Private optimization in the interpolation regime: faster rates and hardness results


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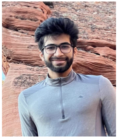
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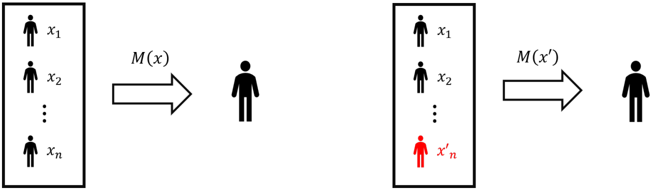
Differential Privacy Definition



- A mechanism M is (ε, δ)-*differentially private* if for every set S and for every pair of datasets x, x' differing by one entry

$$\mathbb{P}\left(M(x) \in S\right) \leq e^{\varepsilon} \mathbb{P}\left(M(x') \in S\right) + \delta.$$

Differentially Private Stochastic Convex Optimization

$$\begin{aligned} &\text{minimize} \;\; f(x) = \mathbb{E}_P[F(x;S)] \\ &\text{subject to} \;\; x \in \mathcal{X} \end{aligned}$$

- $F(\cdot; s)$ is a convex loss function, L-Lipschitz and H-smooth
- $\mathcal{X} \subset \mathbb{R}^d$ is the parameter space (diameter D)

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Privacy comes at a price in SCO!

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Definition (Interpolation Problem)

An interpolation problem is one where there exists $x^\star$ such that $\nabla F(x^\star; s_i) = 0$ for all $i \in [n]$.

Problem statement summary

Given $S_1^n \stackrel{\text{iid}}{\sim} P$, develop private algorithms that solve

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Similar improvments in private optimization?

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4. Optimality and the price of adaptivity

Please visit our poster #1011 tonight to learn more!

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Thank you!