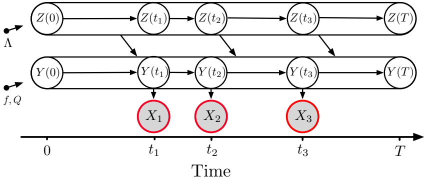
Markov Chain Monte Carlo for Continuous-Time Switching Dynamical Systems

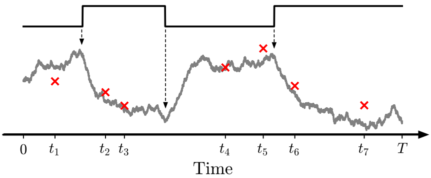
TECHNISCHE UNIVERSITÄT DARMSTADT

Lukas Köhs, Bastian Alt, Heinz Koeppl

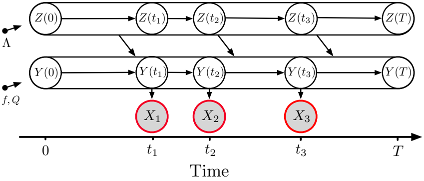
Department of Electrical Engineering and Information Technology
Technische Universität Darmstadt, Germany

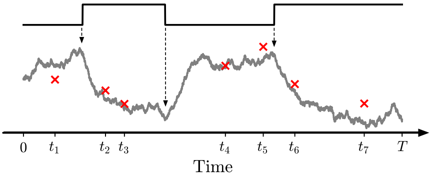
Model & Motivation





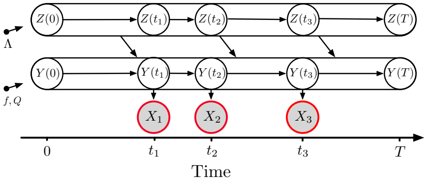
Model & Motivation

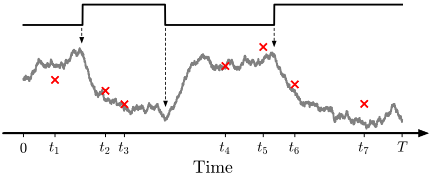




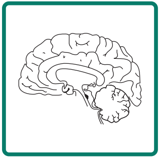
Goal: sample from the posterior $Y_{[0,T]}, Z_{[0,T]} \mid x_{[1,N]}$.

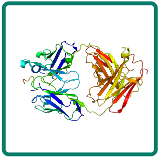
Model & Motivation





Goal: sample from the posterior $Y_{[0,T]}, Z_{[0,T]} \mid x_{[1,N]}$.





Gibbs Sampling

$$Y_{[0,T]} \sim \mathsf{P}(Y_{[0,T]} \in \cdot \mid z_{[0,T]}, x_{[1,N]}, \theta), \tag{1}$$

$$Z_{[0,T]} \sim \mathsf{P}(Z_{[0,T]} \in \cdot \mid y_{[0,T]}, x_{[1,N]}, \theta), \tag{2}$$

$$\Theta \sim \mathsf{P}(\Theta \in \cdot \mid z_{[0,T]}, y_{[0,T]}, x_{[1,N]}). \tag{3}$$

Gibbs Sampling

$$Y_{[0,T]} \sim \mathsf{P}(Y_{[0,T]} \in \cdot \mid z_{[0,T]}, x_{[1,N]}, \theta), \tag{1}$$

$$Z_{[0,T]} \sim \mathsf{P}(Z_{[0,T]} \in \cdot \mid y_{[0,T]}, x_{[1,N]}, \theta), \tag{2}$$

$$\Theta \sim \mathsf{P}(\Theta \in \cdot \mid z_{[0,T]}, y_{[0,T]}, x_{[1,N]}). \tag{3}$$

(1) Conditional Diffusion

1. Backward filtering:

$$\beta(y,t) \coloneqq p(x_{[k,N]} \mid y, t),\ t_k \geq t, \quad \partial_y \log \beta(y,t) = -I(t)y + a(t),$$

$$\frac{\mathrm{d}}{\mathrm{d}t} I = \dots, \quad \frac{\mathrm{d}}{\mathrm{d}t} a = \dots$$

Gibbs Sampling

TECHNISCHE UNIVERSITÄT DARMSTADT

$$Y_{[0,T]} \sim \mathsf{P}(Y_{[0,T]} \in \cdot \mid z_{[0,T]}, x_{[1,N]}, \theta), \tag{1}$$

$$Z_{[0,T]} \sim \mathsf{P}(Z_{[0,T]} \in \cdot \mid y_{[0,T]}, x_{[1,N]}, \theta), \tag{2}$$

$$\Theta \sim \mathsf{P}(\Theta \in \cdot \mid z_{[0,T]}, y_{[0,T]}, x_{[1,N]}). \tag{3}$$

(1) Conditional Diffusion

1. Backward filtering
2. Forward sampling:

$$\mathrm{d}Y(t) = \{f(Y(t), t) + D(t)\partial_y \log \beta(Y(t), t)\} \, \mathrm{d}t + Q(t) \, \mathrm{d}W(t),$$

Gibbs Sampling

TECHNISCHE UNIVERSITÄT DARMSTADT

$$Y_{[0,T]} \sim \mathsf{P}(Y_{[0,T]} \in \cdot \mid z_{[0,T]}, x_{[1,N]}, \theta), \tag{1}$$

$$Z_{[0,T]} \sim \mathsf{P}(Z_{[0,T]} \in \cdot \mid y_{[0,T]}, x_{[1,N]}, \theta), \tag{2}$$

$$\Theta \sim \mathsf{P}(\Theta \in \cdot \mid z_{[0,T]}, y_{[0,T]}, x_{[1,N]}). \tag{3}$$

(2) Conditional Switching

1. Forward filtering

$$p_f(z,t) := p(z,t \mid y_{[0,t]})$$

$$\mathrm{d}p_f(z,t) = \dots$$

Gibbs Sampling

TECHNISCHE UNIVERSITÄT DARMSTADT

$$Y_{[0,T]} \sim \mathsf{P}(Y_{[0,T]} \in \cdot \mid z_{[0,T]}, x_{[1,N]}, \theta), \tag{1}$$

$$Z_{[0,T]} \sim \mathsf{P}(Z_{[0,T]} \in \cdot \mid y_{[0,T]}, x_{[1,N]}, \theta), \tag{2}$$

$$\Theta \sim \mathsf{P}(\Theta \in \cdot \mid z_{[0,T]}, y_{[0,T]}, x_{[1,N]}). \tag{3}$$

(2) Conditional Switching

1. Forward filtering
2. Backward sampling

$$\frac{\mathrm{d}}{\mathrm{d}t} p_s(z,t) = -\sum_{z' \in \mathcal{Z}} \tilde{\Lambda}(z', z, t) p_s(z', t), \quad \tilde{\Lambda}(z', z, t) = \frac{p_f(z', t)}{p_f(z, t)} \Lambda(z, z'),$$

with condition $p_s(z,T) = p_f(z,T)$.

Gibbs Sampling

$$Y_{[0,T]} \sim \mathsf{P}(Y_{[0,T]} \in \cdot \mid z_{[0,T]}, x_{[1,N]}, \theta), \quad (1)$$

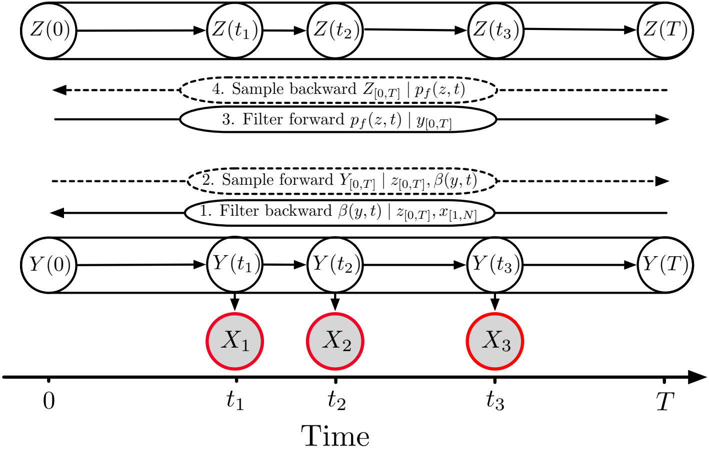
$$Z_{[0,T]} \sim \mathsf{P}(Z_{[0,T]} \in \cdot \mid y_{[0,T]}, x_{[1,N]}, \theta), \quad (2)$$

$$\Theta \sim \mathsf{P}(\Theta \in \cdot \mid z_{[0,T]}, y_{[0,T]}, x_{[1,N]}). \quad (3)$$

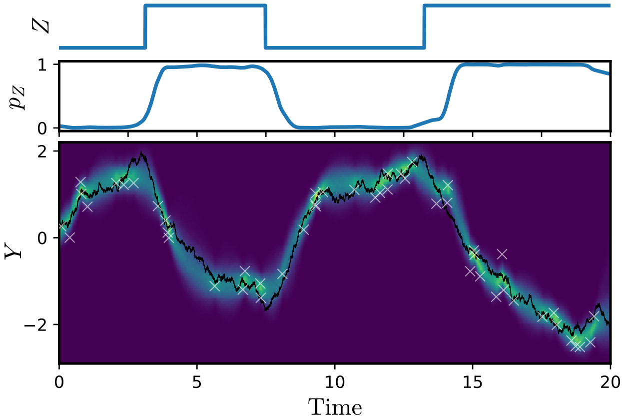
(3) Parameters

Conjugate priors $\rightarrow$ hyperparameter updates

Gibbs Sampling Sketch

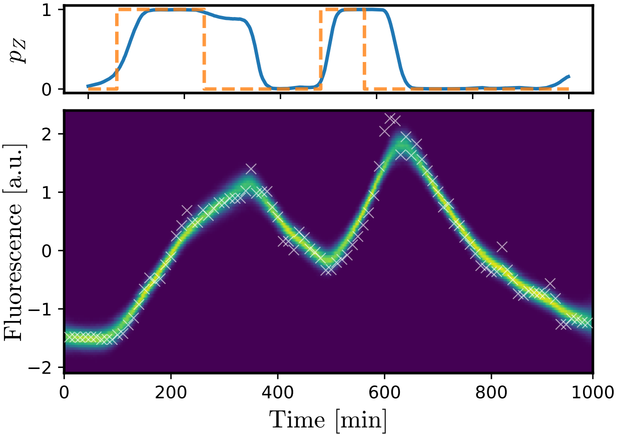


Experiments
Model Validation on Ground-Truth Data



Experiments

Inferring Gene-Switching Dynamics



Summary

TECHNISCHE UNIVERSITÄT DARMSTADT

- We present the first sampling-based path-space inference scheme for switching MJP-SDEs
- Sampling yields accurate results for posterior paths and parameters
- Application to real-world problem shows practical utility

Thank you for your attention.