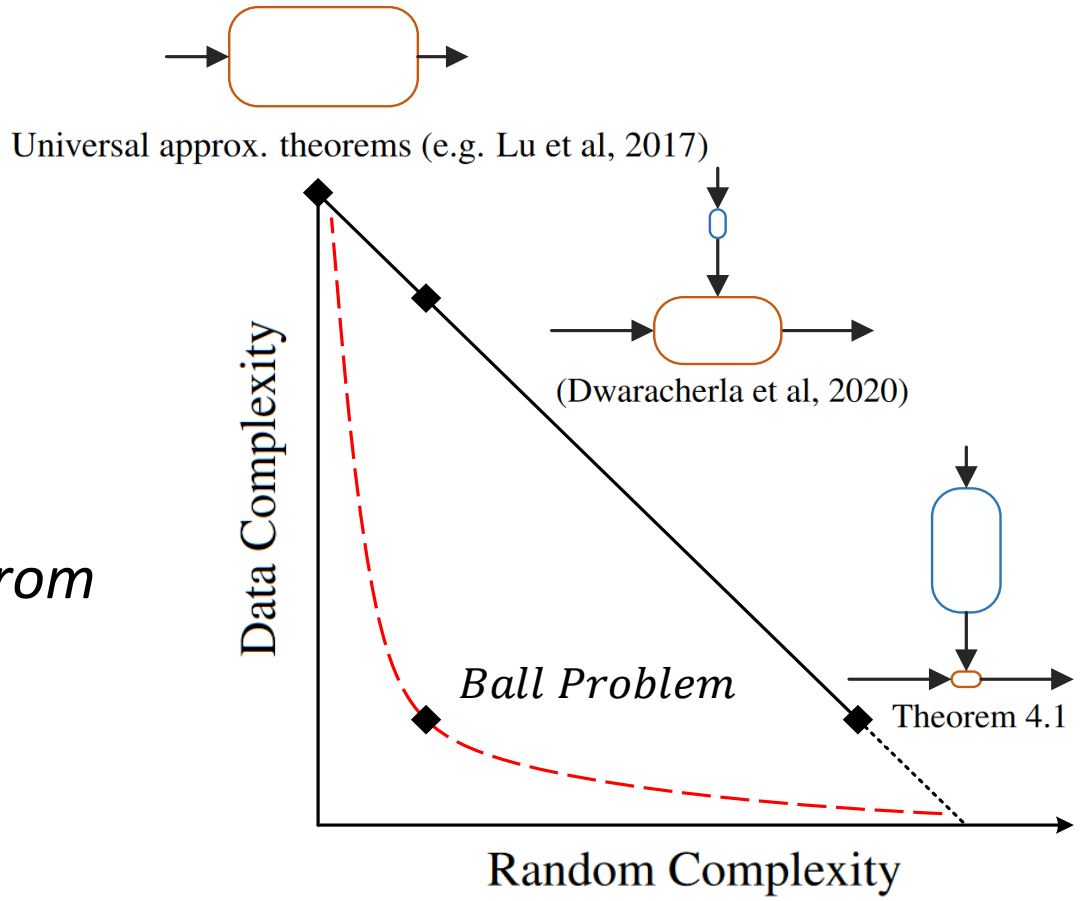


DC vs. RC tradeoff

- *Exact* complexity tradeoff between DC and RC
- Some problems show significant improvement
  - E.g. our Ball Problem
- Conjecture:

Most classification problems will benefit from CFNN

- e.g. functions easy to sample but hard to compute



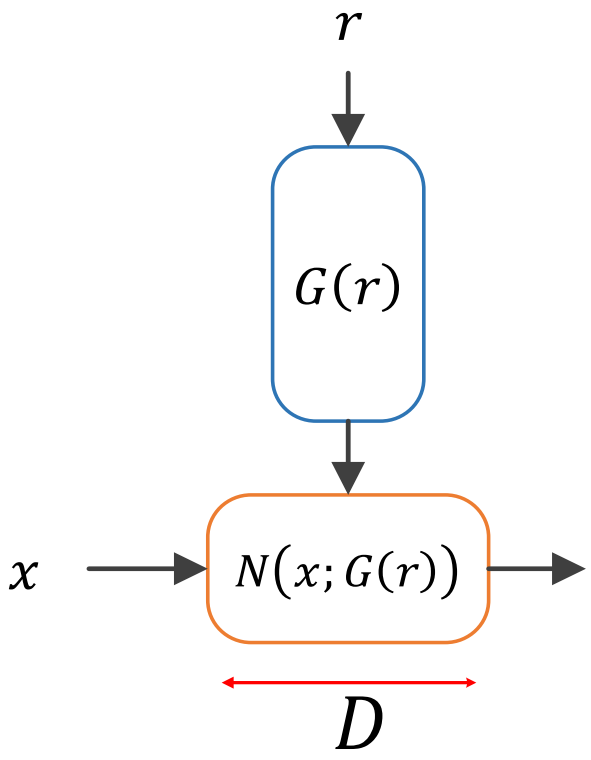
Empirical Study

- Two architectures
- Hypernetwork CFNN
- Resnet20 + Dropout as CFNN

ARCH.	DET.	CFNN	Δ
	CIFAR10		
HYPERNET	79.71 ± 0.5	81.45 ± 0.2	$+1.74 \pm 0.6$
RESNET20	87.82 ± 0.6	90.49 ± 0.1	$\mathbf{+2.67 \pm 0.7}$
	CIFAR100		
HYPERNET	58.09 ± 0.2	61.24 ± 0.2	$+3.15 \pm 0.3$
RESNET20	50.90 ± 0.1	60.16 ± 0.6	$\mathbf{+9.25 \pm 0.7}$

Testing the Tradeoff

D	CFNN	CIFAR10 Det.	Δ
2	44.05	42.14	+1.91
6	**81.45 ± 0.24**	**79.71 ± 0.52**	**+1.74± 0.57**
10	89.37	88.41	+0.96
14	91.81	91.42	+0.39
20	92.37	92.89	-0.52
		CIFAR100	
6	45.29	44.23	+1.06
10	**61.24± 0.41**	**58.09 ±0.22**	**+3.15 ±0.29**
14	66.48	65.16	+1.32
20	69.26	67.68	+1.58
56	72.00	72.23	-0.23



Comparing to MCDropout

METHOD	CIFAR10	CIFAR100
STANDARD TRAIN, STANDARD TEST	87.82 ± 0.65	50.90 ± 0.03
STANDARD TRAIN, MEAN IN TEST (MCDROPOUT)	88.61 ± 0.10	53.66 ± 0.24
STANDARD TRAIN, MAJ IN TEST	88.37 ± 0.18	53.31 ± 0.16
MAJ TRAIN, STANDARD TEST	89.70 ± 0.39	47.92 ± 1.78
MAJ TRAIN, MEAN IN TEST	$\mathbf{90.49 \pm 0.12}$	56.49 ± 0.57
MAJ TRAIN, MAJ TEST (CFNN)	90.20 ± 0.14	$\mathbf{60.16 \pm 0.67}$

Summary

- CFNNs: *Stochastic NNs + Amplification*
- Ball problem – exponential complexity improvement!
- New DC vs. RC tradeoff for hypernetworks
- Empirically verified

Amplification in Inference helps!

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Thank you!

Hope you enjoyed the talk